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## CORRIGENDUM.

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In Vol. XXXVI of the *Proc. R.S.E.*, on p. 238, delete lines 17 and 18 from the foot, and *read*—

Thus we have

$$y_{r+1} = \left( -t \frac{d^2 P}{dt^2} \right)_{r+1};$$

and line 11 from the foot—

$$\text{for } \frac{1}{2} t^2 \frac{d^2 P}{dt^2} \quad \text{read} \quad t \frac{d^2 P}{dt^2}.$$

$$\begin{array}{r} 31 \\ 63 \\ \hline 116 \end{array}$$

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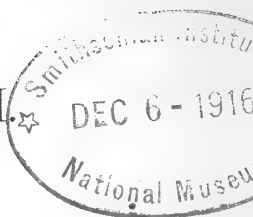


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OF THE

## ROYAL SOCIETY OF EDINBURGH.

SESSION 1915-16.



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## REGULATIONS REGARDING THE PUBLICATION OF PAPERS IN THE PROCEEDINGS AND TRANSACTIONS OF THE SOCIETY.

THE Council beg to direct the attention of authors of communications to the Society to the following Regulations, which have been drawn up in order to accelerate the publication of the Proceedings and Transactions, and to utilise as widely and as fairly as possible the funds which the Society devotes to the publication of Scientific and Literary Researches.

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2. METHOD OF PUBLICATION.—As soon as the final revise of a Transactions paper has been returned, or as soon as the sheet in which the last part of a Proceedings paper appears is ready for press, a certain number of separate copies or reprints, in covers bearing the title of the paper and the name of the author, are printed off and placed on sale. The date of such separate publication will be printed on each paper.

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4. BRIEF ABSTRACTS OF TRANSACTIONS PAPERS will be published in the Proceedings, provided they are sent along with the original paper.

5. SPECIAL DISCUSSION OF PAPERS ACCEPTED FOR PUBLICATION.—Where a paper has been accepted for publication, the Council may, with the consent of the author, select this paper for Special Discussion. In the case of such papers advanced proofs will be sent to the members of the Society desiring copies, and copies will be supplied to the author for distribution. A paper selected for Special Discussion will be marked with an asterisk (\*) and placed first on the Billet for the day of reading. Any following papers for that day may be adjourned or held as read if the discussion prevents their being read.

6. COMMUNICATIONS NOT SUBMITTED FOR PUBLICATION, such as Demonstrations of Experiments, Statement of Scientific Problems, etc., may be received by the Council, and may also be selected for Special Discussion. The Council does not undertake to publish any notice of such communications in the Proceedings or Transactions of the Society.

*[Continued on page iii of Cover.]*







*Photo by*

*Horsburgh.*

JAMES GEIKIE, LL.D., D.C.L., F.R.S., F.G.S.,  
Emeritus Professor of Geology in the University of Edinburgh.  
President of the Royal Society of Edinburgh.



PROCEEDINGS  
OF THE  
ROYAL SOCIETY OF EDINBURGH.

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VOL. XXXVI.

1915-16.

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I.—The Influence of James Geikie's Researches on the Development of Glacial Geology. Opening Address by John Horne, LL.D., F.R.S., President.

(November 1, 1915.)

My first duty is to express my cordial thanks to the Council and Fellows of this Society for electing me as President in succession to the late Professor Geikie. I have accepted the honour with great reluctance, for I am conscious of the fact that others have far stronger claims, and have waived their claims under very exceptional circumstances. But I may take this opportunity of assuring the Fellows that since they have been pleased to confirm the recommendation of the Council, I shall take an active interest in the affairs of the Society and do my best to promote its welfare.

The list of papers read during the past session shows that important contributions relating to different branches of science have been made to the Society. Notwithstanding the strain caused by the war, the output of original work is noteworthy. In this connection reference may be made to some researches having a direct bearing on the war. Last session the Council appointed a Committee, composed of leading scientists, to conduct investigations in connection with submarines, aeroplanes, asphyxiating gas, and high explosives. The experimental work has been carried out with the financial aid of an anonymous donor, whose generosity and patriotic spirit have been highly appreciated by the Council. These researches are still in progress, and are likely to result in suggestions that may be of practical value to the State.

Important changes affecting the mode of election of Fellows are worthy

of special mention. The Council carefully considered certain proposals which would bring our procedure as to the election of Fellows more into line with that of the Royal Society of London. These were: (1) that instead of each candidate applying for admission, his candidature should be promoted by several ordinary Fellows; (2) that the certificates should be handed to the Secretary before the end of each year, and that an annual election be held early in the following year; (3) that the Fellows admitted yearly should be restricted to a definite number. The Council approved of the first and second of these proposals, and as these have been adopted by the Fellows, they will come into operation during the present session.

I have chosen as the subject of my address "The Influence of James Geikie's Researches on the Development of Glacial Geology," because these researches are the most striking feature of his scientific career. It was the sphere in which he laboured with conspicuous success. His work in this department was characterised by marked originality and imaginative power. They stimulated inquiry, and at the same time they aroused keen opposition. But, though involved in controversy throughout his life, he never failed to win the respect and esteem of his opponents as well as his followers. My aim, therefore, is to indicate some of the essential points of his teaching which marked him out as one of the foremost leaders of a distinct school in glacial geology.

In order to define clearly his attitude to this branch of inquiry in his early life, it is necessary to refer to the position of glacial research before he began to investigate these deposits. The views of Sir Charles Lyell regarding glacial phenomena, supported by Charles Darwin, de la Beche, and Murchison, had been widely accepted. Lyell advocated the theory that the transported blocks, the striated rock surfaces, the stony clays, sands, and gravels had been the work of floating icebergs, which seemed to be supported by the occurrence of marine shells in these deposits at a few localities at high levels.

The clue to the correct interpretation of these phenomena was furnished by Agassiz. Having carefully studied the effects of glacier action in the Alps, which had also been investigated by de Saussure, Venetz, de Charpentier, and others, he wished to examine regions in temperate latitudes where glaciers no longer exist. In 1840, he visited a considerable part of Scotland, the north of England, and a large part of Ireland. From the characters of the superficial deposits and erratic blocks, and from the polished and striated surfaces of the rocks *in situ*, he inferred that masses of land ice, and subsequently glaciers, formerly existed in those parts of the United Kingdom. He also threw out the suggestion that the Parallel

Roads of Glen Roy had been formed by glacier lakes, the view which is generally accepted at the present day.

The land-ice theory thus propounded by Agassiz was adopted and confirmed by Buckland, Robert Chambers, T. F. Jamieson of Ellon, Sir Andrew Ramsay, and Sir A. Geikie in this country, and by Otto Torell in Sweden. But the iceberg hypothesis died slowly. It had gained a firm hold in the geological world. It deterred many stratigraphical geologists from spending time on what seemed to be an unprofitable task. Nearly a quarter of a century elapsed before it was adequately recognised that Agassiz had placed glacial research on a sound and permanent basis.

Such was the stage of inquiry when James Geikie began to map these deposits in Scotland in 1861, as a member of the staff of the Geological Survey. As the work proceeded he evolved certain ideas regarding changes of climate in Pleistocene time, based on the succession of boulder clays with intercalations of sand, gravel, and peat, and on the cave deposits and Palæolithic gravels of the south of England. He first gave expression to his views during a discussion at the British Association meeting in Edinburgh in 1871. This discussion followed the reading of a paper by Professor Boyd Dawkins on "The Relation of the Quaternary Mammalia to the Glacial Period." This communication was of prime importance, because it furnished a classification of the Quaternary Mammalia and attempted to explain the commingling of Arctic and Southern forms in the deposits in the south of England. He arranged the mammals in five groups, calling special attention to those indicating a cold climate, and those which are now only found in hot regions, such as the hyæna and hippopotamus. Boyd Dawkins maintained that the only mode of explaining this anomalous assemblage is to suppose that in those days the winter cold was very severe and the summer heat intense. Hence, in the summer the animals now found in warmer regions migrated northwards, and in the winter those now found in the Arctic regions moved southwards.

The theory of seasonal migrations of Arctic and Southern mammals advocated in this paper was opposed by James Geikie, who maintained that the phenomena pointed to fluctuations of climate. The discussion impelled him to give an outline of his views in a series of articles in the *Geological Magazine*. Fully convinced of the truth of his opinions, he lost no time in expanding those articles and developing them in detail in his volume *The Great Ice Age*, which appeared in 1874.

What, we may ask, were the distinctive features of this epoch-making volume, which immediately arrested the attention of geologists all over the globe? For the first time it gave a systematic account of the phenomena

of the Glacial Epoch with special reference to its changes of climate. Selecting Scotland as the region which he had closely studied, he described in detail the evidence furnished by the rock striations; the successive boulder clays; the deposits of silt, clay, sand, and gravel, with land plants and mammalian remains, and occasionally with marine shells; the rock-bound basins; the ancient glacial lake of the Clyde with its overflow channels; the raised beaches and the peat mosses.

From these various lines of evidence he inferred that, during the Ice Age, Scotland witnessed several revolutions of climate. The boulder clays, from their inherent character, pointed to the operation of land ice. The interglacial beds indicated the recurrence of milder conditions, not once only, but several times, when the ice retired, if it did not wholly disappear. In this connection he laid special emphasis on the section at Woodhill Quarry, Kilmaurs, in Ayrshire, where a thin peaty layer intercalated in boulder clay yielded the remains of mammoth and reindeer, the peat being overlain by a band containing Arctic marine shells. As regards the disputed question of the origin of rock-bound basins, he was a strenuous supporter of Ramsay's theory that they had been eroded by glacial action. He further maintained that in post-glacial time Britain was joined to the continent by the uprise of the floor of the North Sea. The climate then was continental. Ancient forests flourished whose remains are now found in our peat mosses. Such evidence was regarded as proving oscillations of climate after the ice-sheets had passed away.

This detailed account of the Ice Age in Scotland was of the highest value, because it revealed the mode of investigating and the principles of interpreting glacial phenomena in any highly glaciated region where glaciers no longer exist. Moreover, through the whole historical account runs the principle which James Geikie believed to be fundamental, that the Glacial Epoch was not one continuous age of ice, but consisted rather of a series of alternate cold and genial periods.

These alternations of climate during the Ice Age seemed to harmonise with Croll's theory of the cause of the great lowering of temperature in late Tertiary and post-Tertiary time. Croll's hypothesis was founded on variations in the eccentricity of the earth's orbit, combined with the precession of the equinoxes, together with certain physical agencies, such as the deflection of ocean currents, which arise indirectly from these cosmical causes. It was adopted by James Geikie and expounded by him in the successive editions of his *Great Ice Age*. It was supported by Sir Robert Ball, and, in a modified form, by Alfred Russel Wallace. For a time it was widely accepted in Europe, and to a limited extent in America. But

the destructive criticism of Professor Simon Newcomb and E. P. Culverwell has considerably shaken the confidence of geologists in its efficacy.

Having furnished a detailed account of the glacial and post-glacial deposits in Scotland, James Geikie proceeded to describe the relics of post-Tertiary time in England and Ireland, Scandinavia, Switzerland, and North America, where he found similar evidence of interglacial mild conditions amongst the oldest glacial deposits. He laid special emphasis on the lignite at Dürnten and Wetzikon in the Canton of Zurich, which rests on ground moraine and is overlain by sand and gravel with Alpine erratics. From the plants found in the lignite, Professor Heer concluded that the climate during its formation was similar to what it is now. This interglacial bed also yielded the bones of the Asiatic elephant, a species of rhinoceros, the urus or great ox, the stag, and the cave bear. From a consideration of the whole evidence that had been accumulated up to 1874 in those countries where glacial phenomena had been studied, James Geikie inferred that there is clear proof of a mild interglacial period in the later stage of the Glacial Epoch.

James Geikie had now reached a critical stage in his investigations as presented in *The Great Ice Age*, for he had to face the question of the antiquity of man, which was intimately associated with the age of the Palæolithic gravels and cave deposits in the south of England. He was convinced that it was necessary to establish the sequence of events in the Glacial Epoch before the age of the Palæolithic gravels could be defined. He called attention to the great contrast between the fauna of the post-glacial and recent beds in Scotland, the north of England, Wales, and Ireland on the one hand, and that of the Palæolithic gravels and cave deposits on the other. Sir Charles Lyell and others held the opinion that all the Palæolithic gravels were post-glacial. Godwin-Austin suggested that they might be the equivalents of glacial deposits elsewhere. Boyd Dawkins maintained, as we have seen, that man and the extinct mammals lived in the south of England, when the greater part of Britain was covered with ice and snow, when the summers were warm and the winters severe. The peculiar assemblage of northern and southern forms pointed to seasonal migrations. James Geikie advocated the view, which was supported by Sir John Lubbock and Dr Croll, that the phenomena pointed to changes of climate.

In dealing with this question he indicated the three groups of mammals found in the cave deposits and river gravels associated with implements of the old stone men. The southern group comprised the lion, the tiger, the spotted hyæna, two extinct species of elephant, the hippopotamus, and

other forms. The northern group included the reindeer, the musk sheep, the Alpine hare, the lemming, the extinct mammoth, and woolly rhinoceros. The temperate group contained the Irish elk, the bison, the grizzly bear, the cave bear, and other forms.

His arguments against the migration theory are best given in his own words:—

“The migrations of land animals in Northern Asia and equivalent latitudes in North America take place between Arctic and temperate regions. This is simply the case of adjacent provinces overlapping one another. Inasmuch, therefore, as the migration theory asks us to believe that widely separated zones overlapped across the whole breadth of the temperate provinces, it is unreasonable, and not supported by our knowledge of what actually occurs in nature.

“The general character of the southern group of mammalia, as exhibited in cave deposits and river gravels, is non-migratory.

“The union of Britain and Ireland to the continent, across the upraised beds of the English Channel and the Irish Sea, and a great increase of land within the area of the Mediterranean, could not confer upon Europe a climate in any degree approaching to that of Siberia or British America. The climate of our continent would still be insular, and consequently great migrations could not take place.

“During the last continental condition of our islands, snow-fields and glaciers existed in our mountain regions, betokening a climate quite unsuited to the needs of the southern mammalia. The winters at that period must have been excessive, and the summers cold and ungenial.

“Lastly, so long as the Atlantic continues to wash the coast of Europe, and so long as the present configuration of the land endures, our continent must continue to enjoy an insular climate, and there is not the slightest physical evidence to show that it possessed any other kind of climate during the period that the southern mammalia inhabited Britain.”

With reference to the age of the Palæolithic deposits, James Geikie concluded that they are preglacial or interglacial and not post-glacial. In the midland and northern counties of England, in Wales, Scotland, and Ireland, which had been subjected to the grinding action of land ice, these deposits are either absent or sparingly represented. But in those regions, such as the Thames valley, which had not been overridden by the ice, the valley gravels furnish a continuous record from preglacial times to the present day. Between Palæolithic and Neolithic times there was a great gap which coincided with a period of submergence. Thereafter, the new stone men entered Britain accompanied by a distinct mammalian



fauna. The Pleistocene mammals disappeared with Palæolithic man, and, in their place, we find dogs, horses, pigs, several breeds of oxen, the Irish elk, the red deer, and other forms. The implements of the new stone men are found all over the British Islands, showing that their distribution was very different from that of their predecessors.

The second edition of *The Great Ice Age*, which appeared in 1877, the volume on *Prehistoric Europe* (1881), the third edition of *The Great Ice Age* (1894), and the Munro Lectures on *The Antiquity of Man in Europe* (1914) mark successive stages in the evolution of James Geikie's views. His great aim was to keep himself abreast of the increasing volume of research in glacial geology, the results of which were communicated to him by investigators all over the globe. From time to time he modified his opinions regarding the interpretation of particular details in the history of the period. But the fundamental points in his teaching, viz. that the Ice Age was characterised by a succession of cold and genial periods, and that man then lived in Europe, were never abandoned by him.

His classification of the glacial succession in its final form was presented in the Munro Lectures (*The Antiquity of Man in Europe*, 1914). It is given in the subjoined table:—\*

Upper Turbarian	6th Glacial Epoch.
<i>Upper Forestian</i>	<i>5th Interglacial Epoch.</i>
Lower Turbarian	5th Glacial Epoch.
<i>Lower Forestian</i>	<i>4th Interglacial Epoch.</i>
Mecklenburgian	4th Glacial Epoch.
<i>Dürntenian</i>	<i>3rd Interglacial Epoch.</i>
Polonian	3rd Glacial Epoch.
<i>Tyrolian</i>	<i>2nd Interglacial Epoch.</i>
Saxonian	2nd Glacial Epoch.
<i>Norfolkian</i>	<i>1st Interglacial Epoch.</i>
Scanian	1st Glacial Epoch.

A brief outline will now be given of the evidence which led James Geikie to establish this classification.

He maintained that the Scanian or First Glacial Epoch (the Günzian stage of the Alps) is represented in Britain by the Chillesford Clay and

\* In the Munro Lectures (1914) James Geikie made some changes in the nomenclature of his glacial and interglacial periods. The Second Interglacial Period, previously termed by him the Helvetian, was renamed the Tyrolian; and the Third Interglacial Phase, formerly designated the Neudeckian, became the Dürntenian. He also suggested the substitution of the term Polonian for Polandian for the Third Glacial Epoch.

Weybourne Crag, which, from the presence of Arctic species among the mollusca, point to the advent of cold conditions. No morainic deposits in the British Islands can be definitely referred to this stage, but Scandinavia nourished an ice-sheet which overflowed Scania, occupied the basin of the Baltic, and extended as far south as Hamburg and Berlin.

The Norfolk Forest-bed series contains the British records of the Norfolkan or First Interglacial Epoch. They consist of fresh-water and estuarine beds which are supposed to have been laid down when the southern part of the North Sea basin was dry land and Britain was joined to the continent. The flora points to a temperate climate similar to that of Norfolk at the present day. The fauna yields remains of elephants (*Elephas meridionalis*, *E. antiquus*), hippopotamus, Etruscan rhinoceros, deer, bison, the sabre-toothed tiger, and two northern forms, the glutton and musk ox. Some of the species are of distinct Pliocene types, and many of the large mammals found in the Forest-bed series did not survive that stage. The Forest-bed series is overlain by the *Leda myalis* bed and a layer of flood loam with Arctic plants (*Salix polaris*, *Betula nana*), indicating glacial conditions.

The Chillesford Clay, the Weybourne Crag, and Norfolk Forest-bed have been usually regarded as of Pliocene age by British geologists, but James Geikie contended that the temperate flora found in the Forest-bed pointed to an oscillation of climate between the overlying Arctic plant zone and the underlying Weybourne Crag with its northern species. Hence he regarded them as forming stages in his glacial succession.

On the continent, representatives of the First Interglacial Epoch were believed by James Geikie to occur in the Low Countries, France, Germany, and Italy. This correlation was based mainly on their mammalian remains, which closely resemble those of the Forest-bed series. The interglacial fresh-water shell bed, proved in borings near Berlin, are referred to this stage. From the descriptions by Wahnschaffe it appears that this deposit rests in places on boulder clay of considerable depth and is covered by drift. The most abundant species is *Paludina diluviana*, whose present habitat is far to the south on the borders of the Black Sea. Among the alluvia of the First Interglacial Epoch he grouped the sand beds at Mauer, near Heidelberg, which have yielded a lower jaw of early Pleistocene man. He admitted that the geological and palæontological evidence supporting this correlation was not quite decisive.

The Saxonian or Second Glacial Epoch (Mindelian of the Alps) was coincident with the maximum glaciation. During this period the ice radiating from Scandinavia spread over the northern plains of Europe to

the slopes of the Harz Mountains, the Erzgebirge, the Sudetes, and across a large part of Russia. The British and Scandinavian ice-sheets coalesced on the floor of the North Sea, and the united ice-field moved westwards towards the limit of the continental shelf. The lower boulder clays of Holland, North and South Germany, Poland, and Central Russia were then accumulated together with the lower boulder clay of Britain. He believed that a cold current washed the western coasts of Europe, and that Arctic molluscs entered the Mediterranean.

The Second or Tyrolian Interglacial Epoch was characterised by a temperate flora and fauna, and the interior of Europe was affected by a mild oceanic climate. This interval was supposed to be of long duration. The palæontological evidence furnished by deposits of this age in the Alps is of the highest importance in relation to oscillations of climate, and will be referred to in the sequel in connection with the researches of Penck and Brückner. In the North German plain fossiliferous beds, referred to this Interglacial Epoch, have been found at numerous localities, and have been described in detail by Wahnschaffe. They have yielded a rich mammalian fauna comprising southern forms, and some suggesting a colder climate than the present. In certain peat beds, supposed to belong to this horizon, Stoller found plant remains indicating a mild climate and none of boreal aspect. James Geikie believed that during this phase one or more land bridges existed across the Mediterranean, which enabled the southern forms to migrate northwards into Europe. Britain was then joined to the continent, and a large part of the North Sea may have been land.

The Chellean and Acheulian culture stages of Palæolithic man were referred by James Geikie to the Second Interglacial Epoch. The rude stone implements of Chellean man are to be found in the river drifts of that period, showing that he frequented the river valleys under genial climatic conditions.

A noteworthy feature of this Interglacial Epoch is the evidence of prolonged denudation of the land during the Chellean culture stage. Valleys were excavated and the general surface of the land was lowered. These topographical changes were accompanied by crustal movements which led to the submergence of the Baltic coast lands and parts of the Mediterranean region.

During the Polonian or Third Glacial Epoch (Rissian of the Alps), Scandinavia and most of the British Isles were again buried under one continuous ice-field, but the ice-sheet did not cover such an extensive area in Britain, North Germany, and Russia as in the Saxonian Epoch. The

direction of the ice-flow was influenced to some extent by the Baltic basin, the movement in that region being westerly and south-westerly. Though the ice-field did not extend farther south than the Midlands of England, James Geikie maintained that the loamy deposits, with characteristic Arctic plants in the Hoxne section, indicated an Arctic climate in the south of England during this period. They rest upon interglacial fresh-water beds with a temperate flora, which are underlain by the chalky boulder clay, which he regarded as the Saxonian ground moraine. Palæolithic man of the Mousterian culture stage witnessed the advance of the Polonian ice-sheet, and was contemporaneous with the mammoth, woolly rhinoceros, reindeer, Arctic fox, and other forms. The glacial conditions evidently led to his occupation of the caves of North-West, Central, and Southern Europe.

The Dürntenian or Third Interglacial Epoch was characterised by temperate conditions, and a forest fauna spread throughout Europe. This stage derives its name from the lignite beds at Dürnten in Switzerland, which will be referred to in connection with the researches of Penck and Brückner in the Alps. Mousterian man is believed to have existed during this period.

The Mecklenburgian or Fourth Glacial Epoch (Würmian of the Alps) had certain characteristic features which distinguish it from preceding periods of glaciation. The Scandinavian ice was no longer confluent with that of Britain. The basin of the Baltic was occupied by an ice-sheet which radiated from Scandinavia and Finland, and deposited terminal moraines in Denmark and in the northern provinces of Germany and Russia. As the ice retreated northwards the land sank to a limited extent, the sea advanced, and the *Yoldia* clays were laid down.

In North Britain glaciers still occupied the mountain valleys, and, in places, reached the sea-level, the land around the Scottish coast being then about 100 feet lower than at present. The deposits of the 100 feet beach belong to this period. The dominance of Arctic conditions is also indicated by the occurrence of Arctic plants in the sediments of some Scottish fresh-water lakes, together with the remains of a phyllopod (*Apus glacialis*), now met with only in Greenland and Spitzbergen.

The Aurignacian and Solutrén culture stages probably belonged to the earlier part of the Fourth Glacial Epoch, and these were followed by the Magdalenian, which marked the closing phase of Palæolithic man in Europe.

At an early stage in his career, James Geikie called attention to the evidence of climatic changes, furnished by the later Pleistocene deposits of Scotland, which are associated with Neolithic man. He described the

occurrence of two forest beds in our Scottish peat mosses, separated by a layer of peat, and with an overlying sheet of peat. He inferred that the forest growths indicated dry and continental conditions, while the peat layers implied colder and wetter conditions. The botanical evidence was subsequently dealt with by Dr Francis I. Lewis, who published the results of his detailed investigations in the *Transactions of the Royal Society of Edinburgh*. The various stages identified by Dr Lewis between the south of Scotland and the Shetland Isles, and in the Outer Hebrides, are given in the following table in descending order:—

9. Recent peat.	
8. Forest bed.	
7. Peat-bog plants with Arctic plants.	} Upper Forestian.
6. Forest bed.	
5. Peat-bog plants.	Upper Peat Bog.
4. Arctic plant bed.	Second Arctic Bed.
3. Peat-bog plants.	Lower Peat Bog.
2. Forest bed.	Lower Forestian.
1. Arctic plant bed.	First Arctic Bed.

The First Arctic Bed rests on glacial deposits and yields *Salix reticulata*, *S. herbacea*, and *Betula nana*. The Lower Forestian stage is characterised by birch, hazel, and alder, with temperate plants. The Second Arctic Bed, intercalated in peat, is composed mainly of Arctic plants and is overlain by an upper forest, which in the south of Scotland consists of *Pinus sylvestris*, except in Tweedsmuir, where the white birch takes the place of the pine. In the Highland areas the upper forest appears as two growths of pine, separated by a layer of peat from 1 to 3 feet thick, the succession being capped by recent peat. Dr Lewis points out that the flora of the two Arctic beds descends to within 150 feet of the sea-level—a wide divergence from the present lower limit of Arctic-Alpine vegetation, which is about 2000 feet. No trace of Arctic plants has been detected in the intervening Lower Forestian zone with a temperate flora. The evidence furnished by the forest growths is no less interesting. In the Lower Forestian stage the trees rise to 1500 feet, and in the Upper Forestian to about 3500 feet, the present upper limit of pine and birch being 2000 feet.

The definite succession of plant remains in the Scottish peat mosses confirms the conclusions reached by James Geikie on other grounds. The First Arctic Bed marks the passage from the Mecklenburgian Glacial Stage

to the Lower Forestian or Fourth Interglacial Epoch with its temperate flora. The Second Arctic Bed is correlated with the Lower Turbarian or Fifth Glacial Period, and the Upper Forest with the Fifth Interglacial Stage. Dr Lewis calls special attention to the continuity of these horizons in the peat throughout Scotland, and to the widespread alternate depression and elevation of the Arctic-Alpine vegetation after the Fourth Glacial Period. They indicate climatic changes sufficiently pronounced to affect the distribution of the flora in the north of Britain.

James Geikie regarded these fluctuations of climate as of considerable amplitude. Hence he included them in his glacial succession and described them as glacial and interglacial. He referred to the researches of Dr Hans Schreiber in the peat bogs along the northern border of the Eastern Alps, which furnish evidence of climatic change similar to that of our Scottish peat mosses. But, in my opinion, it is extremely doubtful if the fifth and sixth cold phases were of sufficient severity to be ranked as "Glacial Epochs." He admitted that the preceding glacial and interglacial periods were more prolonged and more strongly contrasted than the post-Mecklenburgian series. All the available evidence indicates that the glaciation which accompanied the fifth glacial phase was restricted to mountainous areas in the Highlands, where small glaciers, in certain localities, reached the sea-level and deposited their moraines on the 50 feet beach. Regarding the sixth glacial phase, high-level corrie glaciers may have existed during this period, but there is no evidence to show that moraines of this stage are associated with the 25 feet beach. The peat situated in corries between the 3000 feet and 4000 feet level was not examined by Dr Lewis, but from the evidence collected in the peat at lower levels he inferred that, during the last glacial stage, the north of Scotland was not affected by great cold.

In view of this evidence it seems more reasonable to compare the fluctuations of climate experienced in Scotland in post-Mecklenburgian time with the stadial and interstadial phases of the Alps, which are later than the Würmian or Fourth Glacial Epoch. If we eliminate the fifth and sixth glacial periods, then James Geikie's classification agrees with that of Penck and Brückner, based on their detailed researches in the Alps.

We have now given a brief outline of the glacial succession as propounded by James Geikie, from which it will be seen that Pleistocene time was characterised by a series of cold and mild periods. His views have been persistently opposed by the monoglacialisists, who regard the intercalated deposits in the drifts as indicating local recessions of the ice during one

prolonged period of glaciation. It must also be admitted that many glacialists who adopt the interglacial theory in a modified form have given a different interpretation of some of the details from that advanced by James Geikie. At the same time, it is a remarkable fact that the careful mapping of glacial deposits in the Alps and North America has led distinguished investigators to similar conclusions regarding the sequence of events in the Ice Age.

The elaborate investigations of Penck and Brückner, described in their monumental work *Die Alpen im Eiszeitalter*, have demonstrated four glaciations of the Alps, separated by interglacial periods. The evidence is based on a succession of gravels at different levels, stretching down the valleys. When traced upwards towards the mountains, these sheets of gravel are associated with moraines. Their classification is given below in descending order :—

4. The Würm Glaciation, associated with the Lower Terrace.
3. The Riss Glaciation, associated with the Higher Terrace.
2. The Mindel Glaciation, associated with the Younger Deckenschotter.
1. The Günz Glaciation, associated with the Older Deckenschotter.

The evidence points to a severe glaciation of the Alpine region during the First or Günzian Epoch. Glaciers descended the mountain valleys, indicating a depression of the snowline of about 4000 feet. The gravel outwash (Older Deckenschotter) has been extensively denuded, being preserved in narrow strips and outliers at considerable altitudes above the present lines of drainage. This deposit has been widely distributed on the northern side of the Alps, where it reaches a thickness of about 100 feet. At certain localities it has been cemented into a hard conglomerate, and, at the surface, shows the effects of weathering and solution.

The extensive erosion of the Older Deckenschotter implies a long interval of deglaciation during the First or Günz-Mindel Interglacial Period. But it is extremely difficult to identify interglacial deposits of this age beneath the later glacial materials. Some have referred the lignite beds of Leffe with remains of *Elephas meridionalis* and *Rhinoceros leptorhinus* in Northern Italy to this stage, but this correlation is extremely doubtful.

The renewed advance of the glaciers during the Second or Mindel Glaciation is proved by the development of moraines and the accompanying Younger Deckenschotter. Along part of the northern border of the Alps they are the most extensive of the glacial formations; in other areas they nearly coincide with the limits of the Riss drift of the Third Glacial

Epoch. The outwash gravels are cemented and weathered, but the pebbles are not so highly decomposed as those in the Older Deckenschotter.

The deposits of the Mindel-Riss or Second Interglacial Epoch separate the older from the younger drifts of the Alps. According to Penck and Brückner it was probably the longest of the interglacial periods in the Alpine region, and was characterised by a warmer climate than the present. The well-known Hötting Breccia near Innsbruck in the Tyrol has been latterly referred by Penck to this stage, though he formerly assigned it to the Third Interglacial Period. The plants contained in it furnish important evidence regarding climatic change. Forty-two species of plants have been identified, four of which are new. Thirty species still survive in the neighbourhood. Three species are of special importance—a rhododendron (*R. ponticum*), now living in the Caucasus in a climate about 3° C. warmer; a buckthorn (*Rhamnus hættingensis*), related to *R. latifolia* from the Canaries; and the box (*Buxus sempervirens*), a southern species. From the evidence Penck infers that the Hötting flora indicates a climate 2° C. warmer than now, when the Alps were comparatively free of ice and snow.

The morainic material and high terrace gravels accumulated during the Third or Rissian Glacial Epoch are not weathered and cemented to the same extent as the older glacial deposits. The detailed mapping of the glacial drifts has shown that, in Switzerland, France, and in the Po districts, the Riss glaciation exceeded that of the preceding Mindelian period, while in the valleys of the Inn, the Salzach, and the Iller, the reverse was the case. In order to account for this distribution of the glaciers, Penck has suggested that differential crustal movements may have produced a greater depression of the snowline in the northern part of the Eastern Alps.

The Riss-Würm or Third Interglacial Period is proved by the occurrence of interglacial deposits yielding evidence bearing on climatic change. Of these, reference may be made to the lignite or brown coal of Dürnten and Unter-Wetzikon. At both localities they are overlain by the moraine of the Fourth Glacial Epoch, and are underlain by glacial accumulations. Eighteen species of plants have been found at Dürnten, comprising, among others, the yew, the Swiss fir, white birch, sycamore, hazel, and water lily. The presence of the yew indicates that the climate at this stage must have been warmer than that now prevailing in the same region. The mammalian remains found at Dürnten tend to support this conclusion. These include *Elephas antiquus*, *Rhinoceros merckii*, *Bos primigenius*, and *Cervus elephas*.



The lake deposits of Pianico, which are underlain by moraine matter of the Third Glacial Epoch and are covered by moraine of the Würm Glaciation, have also yielded plant remains. The larger number still live in that region, but a certain proportion have a south-easterly distribution.

Other examples might be quoted, such as the brown coal of Uznach, the plant-bearing clays of Re and Calprino; but sufficient evidence has been adduced to prove important climatic changes during the Third Interglacial Period.

The Fourth or Würmian Glaciation of the Alps is marked by prominent moraines strewn with boulders, which show slight surface weathering. Associated with these morainic ridges there are outwash gravels (lower terrace), which are conspicuously developed round the Alpine region. They consist of coarse gravels, which, like the moraines, are not much decomposed. The moraines and gravels of the Fourth Glaciation have no covering of loess—a feature which distinguishes them from the older glacial deposits of the Alps.

In post-Würmian times in the Alps, Penck and Brückner have obtained evidence of minor climatic oscillations which they have termed stadial and interstadial stages. The end-moraines of the “retreat stadia” fall into three groups, corresponding to depressions of 900, 600, and 300 metres below the present snowline, which have been named the Buhlstadium, the Geschnitzstadium, and the Daunstadium. It is probable that the moraines of these stages may mark periods of readvance of the ice, for in the case of the two younger stadia (Geschnitzstadium and Daunstadium) the moraines overlie deposits of gravel. The interstadial phases were evidently characterised by milder conditions. According to Schreiber the snowline in the period following the Buhlstadium rose 100 metres above its present level, equivalent to a rise of 3250 feet from the preceding cold stage. Further, the moraine of the Daunstadium rests on calcareous tufa, which yields a flora indicating conditions similar to those of the present day.

In North America a prominent school of interglacialists has reached similar conclusions regarding the characteristic features of the Ice Age. They maintain that the Glacial Epoch was marked by a series of ice invasions, separated by intervals of time during which the ice retreated, and, in some cases, disappeared over large areas under conditions resembling those of the present day. These interglacial periods are proved by the denudation of the older sheet of drift before the advance of the succeeding one, by the depth of decomposition of the glacial materials before the

deposition of the overlying series, and by the accumulation of peat, soils, sands, and gravels between successive drift sheets.

The detailed classification advanced by Chamberlin and Salisbury is given below (*Text Book of Geology*, 1906):—

- XIII. The Champlain sub-stage (marine).
- XII. The Glacio-lacustrine sub-stage.
- XI. The Later Wisconsin, the Sixth Glacial Advance.
- X.      The Fifth Interval of Deglaciation.
- IX. The Earlier Wisconsin, the Fifth Glacial Invasion.
- VIII.      The Peorian, the Fourth Interglacial Interval.
- VII. The Iowan, the Fourth Invasion of the Ice.
- VI.      The Sangamon, the Third Interglacial Interval.
- V. The Illinoian, the Third Glacial Invasion.
- IV.      The Yarmouth, the Second Interglacial Interval.
- III. The Kansan or Second Glacial Invasion.
- II.      The Aftonian, the First Interglacial Interval.
- I. The Jerseyan, the First Glacial Advance.

Pirsson and Schuchert (*Text Book of Geology*, 1915) give the following table of divisions of Pleistocene time in North America, based on the researches of Chamberlin and Salisbury, and O. P. Hay:—

Post-glacial or present time	{	Vanishing of ice-sheets. Champlain marine invasion. Lowering of water-level of Great Lakes. Gradual amelioration of climate. Gradual extinction of elephants, mastodons, <i>Megalonyx</i> , musk-oxen, etc.
Fifth or Wisconsin Glacial Stage	{	Spread of ice-sheets and drift. Fauna and flora driven south.
Würm Stage in Europe		
Fourth or Peorian Interglacial Stage		Record not well determined. Formation of peat beds and soils. Wide distribution of loess.
Fourth or Iowan Glacial Stage		Spread of ice-sheets and drift. Record not well determined.
Third or Sangamon Interglacial Stage		Accumulation of peats, soils, and loess. Horses, elephants, mastodons, bison, peccaries, and tapirs probably present.
Third or Illinoian Glacial Stage	{	Spread of ice-sheets and drift. Deposition of loess. Apparently 60 per cent. of present land fauna then living. Mastodons, mammoths, horses, tapirs, bison, deer, and sabre-tooth tigers.
Riss Stage in Europe		
Second or Yarmouth Interglacial Stage		Formation of peats, soils, and bluish loess. Animals about as in Illinoian stage.
Second or Kansan Glacial Stage	{	Spread of ice-sheets and drift. Extinction of certain camels and horses.
Mindel Stage in Europe		
First or Aftonian Interglacial Stage	{	Great abundance of mylodons, megatheres, <i>Megalonyx</i> , mastodons, elephants (3 species), horses (6 species), camels (4 species), sabre-tooth tigers, bears, etc. A warm-temperate fauna.
First or sub-Aftonian Glacial Stage		Spread of ice-sheets and drift. Includes pre-Kansan, Nebraskan, and Albertan drifts.

Many of the interglacial deposits in North America do not furnish definite palæontological evidence regarding fluctuations of climate during the Ice Age. The best sections are those near Toronto, Ontario, which have been minutely examined and described by Professor Coleman. They are exposed in the Don river valley and in the Scarborough cliffs, where the fossiliferous interglacial deposits reach a maximum thickness of 150 feet, the lower portion being visible in the Don valley and the upper in the Scarborough cliffs. They rest on boulder clay which has been referred to the Iowan Glacial Stage, though it might be of older date. Above the interglacial beds come sheets of boulder clay and assorted drift which are grouped with the Wisconsin stages.

The flora and fauna obtained from the Don section indicate a warm climate, or, at least, conditions more genial than those of Toronto at the present day. The flora, according to Penhallow, who has identified a large number of species, points to a climate similar to that of the Central United States from 3° to 5° farther south. It includes the pawpaw (*Asimina triloba*), the Osage orange, both of which now flourish in more southerly regions, the maple, the elm, ash, oak, hickory, and basswood. The fauna contains eleven species of unios, four of which are not recorded in the St Lawrence basin, but are now found farther south in the Mississippi basin.

The flora and fauna in the higher interglacial beds on Scarborough cliffs imply a cold temperate climate resembling that now prevailing in the region north of Lake Superior. They doubtless indicate the gradual return of glacial conditions which characterised the earlier Wisconsin Glacial Stage.

The older interglacial periods, viz. the Aftonian, the Yarmouth, and the Sangamon, are characterised by deposits of peat, indicating growth of vegetation, while the leaching of the materials and the denudation of the older drift imply prolonged intervals between the successive glacial stages.

From the evidence summarised in the course of this address it is clear that the views advanced by James Geikie regarding oscillations of climate in Pleistocene time have been adopted by many investigators in Europe and America. Future research will show whether his elaborate classification of glacial and interglacial periods will have to be modified. Great diversity of opinion still exists regarding the interpretation of many glacial phenomena, the correlation of deposits in widely separated regions, and the sequence of conditions. But, in my opinion, sufficient palæontological evidence has been obtained to establish the general principle of oscillations of climate in the Glacial Epoch, though the number of interglacial periods may remain a subject of controversy.

## HISTORICAL SKETCH.

James Geikie was born in Edinburgh in 1839, and educated at the High School and University of his native city. In his school-days he was interested in the geological features of Edinburgh and its surroundings, and made excursions to sections which he expounded to his students in after years. He joined H.M. Geological Survey in 1861, and was promoted to the rank of District Surveyor in 1869, when his brother, Sir Archibald, was Director of the Scottish staff.

During his service he mapped large areas of the Scottish coalfields, especially in Lanarkshire, which gave him excellent opportunities for studying the geological structure of the Scottish Carboniferous rocks. He thus acquired a keen interest in tectonics, which he retained to the last, and was led to discuss the probable physical conditions that prevailed during the deposition of the coal- and ironstone-bearing strata in the west of Scotland. He also surveyed tracts of Silurian and Old Red Sandstone rocks along the northern margin of the Southern Uplands between the Cheviots and Ballantrae. Adopting the views then in the ascendant, he speculated on the metamorphic origin of granites and contemporaneous volcanic rocks in these older Palæozoic formations. For a time he was stationed in Perth, from which centre he mapped tracts of Old Red Sandstone and of the metamorphic rocks of the Highland border.

Throughout his field work he paid special attention to the glacial and post-glacial deposits of the country, and was led to study their development throughout Europe and North America. Ultimately he arrived at certain conclusions regarding climatic changes in Pleistocene time, which were expounded with great clearness and force in his volume *The Great Ice Age*, published in 1874. Its success was so marked that a second edition was called for within three years, and the third edition, brought up to date, appeared in 1894. It has been universally recognised as one of the standard works of reference relating to the history of glacial and post-glacial time.

In 1882 he issued a volume on *Prehistoric Europe*, which was supplementary to his earlier work. In it he dealt with the cave and river accumulations which had yielded traces of man and the Pleistocene mammals. He described the physical changes that characterised the post-glacial and recent periods, especially in the British Islands. In the same year, on the promotion of his brother, Sir Archibald Geikie, to the post of Director-General of the Geological Surveys of the United Kingdom, he was appointed to the Chair of Geology and Mineralogy in the University

of Edinburgh. On leaving the Survey he was specially thanked by the Department for his distinguished services, extending over a period of twenty-one years. In his application for the chair he received the cordial support of Charles Darwin.

During his tenure of the Professorship he brought the geological department to a high state of efficiency. Owing to the lack of adequate endowment he encountered great difficulties in realising his ideals. Ultimately he succeeded in establishing a lectureship in petrology, a lectureship in palæontology, and a museum collection for teaching purposes. By the encouragement of research in his laboratories and in the field, he sent forth students who have made important contributions to geology and who hold prominent positions at home and abroad. As an administrator within the University he was no less successful, for when the Science Faculty was established in 1894 he was appointed the First Dean—a position which he held for nineteen years.

While geological research was his great aim, he also rendered valuable service to the cognate science of geography. In 1884, when Dr Bartholomew—then a student in geology—suggested to Professor Geikie that the time was ripe for the foundation of a Scottish Geographical Society, he leapt at once to the idea. At the original meeting, which took place in the hall of the Chamber of Commerce in 1884, it was he who proposed the resolution: “That this meeting, recognising the scientific and general utility of a national society for the promotion of geography, resolves that a Geographical Society for Scotland be now formed.” During the thirty years that have elapsed since its foundation, this Society has achieved remarkable success. The great aim of the Council has been to improve the teaching of geography in schools and to establish lectureships in Scottish Universities. Professor Geikie was one of the first vice-presidents, and he held this office till his death, except during his occupancy of the presidential chair from 1904 to 1910. In 1888 he became honorary editor of the Society’s Magazine, and contributed articles to its pages. He was awarded the gold medal of the Society, and on his retirement from the presidential chair he was presented with his portrait in recognition of his devoted service.

In addition to the volumes already mentioned, James Geikie produced others which show his power as a clear expounder and original thinker. In 1898 he issued a volume on *Earth Sculpture, or the Origin of Land-forms*, and in 1913 another work on *Mountains: their Origin, Growth, and Decay*, both of which dealt with the borderland of geology and geography. In 1898 he produced a text-book on *Structural and Field Geology*, which passed through three editions, thus furnishing striking

proof of its practical value to students. In 1914 appeared his volume on *The Antiquity of Man in Europe*, which might be regarded as a further development of his work on *The Great Ice Age*. Its distinctive feature was the correlation of his series of Interglacial Periods with the culture stages of Palæolithic and Neolithic man.

On the retirement of Sir William Turner from the Presidential Chair of this Society in 1913, he was elected as his successor—a fitting acknowledgment of his eminence as one of the leading geologists of his time. Among the honours which fell to him in recognition of his scientific researches may be mentioned the Makdougall-Brisbane Medal, awarded by the Royal Society of Edinburgh, and the Murchison Medal, by the Geological Society of London. This brief sketch of his career would be incomplete without a reference to his strong personality, to his literary tastes, which were shown in his volume of translations of Heine's *Songs and Lyrics*, and to his rich vein of humour, which made him a delightful companion in the field and around the festive board.

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(Issued separately May 8, 1916.)

## II.—Notices of Fellows, Honorary and Ordinary, recently deceased. By The General Secretary.

EMILE-HILAIRE AMAGAT was born in 1840. When Professor of Physics at l'Ecole Normale at Cluny, he began in 1867 his investigations of gases under high pressures. Later at Lyons, where he was Professor at the Catholic University, he utilised the tower of one of the churches as the site for a manometer giving pressures up to 80 atmospheres, and later constructed in one of the coal mines of St Etienne a Boylean tube measuring up to 430 atmospheres. In some of his experiments he applied with great success a suggestion of Tait's for recording volumes by means of mercury coming in contact with platinum wires at different heights in the tube. His results in the gaseous laws for hydrogen, nitrogen, and carbonic acid and other vapours are recognised as the most authoritative we have.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1897, and died at his country estate at St Satur in the Département of Cher in March 1915.

GEORGE FRIEDRICH JULIUS ARTHUR AUWERS was born at Göttingen on September 12, 1838. He began his astronomical career at the Observatory of Göttingen, and in 1859 became assistant at Königsberg. In 1862 he was transferred to Gotha, and finally in 1866 was appointed to Berlin.

In his early days he made many excellent observations of variable stars and of double stars, and investigated very fully the irregularities in the proper motions of Sirius and Procyon. His re-reduction of Bradley's observations occupied him ten years, and formed the foundation for his great catalogues of stars, for which he was awarded the gold medal of the Royal Astronomical Society in 1888.

Auwers took a large share in the observations of the transit of Venus in 1874 and 1882, and visited South Africa in 1889, when he shared with Gill the observations of Victoria for the determination of the solar parallax.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1900, and died on January 24, 1915.

PAUL EHRLICH was born on March 14, 1854, at Strehlen, in Silesia, and was educated at Breslau and at Strasburg, where he graduated in medicine. Since 1896 he has been Director of the "K. Institut für Experimentelle Therapie" in Frankfurt-am-Main. Throughout his early career he took the deepest interest in the chemical relationships of living matter

and in the affinities of various reagents for living cells. He devoted much attention to staining methods, and his hæmatoxylin and triacid stains are widely used at the present day. He is perhaps best known for his discovery of salvarsan.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1905, and died on August 20, 1915.

SIR JAMES A. H. MURRAY was born at Denholm near Hawick in 1837, and began life as a schoolmaster. When at Mill Hill School from 1870 to 1885, he conceived the idea of a new historical dictionary of the English language, and, backed by the efforts of the Philological Society, began to collect material, which rapidly outgrew ordinary accommodation. In 1879 the Oxford University Press took up the publication of the Dictionary, and in 1885 Murray moved to Oxford, and there pursued his task to the end.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1908, and died on July 26, 1915, while engaged in editing the words with initial letter "U."

FREDERICK WARD PUTNAM was born in 1839 at Salem, Mass., and studied at Harvard University. He was Emeritus Professor of American Ethnology and Archæology in Harvard University, Honorary Curator of the Peabody Museum, permanent Secretary of the American Association for the Advancement of Science from 1873 to 1898, and President of the Association in 1898, and was distinguished for his contributions to anthropology.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1910, and died at Cambridge, U.S.A., on August 14, 1915.

AUGUST F. L. WEISMANN was born on January 17, 1834, at Frankfurt-am-Main, and was educated at Göttingen. In 1886 he was appointed Professor of Zoology in Freiburg University, and continued throughout his long tenure of this Chair to investigate embryonic and post-embryonic development and metamorphosis of insects, Crustacea, and Hydrozoa. This brilliant work was the foundation on which he afterwards worked out his famous theory of embryology, to which he applied with singular felicity the fundamental principle of Darwinism. With the exception of Darwin himself, no zoologist has had a greater influence than Weismann in moulding scientific opinion on present embryology.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1910, and died at Freiburg on November 5, 1914.

JOHN MACVICAR ANDERSON, Architect, born in 1834, was a native of Glasgow, but he spent the greater part of his life in London. He was

elected an Associate of the Royal Institute of British Architects in 1864, became a Fellow in 1868, and, after filling many offices in the Royal Institute, he occupied the Presidential Chair from 1891 to 1894. As an architect he planned many buildings of banks and insurance offices in the City of London, and was Honorary Architect to the Royal Scottish Corporation and the Caledonian Asylum. He was deeply interested in religious and benevolent works.

He became a Fellow of the Royal Society of Edinburgh in 1893, and died on June 9, 1915.

WILLIAM ANDERSON, F.G.S., born in Edinburgh on January 20, 1860, was the eldest son of Dr Joseph Anderson, the well-known Scottish antiquarian, and was educated at Newington School and at the Edinburgh University. Although he began the study of medicine, he subsequently relinquished it in favour of geology, and in 1886, on the recommendation of Sir Archibald Geikie, took up duties on the Geological Survey of New South Wales. For seven years he continued to work in that Colony, and wrote valuable reports on many points of geological and economic interest. In 1893 he was appointed to the Geological Survey of India as Mining Specialist, and in 1899 became Government Geologist of Natal. He resigned this appointment in 1905, but continued for some years to contribute reports on the geology of South Africa. Failing health compelled him to give up active work, and in June 1913 he took up his residence in Sydney.

He was elected a Fellow of the Royal Society of Edinburgh in 1905, and died in Sydney on May 30, 1915.

J. B. BUIST, M.D., F.R.C.P.E., B.Sc., P.H., graduated M.B. at the Edinburgh University in 1867. He wrote a number of papers on Vaccination, which he taught under the Local Government Board. He was also Lecturer on General Pathology in the Edinburgh School of Medicine. He wrote various papers on Variola and Vaccinia, which were published in the *Edinburgh Medical Journal*, the *Transactions of the Royal Society of Edinburgh*, the *Lancet*, and other medical journals.

He was elected a Fellow of the Royal Society of Edinburgh in 1887, and died on January 29, 1915.

SIR THOMAS SMITH CLOUSTON, Kt., M.D., LL.D., was born in Orkney on April 22, 1840. He studied medicine in the University of Edinburgh, and devoted himself specially to the study of mental diseases. In 1863 he was appointed Medical Superintendent of the Cumberland and Westmoreland Asylum, Carlisle, and ten years later succeeded his former chief,

Dr Skae, as Physician Superintendent of the Royal Asylum, Morningside, Edinburgh. Here for thirty-five years he devoted himself to the study of insanity in all its aspects, and acquired a world-wide reputation as one of the greatest living authorities on the subject. As Lecturer on Insanity in the University of Edinburgh, he came in close touch with the students of that great medical school, and exercised a great influence amongst them. His clinical lectures on mental diseases passed through six editions, and his other books, which all bear upon the question of insanity and its treatment, enhanced his reputation as a physician of the highest order. He retired from his office as Superintendent of the Royal Asylum in 1908, and received the honour of knighthood from the King on the occasion of His Majesty's visit to Edinburgh in 1911.

He was elected a Fellow of the Royal Society of Edinburgh in 1875, and died on April 19, 1915.

ARCHIBALD DAVID CONSTABLE, LL.D., was born in Edinburgh in 1843. He studied at the Edinburgh Academy and at the Universities of St Andrews, Berlin, and Paris. In 1865 he became associated with his father in the printing firm of T. & A. Constable. His intimate knowledge of foreign languages, both classical and modern, and his large acquaintance and personal friendship with men and women of letters, enabled him to exert an important influence on the literary side of his business. His most important piece of literary work was the editing and translating for the Scottish History Society of Major's *History of Scotland*, a work so full and scholarly that the University of St Andrews conferred upon him the honorary degree of LL.D. in 1895.

He was elected a Fellow of the Royal Society of Edinburgh in 1872, and died on January 14, 1915.

SIR JAMES DONALDSON, Kt., LL.D., born April 26, 1831, was educated at the Grammar School and University, Aberdeen, at the New Collège, London, and at the Berlin University. He was appointed Rector of the High School, Stirling, in 1854, but after two years settled in Edinburgh as Classical Master in the Royal High School. In 1866 he was appointed Rector, and during his fifteen years' tenure of this office he gained a wide reputation by his publications on early Christian literature, *The Apostolical Fathers*. In 1881 he was appointed Professor of Humanity in Aberdeen University, and in 1886 was elected Vice-Chancellor and Principal of the University of St Andrews.

He became a Fellow of the Royal Society of Edinburgh in 1867, and communicated to the *Transactions* of the Society an erudite paper on

"The Expiatory and Substitutionary Sacrifices of the Greeks." He died on March 9, 1915.

A. CAMPBELL FRASER was born at the Manse of Ardochattan on Loch Etive on September 3, 1819. At the age of fourteen he entered Glasgow University, and after a single session there was transferred to Edinburgh. In 1838 he passed to the Divinity Hall, where his chief teachers were Chalmers and Welsh, whom he followed in 1843 at the time of the Disruption, and was ordained in 1844 as junior minister of the Free Church at Cramond. In 1846 he was appointed to the Chair of Logic in the New College of the Free Church, and became known to wider circles as editor of the *North British Review*. In 1856 he became Professor of Philosophy in the University of Edinburgh in succession to Sir William Hamilton, and continued for thirty-five years to take a prominent part in the life of the University. After his retirement in 1891 he lived for the most part at Hawthornden, and died in Edinburgh on December 2, 1914, at the great age of 96.

Professor Fraser's chief contributions to philosophical literature were his edition of Berkeley's *Works*, an edition of Locke's *Essay*, with prolegomena and notes, and his Gifford Lectures on the *Philosophy of Theism*.

He was elected a Fellow of the Royal Society of Edinburgh in 1858, served on the Council from 1879 to 1882, and was at the time of his death the senior Fellow of the Society.

JAMES GEIKIE, D.C.L., LL.D., F.R.S., was a distinguished Professor of Geology in the University of Edinburgh, and President of the Royal Society of Edinburgh. He died on March 1, 1915. See the Historical Sketch by Dr Horne, F.R.S., p. 18, above.

D. T. GWYNNE-VAUGHAN, F.L.S., was born at Llandovery in 1871, and educated at Monmouth School and at Christ's College, Cambridge. He began his research work in the Jodrell Laboratory, Royal Gardens, Kew, and in 1896 was appointed assistant in Botany in the University of Glasgow. Here he worked for about ten years, laying the foundation of his unrivalled knowledge of the anatomy of the Pteridophyta. In 1909 he was appointed Professor of Botany in Belfast, and in 1914 took up similar duties at Reading. In addition to his own important series of papers, he co-operated with Dr Robert Kidston in a series of Memoirs on the fossil Osmundaceæ published in the *Transactions of the Royal Society of Edinburgh*. These Memoirs have already taken a high place among botanical classics, and for his share of this work he was awarded



the Makdougall-Brisbane prize by the Council of the Royal Society of Edinburgh.

He was elected a Fellow of the Royal Society of Edinburgh in 1910, and died at Reading on September 4, 1915.

SIR CHARLES AUGUSTUS HARTLEY, Kt., K.C.M.G., was born in 1825. Since 1856, when he was appointed Engineer-in-Chief to the European Commission of the Danube, he was engaged in many very important national and international questions of engineering; for example, he was one of the Congress which met at Paris to decide on the best route for a ship canal across the Isthmus of Panama, and was consulted on such questions as the enlargement of the port of Trieste, the improvement of the Don and Dnieper, the commercial harbours of Constanza, Bourgas, and Varna, etc. etc. He published works on the *Delta of the Danube*, on *Inland Navigations in Europe*, and on the *History of the Engineering Works of the Suez Canal*.

He was elected a Fellow of the Royal Society of Edinburgh in 1869, and died on February 20, 1915.

ARCHIBALD HEWAT, F.F.A., F.I.A., was born in Edinburgh in 1838. He gained his knowledge and experience in various life assurance offices, and finally became closely associated with the Edinburgh Life Assurance Company in his successive capacities of Secretary and Manager. He was author of a number of papers on subjects dealing with Life Assurance, and he also lectured from time to time before various societies throughout the country on similar questions. After his retirement in 1911 from his post as Manager in the Edinburgh Life Assurance Company, he devoted his services to pension schemes in connection with ministers of the Church of Scotland.

He was elected a Fellow of the Royal Society of Edinburgh in 1908, and died in April 1915 while spending his Easter holiday at Keswick.

JOHN HALLIDAY SCOTT, M.D., M.R.C.S., was born at Edinburgh on December 28, 1851. He was educated at the Edinburgh Institution, and after a distinguished career as a medical student graduated Bachelor of Medicine at the University of Edinburgh in 1874. In 1877 he was appointed Professor of Anatomy in Otago University, Dunedin, New Zealand, a position which he held to his death in 1914.

He was elected a Fellow of the Royal Society of Edinburgh in 1880.

## III.—The Torsional Vibration of Beams of Commercial Section.

By **Ernest G. Ritchie**, B.Sc., Engineering Department, University College, Dundee. *Communicated by* Professor A. H. GIBSON.

(MS. received November 13, 1915. Read December 20, 1915.)

WHEN an elastic body is constrained in any manner whatsoever, it is susceptible to vibration, by virtue of its elasticity, when disturbed from its position of equilibrium by an externally applied force. The period and amplitude of such vibration are dependent upon the mass and inertia of the system, the rigidity of the constraints, and upon the nature of the disturbing force.

When a beam of commercial section is loaded centrally, and subjected to vibrations, the frequency of transverse vibration can be readily determined from a knowledge of the dimensions of the beam, its modulus of elasticity, and the conditions of loading. On the other hand, where the loading is eccentric, the transverse vibration is accompanied by a torsional vibration the frequency of which is very much lower than is indicated by the ordinary elastic theory, due to the inefficiency in torsion of beam sections other than circular. In practice it is not always possible to eliminate the eccentric loading of beams, as for instance where power is transmitted through countershafts supported from structural steel-work, and it is with the problem of the torsional vibration of such eccentrically loaded beams that it is proposed here to deal.

## I. THE TORSION OF BEAM SECTIONS.

When a bar of circular section and of length  $L$  is subjected to the application of a static twisting-moment  $T$ , it may be readily shown that

$$CJ = \frac{TL}{\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where  $\theta$  is the angle of twist in radians,  $C$  is the modulus of rigidity, and  $J$  is the polar moment of inertia of the section.

When a bar of non-circular section is subjected to a twisting-moment it has been demonstrated both theoretically and experimentally that the above formula does not truly represent the condition of affairs, the angle of twist for a given torque being greater than would be anticipated from formula (1). The discrepancy is largely due to the inefficiency in torsion



where  $n$  and  $m$  are constants depending upon the type of section. The values of  $n$  and  $m$  for the more important commercial sections were found to be as follows:—

TABLE II.

Type of Section.	<i>n.</i>	<i>m.</i>
I . . . . .	2'00	60
Channel . . . . .	2'30	40
Tee . . . . .	2'30	25
Angle . . . . .	2'30	18

## II. THE TORSIONAL VIBRATION OF BEAMS.

When a bar or beam is subjected to torsional vibration the frequency of vibration can be readily determined, as will be shown later by equating the potential energy of the vibrating system at the point of extreme angular displacement to the kinetic energy of the system in its mean position. Two cases arise,

- (a) The unloaded beam.  
(b) The loaded beam.

The first of these is of very little importance in the subject with which we are dealing, as the conditions necessary for the production of torsional vibrations in an unloaded beam are seldom if ever reproduced in practice, and, moreover, the natural frequency of vibration of such a beam is so high as to preclude any possibility of resonance with machinery in operation.

The problem of the loaded beam is, however, of some importance, and it is intended to treat of this in some detail.

### III. "FIXED-FREE" BEAM WITH SINGLE LOAD AT FREE END.

Let the beam AB (fig. 1) be rigidly fixed at one end and free to rotate about the axis OO at the other end. Furthermore, let the *mass* moment of inertia of the beam be negligible compared with the moment of inertia of the load  $\bar{w}$  whose eccentricity is  $r$ .

Let  $\theta_1$  be the angular displacement due to the *statically* applied torque  $T (=Wr)$ , and let  $\theta$  be the angular displacement on each side of the mean position when the system is vibrating.

From formula (1) we obtain

$$\frac{T}{\theta_1} = \frac{CJ}{L} \quad (3)$$

where  $\frac{T}{\theta_1}$  is the static torque necessary to produce unit angular displacement.

ment and the other symbols have the significance already ascribed to them.

When the system is vibrating, at the point of maximum angular displacement, the whole of the energy is in the form of potential energy and the restoring moment is given by the relationship

$$\text{Restoring moment} = \frac{T}{\theta_1}(\theta_1 + \theta) - \bar{w}r ;$$

but

$$T = \bar{w}r,$$

and from (3)

$$\frac{T}{\theta_1} = \frac{CJ}{L}.$$

$$\therefore \text{Restoring moment} = \frac{CJ}{L}\theta.$$

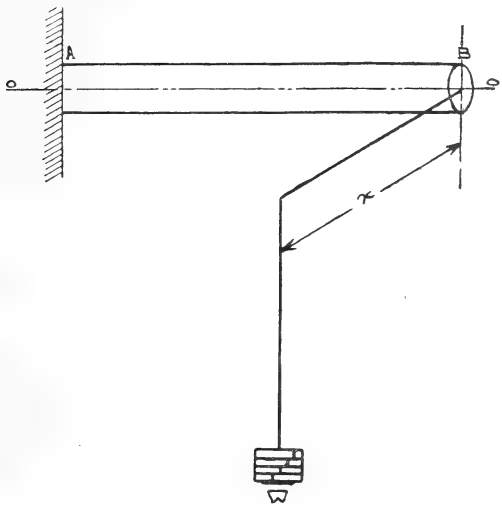


FIG. 1.

Also, it may be shown that the displacing moment

$$= I \frac{d^2\theta}{dt^2}$$

where I is the mass moment of inertia of the load about the axis OO.

Since the sum total of the energy throughout a complete vibration varies in kind but not in amount, the kinetic energy of the system as it passes through its mean position must equal the potential energy of the system at the point of extreme angular displacement. Hence

$$\begin{aligned} I \frac{d^2\theta}{dt^2} &= - \frac{CJ}{L} \theta. \\ \therefore I \frac{d^2\theta}{dt^2} + \frac{CJ}{L} \theta &= 0. \end{aligned} \quad (4)$$

Solving this differential equation, we obtain

$$t = 2\pi\sqrt{\frac{IL}{CJ}}$$

where  $t$  is the periodic time of a complete vibration, from which, if  $n$  is the frequency of vibration, *i.e.* the number of vibrations per second,

$$n = \frac{1}{2\pi}\sqrt{\frac{CJ}{IL}}.$$

For circular sections the above formula applies rigidly, while for sections other than circular the effective value of  $J$ , as obtained from Tables I and II, is to be substituted for  $J$ , giving

$$n = \frac{1}{2\pi}\sqrt{\frac{CJ'}{IL}} \quad \dots \quad (5)$$

It may be noted that equation (5) gives the free or natural frequency of vibration of a loaded beam whether the loading is central or eccentric,  $I$  being in either case the mass moment of inertia of the load.

If a loaded beam is in communication with an external periodic disturbing force the frequency of which is given by equation (5), torsional vibrations will be instituted the amplitude of which will gradually increase until rupture takes place. If the frequency of the disturbing force is not identical with  $n$ , but nearly equal to it, vibration will still occur, and a sufficient number of repetitions of the disturbing force may ensue to cause fracture.

#### IV. "FIXED-FIXED" BEAM WITH SINGLE LOAD.

If the beam of length  $L$  is fixed at both ends and loaded at the middle, the frequency of vibration is to be taken as that of a "fixed-free" beam of half the length and subjected at its free end to a load having half the moment of inertia of the given load, *i.e.*

$$n = \frac{1}{\pi}\sqrt{\frac{CJ'}{IL}} \quad \dots \quad (6)$$

When the point of application of the load is not symmetrical with respect to the fixed ends of the beam, the frequency may be got in a similar manner. Thus if a beam of length  $L$  has a load of moment of inertia  $I$  situated at a point distant  $a$  from one end, the frequency of vibration is the same as a "fixed-free" beam of length  $a$  having a load whose moment of inertia is  $I\left(\frac{L-a}{L}\right)$  at its free end, or a beam of length  $(L-a)$  and a load whose moment of inertia equals  $I\frac{a}{L}$ .

## V. EFFECT OF INERTIA OF BEAM.

If the *mass* moment of inertia of the beam cannot be neglected, but if it is small compared with the moment of inertia of the applied load, a correction can be obtained on the assumption that the angle of twist is proportional to the distance from the fixed end of the beam. Thus in a "fixed-free" beam, if the angle of twist at the free end at any instant is  $\theta$ , the angle of twist at any section of the beam distant  $x$  from the fixed end is  $\theta \frac{x}{L}$ , where  $L$  is the total length of the beam, while the angular velocity at this point equals  $\frac{x}{L} \frac{d\theta}{dt}$ . Hence if  $J$  is the *mass* moment of inertia of the whole beam about the axis of twist, the kinetic energy of a section at a point distant  $x$  from the fixed end and of thickness  $dx$  is given by

$$\begin{aligned} & \frac{1}{2} \frac{J_m}{L} dx \left( \frac{x}{L} \frac{d\theta}{dt} \right)^2 \\ &= \frac{1}{2} \frac{J_m}{L^3} \left( \frac{d\theta}{dt} \right)^2 x^2 dx, \end{aligned}$$

and the kinetic energy of the whole beam is

$$\begin{aligned} & \frac{1}{2} \frac{J_m}{L^3} \left( \frac{d\theta}{dt} \right)^2 \int_0^L x^2 dx \\ &= \frac{1}{3} \left[ \frac{1}{2} J_m \left( \frac{d\theta}{dt} \right)^2 \right], \end{aligned}$$

*i.e.* the dynamic effect of the beam is the same as a mass concentrated at the free end, and having a mass moment of inertia equal to  $\frac{1}{3}$  that of the beam. Hence if the effect of the beam is not negligible, the moment of inertia of the applied load must be increased by an amount equal to  $\frac{1}{3}$  the mass moment of inertia of the beam.

Where the mass moment of inertia of the beam itself is commensurate with the moment of inertia of the applied load, the motion is somewhat complex, and, except for a beam of circular section, a solution is difficult to obtain;\* while for beams of commercial section, I, channel, angle, etc., the analytical solution would seem to be impossible. In practice, however, the only case of importance is that of the loaded beam, in which the dynamic effect of the beam itself is negligible, and in most practical cases the frequency of vibration will be given with sufficient accuracy by equation (5).

\* *Proceedings Inst. Civ. Engrs.*, vol. clxii, p. 382.

## VI. BEAM LOADED AT MORE THAN ONE POINT.

When the beam is loaded at more than one point, the nature of the vibration depends upon the relation of the moments of inertia of the applied loads and the position of these relative to the fixed ends. Suppose the beam AB (fig. 2) is fixed at each end and is acted upon by two loads at C and D. Under these circumstances, the system may vibrate in one of two distinct modes depending upon the relationship between  $I_1$  and  $I_2$  and between  $l_1$  and  $l_2$ .

The lowest frequency of vibration will occur when the frequency amplitude and phase of vibration are the same at the points C and D. Under these conditions, the portion CD of the beam will simply oscillate as a whole about the axis of vibration, the parts AC and DB behaving as

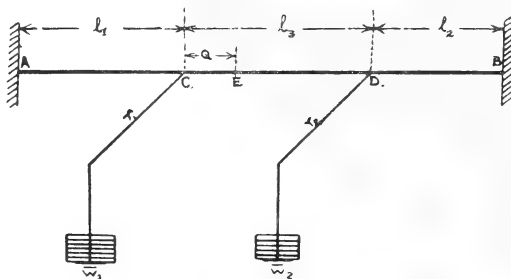


FIG. 2.

“fixed-free” beams of lengths  $l_1$  and  $l_2$  respectively, and having loads at their free ends whose moments of inertia are  $I_1$  and  $I_2$  respectively.

If the frequencies at C and D are to be equal, we have from equation (5)

$$I_1 l_1 = I_2 l_2.$$

Also, if the amplitude at C is to be the same as the amplitude at D, from equation (1)

$$T_1 l_1 = T_2 l_2.$$

If these two conditions are to hold simultaneously, then clearly  $r_1 = r_2$ , i.e. the eccentricity of  $\bar{w}_1$  must equal that of  $\bar{w}_2$ .

If the above conditions are fulfilled, and if the phase of vibration at C is the same as that at D, the beam will vibrate with a frequency given by

$$n = \frac{1}{2\pi} \sqrt{\frac{CJ'}{I_1 l_1}} = \frac{1}{2\pi} \sqrt{\frac{CJ'}{I_2 l_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the above conditions do not hold, a node will be formed at some point between C and D, the position of which may be determined from the



fact that when the system is vibrating the frequency of vibration at C must be the same as the frequency at D.

Let the node be at point E between C and D and distant  $a$  from point C. Then, since no motion ensues at E, the portions AE and BE of the beam will behave as “fixed-fixed” beams of length  $(l_1 + a)$  and  $(l_2 + l_3 - a)$ , loaded at C and D respectively. The position of the node is determined by equating the frequencies of these two “fixed-fixed” beams.

The frequency of the beam AE is given by

$$n = \frac{1}{2\pi} \sqrt{\frac{CJ'}{I_1 \left(\frac{a}{a+l_1}\right) l_1}} \quad \dots \quad (8)$$

and that of the beam BE by

$$n = \frac{1}{2\pi} \sqrt{\frac{CJ'}{I_2 \frac{(l_3 - a)}{(l_3 + l_2 - a)} l_2}} \quad \dots \quad (9)$$

Equating these we have

$$I_1 \frac{al_1}{(a+l_1)} = I_2 \frac{l_3(l_3 - a)}{(l_3 + l_2 - a)} \quad \dots \quad (10)$$

Equation (10) has two real roots corresponding to two possible positions of the node, giving in general a higher and a lower frequency of vibration.

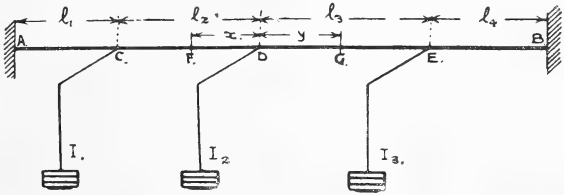


FIG. 3.

The value of  $a$  as obtained from equation (10), when substituted either in equation (8) or equation (9), gives the frequency of vibration of the beam.

When a beam is loaded at more than two points the motion becomes more and more complex as the number of loads increases.

If the beam is loaded at three points C, D, and E, as in fig. 3, a possible mode of vibration is that in which nodes are formed at F and G, under which conditions the three portions AF, FG, and GB of the beam behave as “fixed-fixed” beams. Expressing the condition that the frequency is the same at C, D, and E, we have

$$I_1 \frac{l_1}{(l_1 + l_2 - x)} (l_2 - x) = I_2 \left(\frac{y}{x+y}\right) x = I_3 \frac{l_4}{(l_3 + l_4 - y)} (l_3 - y) \quad \dots \quad (11)$$

which gives us two equations for the determination of the unknowns  $x$

and  $y$ . These having been determined, the frequency can be obtained as before.

If the loading is symmetrical with respect to the ends, *i.e.*

$$\text{if } l_1 = l_4, \ l_2 = l_3, \ I_1 = I_3, \text{ and } r_1 = r_3,$$

if the phase at C is the same as the phase at E, the motion at these two points being opposite to the motion at D, nodes are formed at F and G equidistant from D. If the phase at C is not the same as the phase at E, the vibration is such that a single central node is formed at D.

For more than three loads the motion is somewhat complex, and the equations become difficult to handle. In practice, however, it will usually be found possible to group the loads in such a manner as to allow of solution by one or other of the above methods. Such grouping will not in general lead to errors of large magnitude. Thus in a long beam where two or more of the loads are near one end, these may be assumed to have the same dynamical effect as a single load having the same moment of inertia.

#### VII. BEAM LOADED UNIFORMLY.

When the beam supports a uniformly distributed eccentric load, expressions for the frequency of vibration may be readily obtained. Thus in the case of a "fixed-free" beam of length  $L$  supporting a uniformly distributed load of magnitude  $w$  per unit of length and of constant eccentricity  $r$ , the angle of twist at the free end due to the static application of an element of load distant  $l$  from the fixed end is given by

$$\frac{wrldl}{CJ},$$

and hence the torque at the free end due to the static application of the whole load is

$$\begin{aligned} & \frac{wr}{CJ} \int_0^L ldl \\ &= \frac{wrL^2}{2CJ} \\ &= \frac{1}{2} \frac{\bar{w}rL}{CJ} \text{ where } \bar{w} \text{ is the total load} \\ &= \frac{1}{2} \frac{TL}{CJ}; \end{aligned}$$

*i.e.* the effect of a uniformly distributed load applied statically is the same as that of a load of half the magnitude, and having the same eccentricity, concentrated at the free end. If such a system vibrates, the frequency

will be the same as that of a similar beam having a load at the free end, the eccentricity of which is the same as that of the uniform load, and having a mass moment of inertia equal to half that of the distributed load.

VIII. BEAM OF NON-UNIFORM SECTION.

If the beam is not of uniform section throughout, but is made up of separate sections as in fig. 4, the lengths of which and the effective

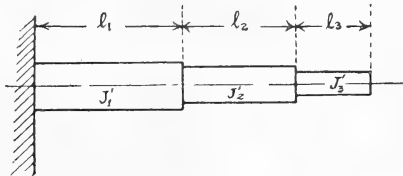


FIG. 4.

polar moments of inertia of which are respectively  $l_1, l_2, l_3$ , etc., and  $J_1', J_2', J_3'$ , etc., the frequency of vibration of the loaded system may be obtained as follows:—

Writing equation (5) in the form

$$n = \frac{1}{2\pi} \sqrt{\frac{C}{I \left( \frac{l}{J'} \right)}}$$

we have for a composite “fixed-free” beam, assuming  $C$  constant throughout,

$$\begin{aligned} n &= \frac{1}{2\pi} \sqrt{\frac{C}{I \left( \frac{l_1}{J_1'} + \frac{l_2}{J_2'} + \frac{l_3}{J_3'} + \text{etc.} \right)}} \\ &= \frac{1}{2\pi} \sqrt{\frac{C}{I \sum \left( \frac{l}{J'} \right)}} \quad \dots \quad (12) \end{aligned}$$

*i.e.* the moment of inertia  $J''$  of a uniform beam of given length which has the same frequency of vibration as the composite beam is given by

$$\frac{L}{J''} = \sum \left( \frac{l}{J'} \right) \quad \dots \quad (13)$$

If the section changes uniformly throughout the length of the beam,  $J'$  will be a function of  $l$ , and the summation in equation (13) can be carried out by integration.

The significance of the above investigation in relation to the vibration of structures will be best understood by the consideration of a specific example. Thus suppose we take the case of an encasté I-section beam 8" deep by 4" flange, and covering a span of, say, 20 feet. Suppose this

beam is subjected to a load of 300 lbs. at its middle point. If the load is central, the natural frequency of transverse vibration is given by the expression \*

$$n_1 = \frac{13.84}{2\pi} \sqrt{\frac{EIg}{wl^3}}.$$

Assuming a value for  $E = 30.0 \times 10^6$  inch lb. units, this gives

$$\begin{aligned} n_1 &= 27.6 \text{ per second} \\ &= 1656 \text{ per minute.} \end{aligned}$$

If the load has an eccentricity equal to  $r$ , besides the above transverse vibration, the system will have a natural frequency of torsional vibration the magnitude of which is obtained from equation (5), making the necessary correction for a "fixed-fixed" beam, viz.

$$n_2 = \frac{1}{\pi} \sqrt{\frac{CJ'g}{wr^2l}}.$$

The value of  $J'$  as obtained from Table II is  $\frac{A^2}{60}$ , which, for the dimensions of a standard commercial  $8'' \times 4''$  I section, is equal to  $.468$  inch<sup>4</sup> units.

Hence, assuming a value for  $C = 12 \times 10^6$  inch lb. units we obtain

$$\begin{aligned} n_2 &= \frac{55.0}{r} \text{ per second} \\ &= \frac{3300}{r} \text{ per minute.} \end{aligned}$$

This shows that the frequency of torsional vibration diminishes as  $r$  increases. When the eccentricity is  $2.0''$ , the frequency is approximately the same as that of the transverse vibrations.

Values of  $n_2$  for various values of  $r$  are tabulated below.

Eccentricity $r$ in inches . . . . .	2"	4"	6"	8"	10"	12"
Frequency in vibrations per minute.	1650.0	825.0	550.0	412.5	330.0	275.0

The above table shows that for quite moderate eccentricities the natural frequency of torsional vibration is considerably greater than the frequency of transverse vibration for the same load applied centrally.

In order to verify the above formulæ, experiments were carried out on two eccentrically loaded I-section beams. These were supported between the centres of a six-foot lathe, one end being secured in the chuck, while the eccentric load was applied at the free end. The system was set vibrating, the number of vibrations over a measured interval of time

\* See Morley, *Strength of Materials*, p. 398.

being noted. The effective polar moments of inertia of the beams were previously determined by measuring the angle of twist on a measured length corresponding to a known torque.

The results of the tests are tabulated below, the frequency as calculated from the *geometrical* polar moment of inertia being tabulated for comparison with the *actual* frequency of vibration.

TABLE III.  
I-section beam 4·78" deep × 1·75" flange, span 57·8".

Moment of inertia of eccentric load in inch lb. units.	37·3	52·7	67·2	83·1
Calculated frequency per minute, using <i>geometrical</i> polar moment of inertia J.	542·5	457·5	406·1	364·0
Calculated frequency per minute, using <i>actual</i> polar moment of inertia J'.	171·9	145·0	128·2	115·5
Observed frequency per minute	165·2	139·2	123·0	113·0

TABLE IV.  
I-section beam 3·01" deep × 3·00" flange, span 57·8".

Moment of inertia of eccentric load in inch lb. units.	57·5	74·1	90·7	107·0
Calculated frequency per minute, using <i>geometrical</i> polar moment of inertia J.	366·0	321·5	291·2	268·5
Calculated frequency per minute, using <i>actual</i> polar moment of inertia J'.	181·2	159·5	144·3	132·6
Observed frequency per minute	176·5	155·5	140·5	129·2

Tables III and IV indicate that the observed frequency of vibration is in close agreement with the frequency as calculated from equation (5). Comparison between the actual frequency and the erroneous frequency as obtained by using the geometrical polar moment of inertia is instructive, as showing the large discrepancy that exists between these two.

The above demonstration has been based upon the supposition that the relationship between torsional stress and torsional strain is a linear one, *i.e.* that stress is proportional to strain. Investigation shows, however, that this law only holds for a small range of stresses, and that strain increases more rapidly than stress. Nevertheless, the above formulæ indicate the frequency of application of a periodically applied force which would *initiate* vibrations in an eccentrically loaded beam.

The investigation leads to the conclusion that beams of commercial section, when loaded non-centrally, may have a period of vibration commensurate with that of machinery in operation.

IV.—The Origin of Oil-Shale. By E. H. Cunningham-Craig,  
B.A., F.G.S. *Communicated by* Dr HORNE, F.R.S.

(MS. received December 1, 1915. Read December 20, 1915.)

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I. INTRODUCTORY.

GEOLOGISTS who have made a special study of petroleum and the conditions under which it occurs have not infrequently to deal with the kindred subject of oil-shales. These subjects, it is true, have always been considered as entirely separate and different, and for all practical purposes they are so, since the mining of oil-shale and the drilling for oil have nothing in common from the engineering point of view; yet the final products of the distillation of natural petroleum and oil-shale are to some extent identical, and it has more than once been suggested that some relation may exist between an oil-rock and an oil-shale.

It is with the idea of reviewing the evidence bearing upon this point that this paper has been written; it must be considered merely as an inquiry, dealing chiefly with geological field evidence, as to whether any light can be thrown upon the origin of oil-shales by a study of petroleum in its broadest and most comprehensive sense. Theoretical disquisitions will be disregarded as far as is possible, and only evidence published by geological surveys and acknowledged authorities, or gleaned from the writer's personal observations in different parts of the world, will be brought forward.

*Definitions.*

To begin with, it is necessary to define the terms which will constantly be used. This presents some difficulty owing to the many varieties of deposits which have been or are being mined and retorted for the extraction of oil. To define an oil-shale as "any rock which yields petroleum on distillation" is not sufficiently precise, as it would include not only oil-sands and the bituminous shales of oil-fields, but also nearly all coals and lignites, bastard coals, "rums" of the Fife coal-fields, natural asphalts, and intrusive bitumens of the series from gilsonite to grahamite.

It is evident that, to exclude coals and lignites, the nature and percentage of the inorganic or inert portion of the deposit must be taken into account. Again, to exclude strata impregnated with crude petroleum, the fact must be emphasised that the oil cannot be obtained by trituration of the rock or by the action of solvents such as petrol or carbon-disulphide.\* To exclude asphalts and solid bitumens, emphasis must be given to the fact that an oil-shale is a stratigraphical deposit, part of a normal geological series.

We arrive then at a definition which, if somewhat clumsy, has at least the merit of being fairly precise:—*An oil-shale is an argillaceous or shaly deposit from which petroleum can be obtained by distillation, but not by trituration or treatment with solvents.*

Even with this definition the distinction between bastard coals, coaly shales, cannel coals and torbanite, and true oil-shales is a very fine one, since a series of more or less intermediate deposits can be brought forward, bridging the gulf between coal and oil-shale.

In practical work, however, a rough-and-ready distinction can easily be made, depending on the percentage of inorganic material present: if a deposit difficult to classify contains too high a percentage of ash to be worked as a coal, and yet yields on distillation a sufficient percentage of oil to be worked as an oil-shale, it must be regarded as an oil-shale; otherwise it may be classified as an impure coal. This still leaves the abnormal deposit torbanite in an equivocal position, with its average of 21 per cent. to 24 per cent. of ash. The true nature of this deposit will be dealt with specially in a later section.

An oil-rock is much more easily defined: it is—*Any rock or deposit impregnated with natural petroleum, which can be extracted by disintegration of the rock or by the action of solvents.* This definition is so wide that it will include not only an intrusive igneous rock which has distilled, from the strata it has passed through, a proportion of mineral oil, and has

\* Broxburn shale yields only 2·04 per cent. soluble in carbon-disulphide, *i.e.* "bitumen."

solidified containing it, but also a recent beach, river alluvium, or other superficial deposit impregnated with petroleum from underlying strata; but in practice it is usual to exclude such cases and to consider an oil-rock as a member of a regular stratigraphical series. Be it noted, however, that it is the impregnation that makes the oil-rock and no other special characteristic of the rock itself, since the same stratum may be oil-bearing, water-bearing, and "dry" within a very short distance.

The problem with which this paper deals may be stated shortly thus: "Is there any essential relationship between oil-shales and oil-rocks as defined above?"

### *Theory of the Formation of Oil-Shale.*

The distinction between oil-rocks and oil-shales set forth above is expressed more briefly by the statement that oil-rocks contain petroleum, and oil-shales contain "kerogen." Kerogen is a term first used by Professor Crum Brown to denote the material which, though not being petroleum itself, yields petroleum on distillation. It is a very useful term, the precise significance of which, however, must be left to the chemist. It will be shown later that, even although we may not be able to state in so many words the actual composition or constitution of kerogen, some evidence can be obtained as to how it has been formed.

In discussing the origin of oil-shales it is chiefly the origin of kerogen that has been considered, possibly to the exclusion of equally valuable evidence.

Though no very definite theory has perhaps been formulated as to how the kerogen has been formed, D. R. Steuart's work, published in Part 3 of the Geological Survey memoir on *The Oil-Shales of the Lothians*, has been generally accepted as summing up what is known upon this subject, and pointing to a fairly definite conclusion, if not actually advancing demonstrative proof. It will suffice to quote a few paragraphs:—

"In the oil-shale, as in the Torbanehill mineral, paraffin and paraffin oil do not exist as such: they are created by the destructive distillation in the retorts. There is very little in shale soluble in petroleum spirit, benzene, carbon-disulphide, ether, and such solvents. Our substance is therefore not of the nature of petroleum, bitumen, or resin. All hydrogen and carbon compounds produced by inorganic reactions are soluble in these solvents, and this makes it almost certain that kerogen is of organic origin.

"The material has probably been deposited, together with clay, at the bottoms of lagoons, and has there been subjected to maceration and limited microbe action. Part would decompose, and only what could withstand the



action of water, etc., would remain. We can imagine a plant or plant organ decaying and leaving only the wax or fat that was originally meant for its protective covering, or perhaps some resinous excretion or secretion. But this would give us materials soluble in our solvents, and might account for the origin of petroleum or bitumen, but not for kerogen.

"Some shales are largely made up of entomostraca, and it is probable that the animal matter has in some cases been converted into kerogen. We can imagine kerogen being produced from any kind of organic matter by the action of microbes under special circumstances, the product being dependent on the microbe. Or, on the other hand, kerogen in some cases may be the remains of certain kinds of vegetable matter, like pine pollen or lycopod spores, perhaps little altered, the product being dependent on the nature of the original organic matter.

"As peat gives paraffin products on distillation very like those of shale, it is probable that shale contains ordinary vegetable or other organic matter that has undergone decay to substances of a humic acid nature, which have been rendered insoluble and preserved by chemical combination with the metallic oxides of the clay or water, the alumina, lime, etc.

"Oil-shale, therefore, may be composed of (1) vegetable matter which has been made into a pulp by maceration in water and preserved by combining with the salts in solution as already mentioned; (2) richer material of many kinds, such as spores, which nature has provided with means of protection against decay; and (3) a proportion of animal matter."

Comment upon this passage is hardly necessary: the accumulation of vegetable material in lagoons, especially such vegetable matter as leaves, may be observed in any favourable locality, but this hardly explains why a carbonaceous shale may be formed in one case and an oil-shale in another.

There are some very fruitful suggestions in the quoted passage, but the proposition is left more "suggested" than "proved."

Other suggestions as to the origin of kerogen need not be considered at length. Professor A. C. Seward and others incline to look towards fresh-water algæ as the source of the organic matter, and suggest that a structureless jelly-like mass might be formed by the decomposition of such organisms. Spores are also suggested, but the absence of resinous compounds in kerogen is against this, and Professor Seward even hints at the possibility of the carbonaceous matter having an inorganic origin.

Mr de Salis suggests that the heat of intrusive rocks may have caused a natural distillation of hydrocarbons from bituminous seams in the Coal Measures, and that the distilled material may have collected in the shales. This theory is interesting since igneous rocks occur in the Scottish Oil-Shale

group, in the oil-shale province in South Africa, and in the neighbourhood of the New South Wales oil-shales. The suggestion, however, only carries the difficulty a step further. Such natural distillations are well known locally, but the concentration of kerogen into seams or shale-beds has to be accounted for as well as the bitumen in the original seams of coal or shale. A more widespread action than any local distillation is required to have caused the great development of oil-shales as known and worked in Scotland.

Professor Quekett, in discussing the origin of torbanite, made one pregnant suggestion in his conclusion that vegetable fossils in the seams of oil-shale or Torbanehill mineral are "accidental," and not necessarily connected with the presence of kerogen.

## II. FIELD EVIDENCE.

### (a) *From Oil-Shale Fields.*

It is necessary now to consider all relevant geological field evidence to ascertain whether any widespread conditions may be taken as favourable to the occurrence of oil-shales, to treat oil-shales not necessarily as separate phenomena *sui generis*, but to learn whether there are any essential associations or modes of occurrence which are typical of oil-shale fields.

Most of our knowledge of oil-shales is derived, perhaps unfortunately, from Scotland. Unfortunately, because in the Scottish shale-fields there may be many conditions which do not obtain in other countries where oil-shales exist, and we may be in danger of regarding as essential matters which are purely accidental, if we only study the Scottish fields.

1. *The Lothians.*—The oil-shales of Scotland occur in the Carboniferous formation, and more especially in that division of it known as the Calcareous Sandstone series.

Thin oil-shales are first seen in the Wardie shales, where there are also thin and unworkable seams of coal. As we ascend in the series, oil-shales become more frequent and thicker, but workable coals do not begin to appear till towards the top of the Oil-Shale group. Above this group, though coals become much more frequent, shale seams are fewer and poorer and generally unworkable; while in the Coal Measures proper, though a few thin oil-shales are known, they are not worked, either on account of poorness or because they are not thick enough. Thus, taking the formation from the horizon where workable oil-shales first appear, we have a shale-bearing group with occasional coals passing up into a coal-bearing group with shales dying out completely towards the top. In other words, dealing

with kerogen and coal, we find the kerogen typical of the lower part, the coal of the upper, with a transition period characterised by both.

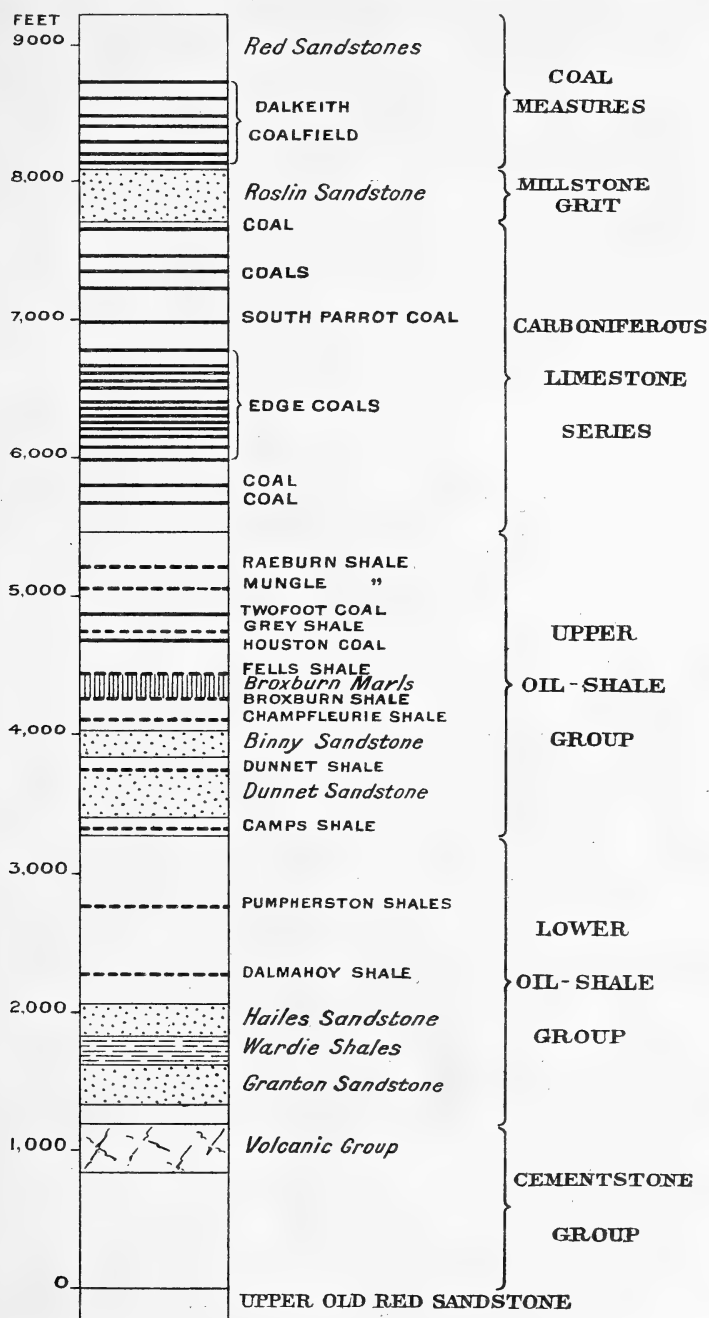


FIG. 1.—Vertical Section of the Carboniferous Formation of the Lothians.  
(From H.M. Geological Survey memoir.)

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The conditions under which the series accumulated were apparently not greatly different: an estuarine and deltaic phase with lagoon and even terrestrial conditions prevailed, broken only by the occasional marine phases which are so typical of all coal-fields from Carboniferous to Tertiary.

Speaking broadly, the Carboniferous areas in Britain are in the form of large and often irregular basins among older strata—broadly synclinal areas, in fact; while in the corresponding anticlinal areas the Carboniferous rocks have been removed by denudation.

Oil-shales in Scotland, however, are not only confined to certain groups in the Carboniferous formation, but to certain regions only where these groups are developed. Thus, though the Coal Measures persist through the west of Scotland, in the Lanarkshire and Ayrshire basins the Calciferous Sandstone series does not contain oil-shales. It is in fact almost entirely in the Lothians that oil-shale has been worked in Scotland, and even in this restricted area the Oil-Shale group does not contain workable seams throughout.

The workable shale-fields are divided by the broad anticlinal area of the Pentland Hills, which, running north-east and south-west, cut off the Midlothian coal-field from the greater shale-fields to the north-west. On the north-western margin of the Midlothian coal-field, a steep and narrow syncline, shale seams have been worked near Straiton and Carlops, where the strata are vertical or dipping very steeply. Where the same strata emerge on the eastern and south-eastern side of the syncline there are no oil-shales.

The broadly anticlinal ridge of the Pentlands, though very steep on the south-eastern side, is much more gently inclined on the north-western, and here a great triangular area from Blackness to Tarbrax and from thence to Dalmeny is characterised by the presence of oil-shales. To the westward this area is bounded by the overlying Carboniferous Limestone series, beneath which workable oil-shales die out. Thus only a narrow strip on the steep flank of the Pentland anticline and a broad area on the gently inclined flank furnish shale-fields. The broad triangular area is much complicated by minor folds, and faults of later date, sharp anticlines, and dome structures of no great size are frequent, and igneous intrusions of Carboniferous age, both sills and irregular masses, are numerous.

A somewhat more detailed account of the flexures is called for, and for this purpose a quotation from the memoir on *The Oil-Shales of the Lothians* will suffice:—

“The flexures are usually normal, but in a few cases the strata become vertical or slightly inverted.

"These folds were established long before the formation of the great east and west faults, and there are three main anticlinal ridges which, with their complementary troughs, can be traced right through the shale area across the faults. . . .

"The Dechmont Arch begins a mile north-west of West Calder, and

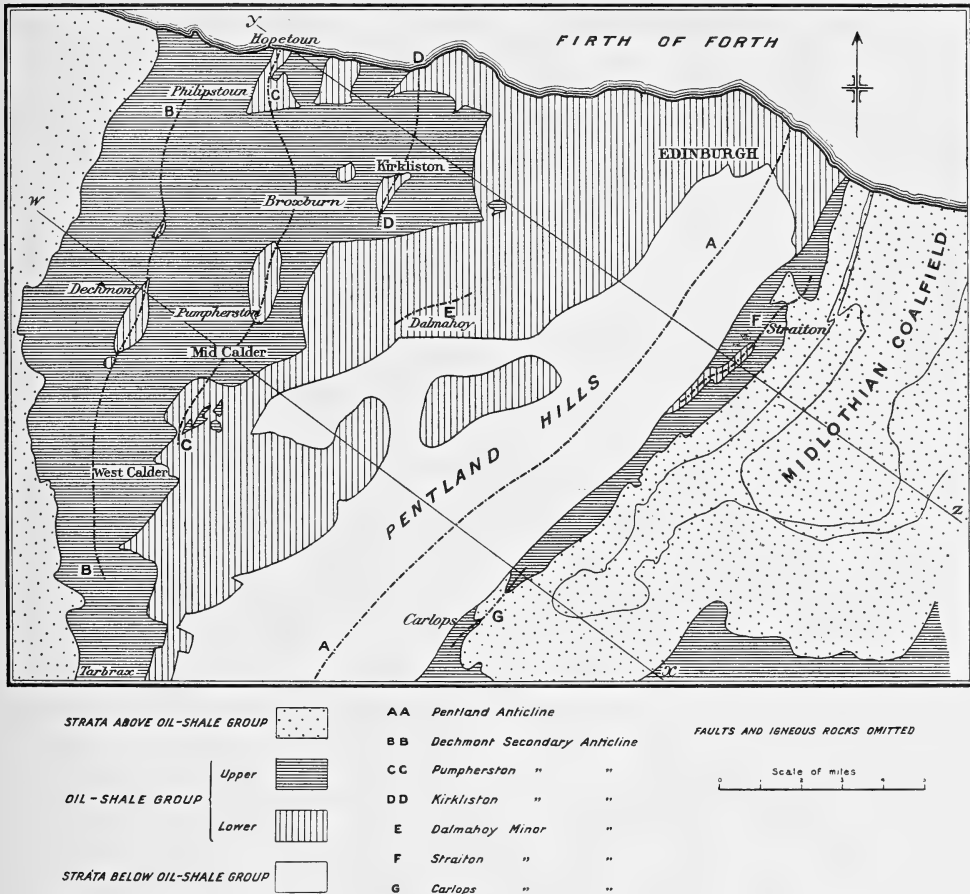


FIG. 2.—Oil-Shale Fields of the Lothians. (Reduced and much simplified from the maps of H. M. Geological Survey.)

runs N.N.E. past Cousland, Barracks, Dechmont, West Binny Reservoir, and Hangingside, ending on the west side of Philipstoun.

"The Pumpherston Arch begins south-west of Mid Calder, and passes northward through Pumpherston, Broxburn, and Winchburgh, reaching the sea-coast at Hopetoun. . . .

"The Kirkliston Arch begins some distance south of Kirkliston, and

can be followed northwards through Humbie to the sea-coast west of Port Edgar, and even across the Firth of Forth to Rosyth. . . .

"Other folds are also developed in the Fife shale-fields, but as they are not widespread they call for no attention here. One exception should be made, however, in favour of the broad anticline at Burntisland, which, as Sir Archibald Geikie long ago indicated, is essentially a prolongation of the Pentland Hills axis.

"A complex series of minor undulations lie amongst these major folds."

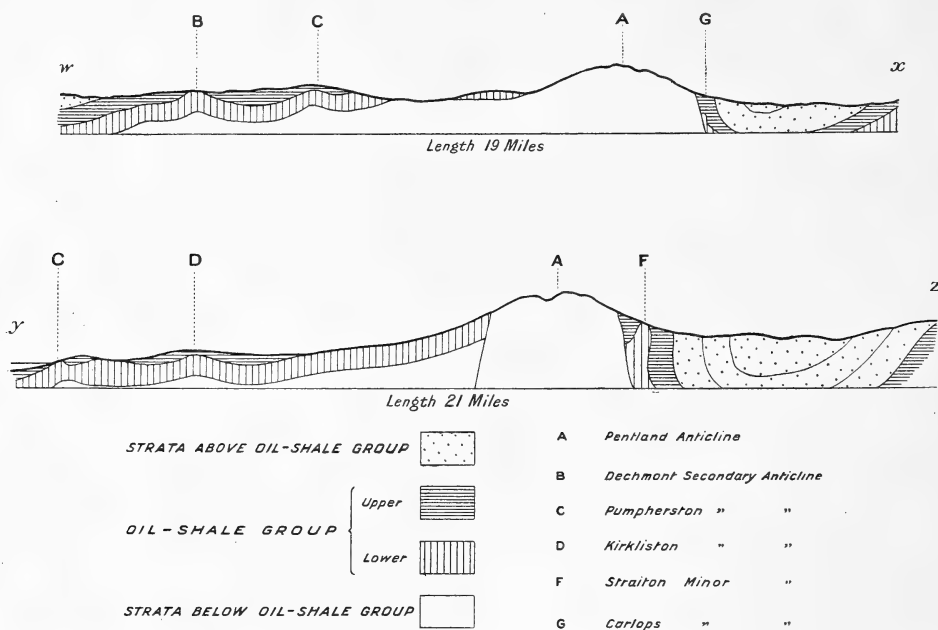


FIG. 3.—Diagrammatic Sections through the Lothians Oil-Shale Field.

These major folds are indicated on the map, fig. 2, by letters and dotted lines AA, BB, CC, and DD. Minor flexures to which I wish to draw attention are seen at Dalmahoy (E), Straiton (F), and Carlops (G), the two latter being merely wrinkles on the steep flank of the Pentland anticline. Oil-shale has been mined on these minor folds, and also at Burntisland, where, however, the shale is not very rich.

In connection with the flexuring, Mr Carruthers, in the memoir above quoted, makes two very significant statements as follows:—"By means of these undulations the oil-shales are spread over a wide tract of country, and the mineral wealth of that district has been greatly increased." "The most striking feature of the West Lothian field, however, is the system of folding, which may be said to characterise the whole area, *differentiating*

*it from any other Carboniferous tract in Scotland*" (the italics are the writer's).

In England, where the Lower Carboniferous rocks are as a rule of marine rather than estuarine and deltaic type, oil-shales are absent. The significance of these facts will appear later.

As regards the oil-shale itself, it varies somewhat in physical character. It is described in the Geological Survey memoir as "a fine black or brownish clay shale, with certain special features which enable it to be easily distinguished in the field." The chief of these characteristics are a brown streak and a toughness and resistance to disintegration by the weather. It is distinguished from ordinary carbonaceous shale—known by the miners as "blaes"—by the latter being far heavier, brittle, and often gritty, while it disintegrates into fragments when exposed and finally "reverts to its original condition of clay or mud."

A good oil-shale, on the other hand, resembles "hard dark wood or dry leather," and it can be "cut or curled up with the edge of a sharp knife." It is "free from grittiness, and is often flexible as well as tough." "In internal structure oil-shale is minutely laminated."

A distinction is sometimes drawn between "plain" and "curly" shales, the latter being contorted or curled and glossy on the surface of bedding planes, while the former variety is flat and smooth. There does not seem, however, to be any essential difference between the plain and curly varieties, and a seam occasionally consists partly of plain and partly of curly shale.

These descriptive notes are taken from the Geological Survey memoir—the recognised work on Scottish oil-shales.

The most significant point with regard to oil-shale is that it may graduate upwards or downwards into bituminous "blaes" and ordinary "blaes" so insensibly that it may be impossible to draw a dividing line between them. Shale seams also, when traced for a distance, may pass into ordinary carbonaceous shales or blaes, and thick seams may contain bands of barren blaes or ribs of hard calcareous ironstone or cementstone.

The roof of a good shale seam is usually impervious blaes, though limestone, sandstones, and "fakes" (sandy shales) may occur as the floor. As a general rule, however, the oil-shale seams are associated with argillaceous rocks, and may be considered as a special modification of the prevailing shaly type of sediment.

Another very significant point of which mention is made in the

Geological Survey memoir is that as a general rule the yield of oil of a shale seam decreases with the depth. This is stated to be very marked in certain cases—*e.g.* in the Linlithgow district in the case of the Broxburn seam,—distinct but not so marked at Broxburn, and much less apparent at Oakbank. It is also stated that this decrease in yield with depth is most noticeable in the north, and less apparent towards the south.

A series of laboratory results from analysis of samples from an incline on the Broxburn shale shows a decrease in the yield of gallons of oil per ton from 31·8 near the outcrop to 13·5 at a depth of something over 1000 feet. It is noted also that at a sharp bend or flexure in a seam the yield of oil often decreases. The possible cause of these phenomena will be dealt with later.

One other characteristic of the Scottish shale-fields requires to be mentioned, and that is the fact that evidence of crude petroleum is not wanting in the strata associated with oil-shales. Natural gas, liquid petroleum, elaterite, ozokerite, and albertite are all known in the shale-fields. The solid minerals usually occur in or near some intrusive igneous mass, where it can be proved that the heat of the intrusion has caused local distillation. In such cases cavities which may be in the solid igneous rock, within a fossil, or on joint faces may be found filled with liquid, plastic, or hard and solid hydrocarbons. Such occurrences are frequent where either coal-fields or shale-fields are invaded by intrusive igneous rocks—*e.g.* in the Karoo and in some parts of the Drakensberg in South Africa,—and they have no particular significance.

But the facts that natural gas, odourless and combustible, has issued from boreholes in the Broxburn district, and that a constant oozing of petroleum and brine was encountered in the Sandhole Pit at a depth of some ten fathoms below the Dunnet Shale, are important. In this case the oil was collected for more than a year, and some 200 barrels were obtained. The oil had the green fluorescence so characteristic of natural petroleum, and a specific gravity of ·830. On analysis it yielded:—

Petroleum spirit . . . . .	10·2
Kerosene . . . . .	34·1
Lubricating oils . . . . .	27·2
Solid paraffin . . . . .	12·5
Loss on analysis . . . . .	16·0

This oil differs in some points from the oil distilled from the shales, notably in the percentage of petroleum spirit; but in the high percentage of solid paraffin it resembles the shale-oil. The loss in analysis is unfortunately very high, making it difficult to compare the oil with other crude petroleum. The oil being associated with brine is another link with normal oil-field conditions.



To sum up the special characteristics of the Scottish oil-shale fields to which attention is called, the points to be noted are :—

- (1) The stratigraphical relations of the oil-shale fields to the coal-fields.
- (2) The fact that oil-shales are a special and not a universal feature of the stratigraphical horizons on which they occur.
- (3) The general and the minor structures of the oil-shale areas, the seams occurring in nutidinal areas, and on the crests and flanks of minor anticlines.
- (4) The decrease in richness of shales with depth in certain cases, *i.e.* down the flanks of the flexures.
- (5) Association of oil-shales with argillaceous deposits, especially as roofs to the seams.
- (6) The passage of oil-shales into normal carbonaceous shales.
- (7) The association of oil-shales in certain cases with phenomena characteristic of oil-fields.

It is now necessary to turn to other countries to glean evidence as to the nature of oil-shales and their stratigraphical relations. For this purpose it is proposed not to consider all known occurrences of oil-shale, but to select certain countries and areas where the writer has had special facilities for observation.

2. *South Africa*.—In the Karoo system, of Permian to Jurassic age, there are numerous seams of what under our definition must be classed as oil-shale. They occur chiefly in the eastern Transvaal and Natal in what is known as the Stormberg series, near the top of the Karoo system and beneath the great volcanic beds of the Drakensberg.

The Karoo system has been variously estimated at from 15,000 to 19,300 feet thick, the upper 4000 feet consisting of the volcanic beds of the Drakensberg. It is in the Stormberg series and in the uppermost beds of the Beaufort series that the well-known Coal Measures of the Transvaal, Natal, and Cape Colony occur, and within the confines of the coal-bearing territory oil-shales have been detected in many localities. The strata of the Karoo system in this region spread with almost perfect horizontality over thousands of square miles, but the plateau being deeply trenched by valleys and carved into great escarpments allows the series to be studied to great advantage. Dikes and sills of dolerite are numerous, but do not cause much disturbance of the bedding except merely locally.

Speaking generally, the Stormberg series is of littoral type, and the rocks are chiefly arenaceous. The coal seams have usually both floors and

roofs of sandstone, but shale beds are fairly numerous, and it is in association with these shale beds that the oil-shales usually occur.

The first point that strikes an observer familiar with the oil-shales of Scotland is the dissimilarity of the South African oil-shales from those of the Scottish fields; indeed, it is often difficult to believe that they can be oil-shales at all till confronted with the results of analysis. As a rule the type approximates to cannel coal or carbonaceous shale, and the writer has only seen one seam, and that unfortunately a thin one, that is "curly," with the horny fracture and glossy surfaces familiar in Scotland. But the streak of these oil-shales is brown and the percentage of volatile matter high. The curly seam above mentioned has yielded from 60 to 85 gallons of oil per ton. Some seams in Natal may be described as black carbonaceous micaceous shales, which pass upwards and downwards into ordinary dark shales, but yet which contain a high percentage of volatile matter.

At one locality not far from Middelburg in the Transvaal a thin seam of oil-shale lies above and in contact with a thin seam of coal: it has yielded 40 gallons of oil per ton. In the Umkomaas and Hlatimbe valleys in Natal thick coaly shale seams, probably upon much the same horizon as the Indwe coals of Cape Colony, have yielded as much as 27 gallons of oil per ton.

It is, however, needless to repeat instances of oil-shale outcrops that have been observed and often traced over wide areas, with but slight variation. The points to be noted are:—

(1) That the geological structure is completely different from that of the Scottish shale-fields, there being no folding or faulting, but the beds lying in horizontal sheets.

(2) That the oil-shales are a feature of the Coal Measures, and are even in cases closely associated with coal seams.

(3) That the series is largely arenaceous, though the oil-shale beds are associated with the more argillaceous groups.

(4) That sudden and rapid variations in the shale seams are rare.

(5) That the types of shale are very different from the oil-shales of Scotland, more nearly approximating to cannel coal.

3. *New South Wales*.—The following particulars with regard to the oil-shale fields of New South Wales are taken from J. E. Carne's memoir on *The Kerosene Shales*.

The shales belong to the Permo-Carboniferous formation, which in the shale-field consists of a lower marine group, the kerosene shale group, characterised by coal seams and shale beds, and an upper group of Coal Measures, the whole overlaid by a thick mass of Triassic sandstones.

Owing to overlap, in some districts the kerosene shale group forms the base of the series lying unconformably upon older rocks. Fresh-water, deltaic, and estuarine conditions are strongly marked in the shale group.

The general geological structure is almost horizontal over a very large area. There are, however, gentle undulations with dips seldom exceeding a gradient of 80 or 90 feet per mile.

In parts of the Capertee Valley, in Hartley Valley, and in several other localities the "kerosene shales" appear to be best developed towards the upper part of gentle monoclines and on the crests of the gentle undulations, and occasionally, though not always, decreasing or disappearing in the gentle troughs.

The oil-shale is very closely associated with coal. Thus a seam frequently consists of a thin bed of impure coal at the base, a band of the shale, and another band of coal at the top. Fireclays, pipeclays, barren shales, and thin bands of ironstone occur above the shale seams, and the thickness of close-grained or impervious strata above the seams is often large.

Such a section as the following is typical:—

Cannel coal	.	.	.	4	inches
"Kerosene shale"	.	.	.	1 foot 8	"
Cannel coal	.	.	.	3	"
"Kerosene shale"	.	.	.	5½	"
					<hr/>
2 feet 8½ inches.					

The workable shale seams are said to be lenticular, and they pass laterally into coals and earthy carbonaceous shales.

The shale itself varies very greatly in quality. It is distinctly laminated, but has a conchoidal fracture, a brown or yellowish streak, and a distinct lustre. It resembles cannel coal in many particulars, and contains vegetable fossils. The richest varieties have a specific gravity as low as 1.008 and a percentage of ash lower than 10, but the amount of inorganic matter is subject to great variations even in different parts of the same seam, and is as high as 56 per cent. in some instances.

The yield of oil in the retorts is very high from the best shales, reaching as much as 160 gallons per ton, but the yield of ammonium sulphate is low as a rule.

There is at least one case recorded of a shale having burnt at outcrop, a phenomenon usually associated with bituminous shales in oil-fields or with the transition stage between the petroliferous and carbonaceous phases.

Thus it is seen that the New South Wales shale-fields have much in common with those of South Africa, both in geological structure and in the

nature of the shales, but that there is little in common with the Scottish shale-fields.

The points to be noted are :—

(1) The occurrence of shales of a cannel coal type closely associated with coals.

(2) The occurrence of a more purely coal-bearing group at a higher horizon.

(3) The passage of oil-shales into carbonaceous shales and impure coals.

(4) The association of the oil-shales with impervious strata, especially above the seams.

(5) The almost horizontal structure.

(6) The occurrence of very gentle undulations where some of the richest shales are developed.

*(b) Evidence from Oil-Fields.*

It has been shown above that oil-shale occurs in very different environments in different parts of the world, and that it varies also in character, possibly according to its environment, though retaining some characteristics in common which we may fairly claim to be essential to its existence.

It now follows to describe briefly some of the essential features of oil-fields, to ascertain whether there are any points of resemblance between conditions typical of oil-fields and oil-shale fields.

In the first place, the stratigraphical relations of coal or lignite seams to oil-bearing rocks may be noted.

In any thick series of strata which contains both coal and lignite seams and oil rocks it is found that the upper zones are characterised by the carbonaceous phase and the lower by the petroliferous phase. This fact has been recorded so often that it is hardly necessary to give examples, but Virginia, Trinidad, Burma, Venezuela, Assam, and even Baku, may be quoted as instances.

It is not necessary here to enter into a discussion as to the origin of petroleum. It is known that oil can be formed from vegetable matter, *e.g.* from peat, and it has been suggested that kerogen has been formed from vegetable debris; but from whatever raw material petroleum may be formed, high pressure, the sealing of deposits by impervious upper strata, fairly low temperature, and the presence of at least a small quantity of water are conditions that must necessarily have obtained. This is not only established by geological field evidence, but is generally accepted by almost every writer on the subject of petroleum.

Any oil-rock to be of value must have a more or less impervious covering.

In examining, then, a thick series in which both the carbonaceous and the petroliferous phase are to be observed, we should expect to find thick impervious beds separating the two phases. In the Yaw River section in Burma and in many coast sections in Trinidad this holds good.

The strata of a thick series may be found to be of the same type throughout—rapidly alternating estuarine and deltaic sediments, with occasional marine bands,—yet the upper beds may be coal-bearing and the lower oil-bearing. The only differences in environment of which we can be sure in such a case are that the lower beds must have been subjected to greater pressure and a slightly higher depth-temperature.

Where thick impervious beds do not intervene between the two phases, we frequently find them overlapping, so that for a certain thickness of strata oil seepages and coal outcrops may alternate, and a bed may be both lignitic and petroliferous. This is to be seen to great advantage in Estado Falcon near the north coast of Venezuela. The lateral passage of a lignitic group into an oil-bearing group has also been described in Trinidad, Assam, and Burma. In such cases, and also in cases of the alternation of the petroliferous and carbonaceous phases, the former is found in argillaceous environment, and especially capped by impervious strata, and the latter in arenaceous environment.

These facts are sufficient to suggest that oil-bearing strata bear somewhat the same relation to coal-bearing strata that oil-shale-bearing strata do. It may be mere coincidence, or there may be something essential in the relation.

To prevent any misunderstanding it must be pointed out that if it be granted that petroleum and lignite or coal can be formed from the same raw material, there is evidence that once the lignitic or coal stage has been reached no conversion into oil is possible short of destructive distillation. Sections in the younger Tertiaries of South America and Trinidad illustrate this point. In one a lignitic seam is exposed on the foreshore with tree roots in position of growth in the underclay. The tree trunks are lignite in the seam, and parts of the roots are lignitic, but the greater part remains wood. A section near Caoderalito in the district of Buchivacoa in Venezuela supplies another link. The strata are dipping at 30° and are somewhat above the middle of a Tertiary series of about 8000 feet in thickness. In a band with somewhat confused bedding several flattened tree trunks are preserved as lignite, while the rest of the band is petroliferous and gives rise to a seepage of asphaltic oil. A thin underclay is beneath the band, while above it is some 10 inches of clay followed by 3 feet of oil-sand, the whole being surmounted by a thick argillaceous band. It is suggested that

in this case the tree trunks had become lignite before the pressure of superincumbent accumulating sediment had reached the stage at which oil formation could commence, but that once this stage had been reached the rest of the vegetable matter present had been converted into petroleum, the lignite remaining unchanged.

This section occurs in what may be called the transition stage; the series above is entirely lignitic, while below, though there are some lignitic bands, the petroliferous phase becomes increasingly pronounced.

Certain phenomena of natural filtration of petroleum and the weathering of oil-rocks must now be considered.

In examining a series for evidence of petroleum it is natural to seek first the most porous beds, as these may be expected to contain the greatest quantity of liquid hydrocarbons. It does not follow, however, that this is the most sensible course to pursue except under certain conditions. If a coast or river section is being examined, and the strata are almost entirely argillaceous and impervious, any bed of sandstone, however thin, will be examined for traces of oil; and if no porous bed be present, ironstone nodules and thin laminae or concretions of limestone will be examined. On breaking these a faint odour of inspissated petroleum may be noticed, or possibly bituminous stains or a film of dried-up petroleum on the joint faces may be observed. Thus where the mass of the strata is too impervious to contain distinct traces of an impregnation, any material slightly more porous may give evidence of having been impregnated by petroleum that has filtered slowly through the to all intents and purposes impervious surrounding rock.

But where the strata to be examined have been fully exposed to weathering agencies, say in an outlier, it will be necessary to examine the less pervious beds. Porous sandstones which may have been oil-sands may have been so thoroughly weathered and lixiviated that not a trace of petroleum can be detected. Beds of clay and sandy clay, however, which are sufficiently impervious to have only been impregnated very slowly, yield their contents of liquid hydrocarbons equally slowly, and may retain traces of petroleum long after all more porous beds have lost every such indication. This phenomenon can often be studied to advantage when an oil-bearing series lies unconformably upon an older series that is not oil-bearing in a primary sense, and where the younger oil-bearing series has since been removed by denudation. The more porous beds of the older series may long have lost all traces of a former impregnation, but the less pervious beds may still be yielding a little oil and gas in seepages, or may

contain the inspissated remnants of a saturation with oil. A notable case in point is the fine-grained rock of San Fernando Hill in Trinidad. This deposit is known as argiline, and, though the oil-bearing Tertiaries have long been removed from above it, at a depth of some 8 or 10 feet from the surface a slight impregnation with dried-up petroleum is very obvious, while more porous sandstones of the same (Cretaceous) series in the neighbourhood show not a sign of oil. Similar evidence both in Trinidad and Venezuela may have caused the Cretaceous formation in these countries to be considered as a primary oil-bearing series.

When oil-seepages are noticed at the surface, or oil-shows struck in a well in such very slightly porous but impregnated strata, it is remarkable that the oil is invariably lighter in gravity than that of the main supply of oil in the district. This is not so much due to the protection against inspissation that the less porous rock affords, as to the natural filtration to which any oil that has migrated through argillaceous or slightly porous rocks is subjected. This filtration removes wholly or in part the heavier fractions of the petroleum, and thus by analysis it is usually a simple matter to detect a filtered oil. The proportions of the other constituents in the petroleum remain relatively to each other much the same, but the "residue" and the heavier lubricating fractions, together with the solid paraffin (if the original or primary oil should contain that product), are greatly reduced, if not actually eliminated. The heavier molecules, as seems only natural, are less active in migration through capillary interstices, or perhaps more correctly have a much lower surface tension, and hence are outstripped by the lighter molecules.

This phenomenon of filtration of petroleum is very familiar to oilmen. A good instance of it is the water-white oil of Kala Deribid in Persia, which seeps slowly from a very compact and fine-grained shaly rock. Its original source is probably in sandstones of a higher horizon, which have many of the characteristics of oil-rocks, but now contain no trace of having been such beyond a little sulphur staining.

Perhaps, however, the most remarkable case of filtration is furnished by the Calgary field, where commercial results have unfortunately as yet failed to equal the results of scientific interest. A series of wells drilled in somewhat complicated structure within a distance across the strike of about half a mile have furnished a series of oils varying from one with 20 per cent. or 30 per cent. of residue to an oil with about  $2\frac{1}{2}$  per cent. of residue, 6 per cent. of lubricating oil, and 72 per cent. of petrol. The oil becomes lighter the further east it is struck and the higher in the geological series. The oils all give clear evidence of a common origin, but on their migratory

journey have left behind their more sluggish molecules in proportion to the distance travelled. The strata travelled through are very largely close-grained calcareous shales.

It will be seen later that similar filtration can be conducted experimentally.

Mention has been made above of the inspissation or drying up of natural petroleum. This process is probably somewhat complicated: it is accompanied by a certain amount of oxidation, and probably also by the formation of unsaturated hydrocarbons, but for all practical purposes we may look upon it as the loss by evaporation of the lighter constituents in the complicated mixture of hydrocarbons which is known as crude petroleum. Thus the butane and propane and such light saturated hydrocarbons are lost, and the residue becomes gradually richer and richer in the heavier members of the series, and more especially in the unsaturated hydrocarbons. At the same time, as will be seen later, the more highly inspissated a petroleum becomes, the richer it will be found in compounds containing oxygen, sulphur, and nitrogen.

The next phenomenon of oil-fields to be considered is the occurrence of intrusive solid bitumens or manjaks. The term manjak is used as the oldest term (dating from the seventeenth century) to include the whole series of minerals to which local names have been given in many parts of the world. Uintaite, grahamite, gilsonite, manjak, glance pitch, wurtzelite, and albertite are some of the names used, and they are classified according to their percentages of fixed carbon and their behaviour under the action of certain solvents.

The following table gives the characteristics of a number of these manjaks:—

Mineral.	Specific Gravity.	Bitumen, Percentage soluble in CS <sub>2</sub> .	Malthenes, Percentage soluble in Petrol.	Fixed Carbon.
Gilsonite . . . . .	1·04	93·4 to 99·5	35 to 72	3·3 to 26·2
Barbados manjak . . . . .	1·08	97·4 to 99·2	15 to 36	25
Egyptian glance pitch . . . . .	1·09	99·7	23·5	15
Trinidad manjak . . . . .	1·09 to 1·1	84 to 96·2	6·3 to 56	24 to 33
Grahamite . . . . .	1·16	94·1 to 98·2	4 to 3·3	41 to 53
Wurtzelite . . . . .	1·05	6·7 to 12·8	...	5·2 to 8·8
Albertite . . . . .	1·07 to 1·2	1·6 to 11·9	Trace to 3·2	29·8 to 54·2

It will be seen that there is a great deal of variation in the constants. This is due not only to there being different percentages of inorganic



impurities, but to many different analyses having been made by different observers, and also to the fact that samples taken from different parts of the same vein often show considerable differences.

But for the fact that albertite and wurtzelite are distinguished from the rest by the very large percentages insoluble in carbon-disulphide, the series shows a fairly complete gradation, the percentages of malthenes or petrolenes (*i.e.* soluble in petroleum spirit) decreasing as the specific gravity increases. The part of the bitumen insoluble in petroleum spirit is called the asphaltenes.

These "native bitumens" occur in veins almost invariably vertical or very highly inclined, though they send off minor branch veins along joint or bedding plains in any direction. Almost all the phenomena of igneous intrusion are simulated: sometimes there is a regular columnar jointing at right angles to the walls of the vein, and sometimes there are selvages with columnar jointing, while the centre of the vein, obviously a later intrusion, is characterised by conchoidal fracture. In such cases the conchoidal variety is always richer in malthenes or petrolenes and has a lower melting-point than the selvages. The material is truly intrusive, and has consolidated without reaching the surface. Weathered specimens taken near the surface are less soluble in petroleum spirit than specimens taken from greater depth; and in fact, though it is evident that the material must have reached its present position in a liquid or semi-liquid state, the process of inspissation is still in progress. In one vein in Marbella Mine, Trinidad, a soft variety of manjak exuded slowly from the centre of a vein; this material could be fractured by bending it sharply and quickly, but if allowed sufficient time it flowed slowly, adapting itself to every irregularity of the surface upon which it was placed. The malthene percentage was as high as 56 in this case.

1. *Barbados*.—In a typical Barbados manjak-field the relations of the solid bitumen to petroleum are admirably illustrated. The structure is anticlinal, but complicated by sharp minor folding in the lower strata (Scotland series) exposed. These strata are unconformably overlaid by the Oceanic beds, also thrown into anticlinal form, though much less pronounced, and the Oceanic beds are in turn unconformably overlaid by the Coral Limestone, which is horizontal. It is with the Scotland series alone that we are concerned. It is obviously in the petroliferous stage, every bed when unweathered having at least a distinct odour of petroleum. The upper beds are chiefly impervious clays, but sandstone beds become more frequent in the lower horizons. It is through these strata that the manjak is intruded in somewhat irregular veins, which vary considerably in thickness and give

off stringers here and there. The material has been mined to a depth of over 300 feet, and a vein has more than once been followed downwards till it breaks up into a network of fine stringers in argillaceous rock and finishes finally in a sandstone saturated with heavy, sticky petroleum. All the fine sandstone beds among the clays are more or less stained or impregnated with inspissated oil, and films of dried-up oil can be detected on joint and slip planes among the clays. There can be no doubt as to the origin of the veins; the manjak in a fluid or semi-fluid state has been forced under great pressure upwards from its source in the parent oil-sands below, and taking advantage of any plane of weakness has formed the main vertical veins

S.E.

N.W.

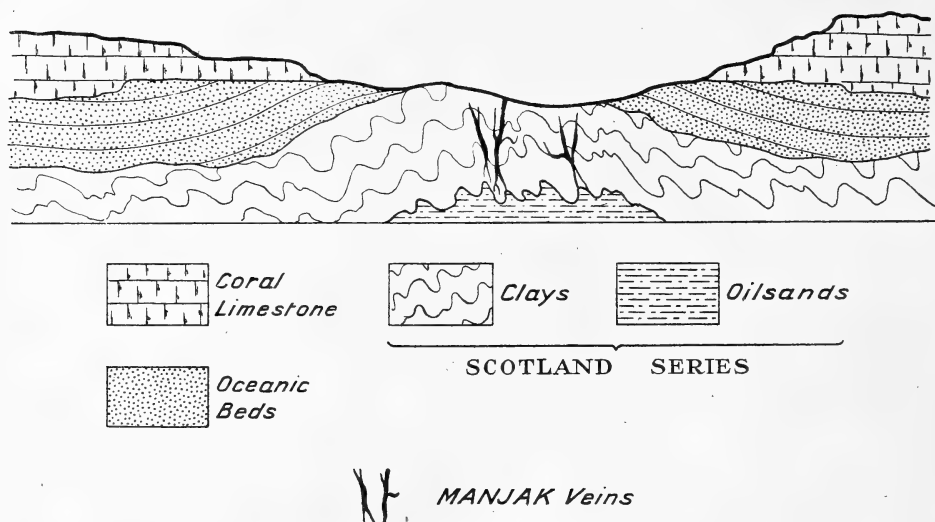


FIG. 4.—Diagrammatic Section through a Barbados Manjak-Field.

with their ramifying branches, and has gradually solidified and dried up or inspissated to its present condition. The manjak veins in Barbados all occur above the main oil-sands, and had there not been a practically impervious cover of argillaceous rock we may safely assume that there would be no veins of intrusive bitumen.

2. *Trinidad*.—In the principal manjak-field in Trinidad, the San Fernando field, the structure is somewhat different. Here we are dealing with the southern lip of a large basin, the inclination of the strata decreasing from  $40^\circ$  to horizontal as we proceed northwards. Along the southern lip of the basin there is an outcrop of oil-sand from which heavy inspissated oil can be collected in excavations. The strata above for some 800 feet are entirely

argillaceous, soft greenish and blue clays of great thickness and without any porous bands, though there are calcareous and ironstone nodules.

The main manjak vein runs in a northerly direction at an angle of approximately  $70^\circ$ . It is lenticular in shape, and reaches a thickness of 33 feet at about 180 feet below the surface. This is, I believe, the thickest vein of similar material ever recorded. There are also other veins in the neighbourhood, some running along the bedding for a considerable distance. The veins have not been worked deeper than 250 feet. The surrounding clays, though almost absolutely impervious to water, show on joint faces and slip planes spots and films of sticky oil, and the ironstone nodules

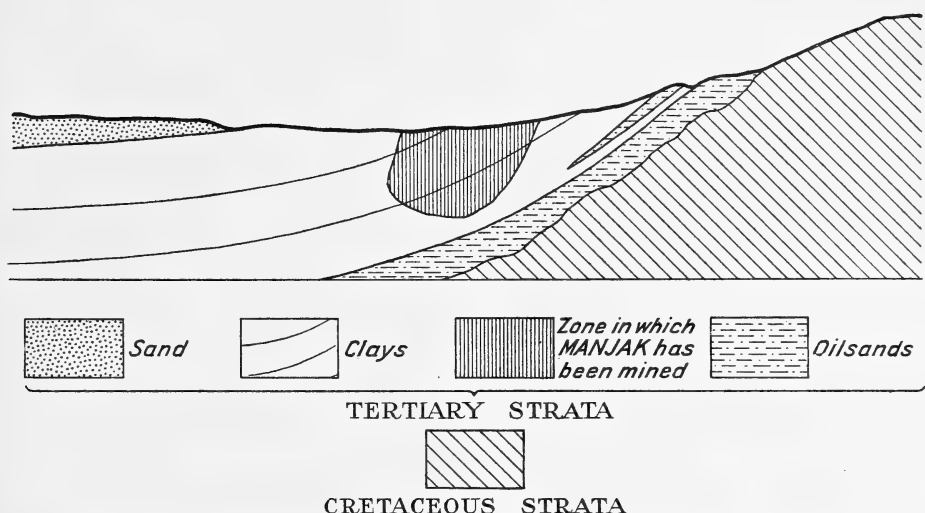


FIG. 5.—Diagrammatic Section through the San Fernando Manjak-Field, Trinidad.

when broken open show similar films of oil and a slight impregnation due to the material being rather more porous than the clay. Firedamp is encountered in the workings, and has in one instance caused a serious explosion.

It is evident from the nature of the strata and the size of the veins that enormous force must have been exerted to cause such intrusions.

These phenomena can be paralleled in many oil-fields where similar conditions obtain, *i.e.* thick argillaceous deposits overlying oil-sands either near the crests or on the flanks of anticlinal structures.

To recapitulate, the special points in connection with oil-fields to which it is wished to draw attention are:—

(1) The stratigraphical relations of coal and lignite seams to oil-bearing strata.

(2) The occurrence of transition groups in which both the petroliferous and the carbonaceous phases are in evidence.

(3) The fact that in such cases the petroliferous phase is always associated with more argillaceous and less pervious strata than the carbonaceous phase.

(4) The fact that when a petroliferous series is thoroughly exposed to weathering, the argillaceous and less pervious beds retain petroleum impregnations long after porous sandstones and limestones are deprived of all traces of petroleum.

(5) The fact that oil can be naturally filtered by migration through very slightly pervious strata, the filtration taking the form of the removal to a greater or less extent of the heavier and less volatile hydrocarbons, especially the unsaturated hydrocarbons.

(6) The nature of the process of inspissation.

(7) The occurrence of intrusive manjaks or "native bitumens" in argillaceous rocks overlying rich and porous oil-rocks.

(c) *From New Brunswick.*

We have now considered some of the characteristics of oil-shale fields and of oil-fields, and have noted certain minor points of resemblance brought out by a study of field relations. It remains now to bring into line and connect the various items of evidence by the study of an area where oil-field and shale-field conditions approach each other very closely.

For this purpose no better field could be selected than New Brunswick.

The oil-shales of New Brunswick have been known for many years, and have been examined by many competent geologists, but have not as yet been mined extensively. They belong to the Devonian formation, which lies upon an irregular surface of pre-Cambrian rocks in this region. It is, however, to one particular locality that attention is called, *i.e.* the Albert Mine, Albert County, whence albertite takes its name.

At this place there is an anticline of shales about 1000 yards long, rising from beneath an unconformable capping of arenaceous strata. The anticline is complicated by sharp minor folding, and the structure is very similar to that described in a Barbados manjak-field above. Almost but not quite along the crest of the anticline a very highly inclined vein of albertite was worked for many years for a distance of some 2800 feet along the strike, and to a depth of 1300 feet, while exploratory workings have been carried to a depth of a further 200 feet, and have found the vein breaking up into a network of fine stringers among shales, and finally reaching sandstone beds, where traces of semi-liquid bitumen were

encountered. There are no branch veins of any importance, but numbers of minute veins of an inch or less in thickness. The workings have been closed for some years, but several tunnels driven across the strike from the surface can be entered and examined so that fairly complete knowledge of the structure and the country rock can be obtained.

The shales surrounding this intrusion of albertite as far as it has been worked, and in every surface opening, outcrop, or exploratory tunnel, are practically *all oil-shales*. That is to say, a mass of strata 3000 feet in length by about 1000 feet broad, and proved to a depth of 1500 feet, is oil-

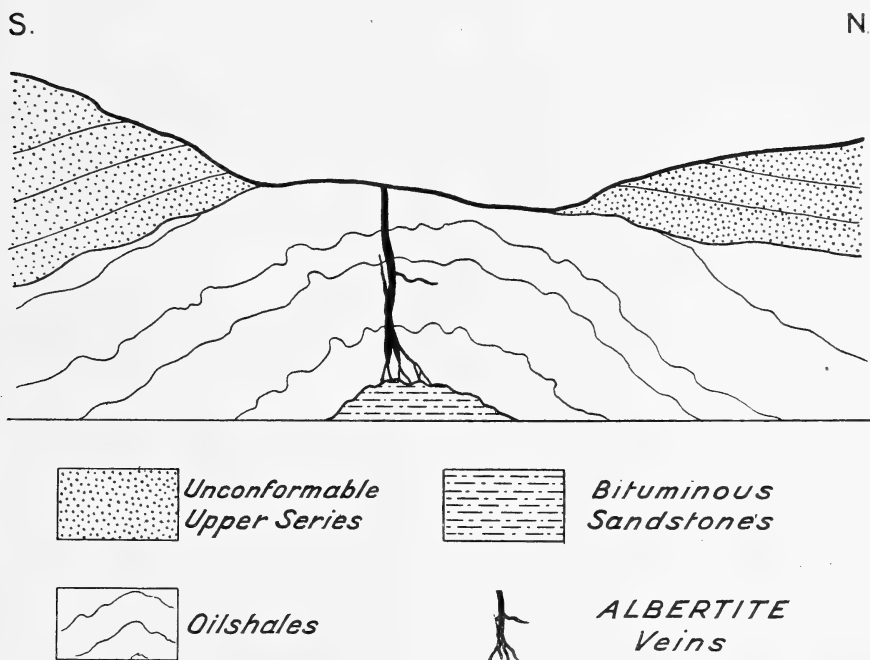


FIG. 6.—Diagrammatic Section through Albert Mine, New Brunswick.

shale—not shales impregnated with petroleum as in the case of Trinidad and Barbados manjak-fields, but true oil-shale full of “kerogen,” and with little or nothing soluble in organic solvents (2·3 per cent. is soluble in petroleum spirit). Though all the strata contain kerogen, some bands are richer than others, and yields of from 27 to 48 gallons per ton have been given in the experimental retort of the Pumpherson Oil Company. It is usual to distinguish two types in this mass of shales—the “paper shales” (a brown shale so fissile that it almost resembles mica), and a more massive and darker variety which shows faint “curly” structures. The latter type is distinctly sandier and more gritty to the touch, and it gives a higher yield

of oil per ton. It has obviously been originally a more porous rock than the "paper-shale."

The yield of ammonium sulphate is no less remarkable than the yield of oil, the experiments shown at the Pumpherstons works giving an average of nearly 77 lbs. per ton. Another series of experiments on shale from the Albert Mine give an average of 59.4 lbs. per ton.

These shales are therefore true oil-shales, though not closely resembling the Scottish shales, and they occur in environment typical, as we have seen, of oil-fields.

Very similar shales in the same series are exposed in several other localities in the district, though in less complicated structure. The beds in these cases are more clearly defined as separate seams, though often of great thickness. Borings for petroleum in the neighbourhood also have proved thick beds of oil-shale or bituminous shale, and have furnished much evidence to connect these occurrences with the phenomena of oil-fields. For instance, west of the Petit Codiac River, a few miles north of the Albert Mine, there is a gas-field where borings through this series supply natural gas in large quantities to the towns of Moncton and Hillsborough. Many borings have been made for oil, and small quantities—"shows" rather than productions—have been recorded near the Memramcook and Petit Codiac rivers: dark bituminous shales occur in all cases above the oil-sands.

In the Albert Mine itself only traces of petroleum impregnation were encountered in the joints of clay-ironstone nodules, and in sandy beds at the deeper levels.

From this field evidence we see that rich oil-shales may occur under conditions which have been shown to be typical of oil-fields, and we have additional instances of the connection between petroleum and oil-shale. The point to be noted specially is that in the case of the Albert Mine, where the anticlinal structure has no doubt caused at one time a great concentration of petroleum and a great intrusion of bitumen, all the country rock, and not only a few selected bands, contains kerogen to a greater or less extent.

### III. FILTRATION, ABSORPTION, AND ADSORPTION.

These facts are at least suggestive of an origin for kerogen, but there are still many problems unexplained. How, for instance, has kerogen been formed from petroleum? Why is one bed of shale an oil-shale while another is barren or carbonaceous? What determines the presence of crude petroleum in one district or formation, and kerogen in another?

And lastly, why can a commercial yield of ammonium sulphate be obtained from an oil-shale and not from an oil-rock?

To attempt to answer these questions we must have recourse to experimental work, and fortunately we find that there is no lack of experimental and laboratory researches bearing upon the subject.

Allusion has already been made to the natural filtration of oils: experiments in artificial filtration will furnish us with a clue to more than one of our difficulties.

Mr Clifford Richardson, the acknowledged authority upon asphalt, and the author of *The Modern Asphalt Pavement*, found that solutions of the bituminous matter in natural asphalt could be filtered and decolorised by continued filtration through clays, the heavier and darker-coloured constituents of the solution remaining in the clay, while the more volatile constituents passed through in solution. The material he experimented with was the bituminous content of Trinidad asphalt from the asphalt lake.

On treating the clay used for filtering with petroleum spirit it was found that the greater part of the bituminous matter could be recovered, but not all. A proportion remained in the clay which could not be extracted by solution, but which could be recovered by distillation. For this property of clay Mr Clifford Richardson proposed the term "adsorption," to distinguish it from the absorptive property. Thus in any clay saturated or in contact with petroleum a portion is *adsorbed* and can only be recovered by distillation. It must be left to the chemist to prove in what this adsorption consists—whether it be due to the formation of insoluble compounds with bases, or some other chemical action. The proportions absorbed and adsorbed vary, naturally, with different clays, but it is only the heavier molecules, the products of inspissation, that are thus removed from solution. This confirms what has been learnt about the natural filtration of petroleum. It is apparently chiefly the unsaturated hydrocarbons that are thus fixed in the argillaceous rock.

The asphalt lake of Trinidad is formed on the outcrop of the La Brea oil-sand, and may be considered as a highly inspissated petroleum. Its average composition is given by Mr Clifford Richardson as:—

Water . . . . .	29 per cent.
Bitumen . . . . .	39 "
Organic matter not bitumen . . . . .	7 "
Inorganic matter . . . . .	25 "

It is a very uniform mixture or emulsion.

The percentage of "organic matter not bitumen," *i.e.* not soluble in carbon-disulphide, is remarkable. In every inspissated oil or asphalt, in every intrusive or native bitumen, and in every specimen of oil-rock taken

at the surface there is a percentage of this insoluble material, and the percentage seems to depend upon the degree of inspissation and on the impurity in the shape of inorganic matter present, especially if the inorganic matter be of an argillaceous nature. Thus we have seen that a Barbados manjak of presumably Tertiary age may contain less than 1 per cent. of this insoluble material, while an albertite of Devonian age may contain 90 per cent.

Similarly, in a manjak vein from the College Mine in Barbados at a depth of 60 feet we find a distinct difference between the columnar selvage of the vein and the centre as follows :—

	Columnar Selvage.	Centre (conchoidal fracture).
Water and volatile matter at 100° C.	1·20	2·16
Ash . . . . .	·44	·44
Bitumen . . . . .	96·00	97·40
Organic matter not bitumen . . . . .	2·36	nil.
	100·00	100·00

That the state of inspissation is much higher in the selvage than in the centre of the vein is proved by the percentages of malthenes, which are 18·20 and 35·60 respectively. The analyses are by Professor Carmody, Government analyst of Trinidad, and the two samples are taken from within a few inches of each other. The conchoidal variety is probably a later intrusion.

Much similar evidence from manjak veins in Trinidad could be adduced, but the point is quite clear that as the material weathers, as shown by the decreasing percentage of malthenes, the percentage of insoluble matter increases.

A very pure asphalt formed by the inspissation of oil from a seepage in the Guapo field, Trinidad, analyses as follow :—\*

Water and matter volatile at 100° C.	18·50
Bitumen . . . . .	80·40
Non-bituminous organic matter . . . . .	·86
Ash . . . . .	·24
	<hr/> 100·00

The malthenes in this case amount to 71 per cent. The percentage of "organic matter not bitumen" cannot in this case be *adsorbed*, as the

\* This analysis and those that follow are by Professor Carmody.



inorganic matter is present in such minute quantity, but must have been formed during the process of inspissation.

In dealing with an asphalt consolidated at the surface, and an oil-rock taken at outcrop, the question of adsorption comes in, since any argillaceous material present will have adsorbed part of the inspissated heavy residues.

Thus three specimens of oil-sands from the Guayaguayare field in Trinidad (analyses by Professor Carmody) give the following interesting results :—

	Per cent.	Per cent.	Per cent.
Water . . . . .	1·320	3·28	4·56
Bitumen . . . . .	14·0	9·6	17·3
Non-bituminous organic matter . . . . .	3·44	5·68	10·02
Silica . . . . .	76·47	73·74	58·37
Fe <sub>2</sub> O <sub>3</sub> and Al <sub>2</sub> O <sub>3</sub> . . . . .	3·8	6·8	7·89
CaO . . . . .	·12	·06	·15
MgO . . . . .	·42	·38	1·04
K <sub>2</sub> O and Na <sub>2</sub> O . . . . .	·25	·35	·45
Sulphuric anhydride . . . . .	·09	·09	·08
Phosphoric anhydride . . . . .	·08	traces	·12
	100·00	100·00	100·00

Thus the oil-rock poorest in silica and richest in basis not only retains the greatest percentage of inspissated bitumen, but by far the largest percentage of “organic matter not bitumen.”

Another very greatly weathered oil-sand from San Fernando, Trinidad, shows on analysis :—

Water . . . . .	0·76
Bitumen . . . . .	4·94
Non-bituminous organic matter . . . . .	2·50
Mineral matter . . . . .	91·80
	100·00

A much more purely arenaceous and porous oil-rock from the Guapo field, Trinidad, gives on analysis :

Water . . . . .	2·60
Bitumen . . . . .	17·50
Non-bituminous organic matter . . . . .	·46
Mineral matter . . . . .	79·44
	100·00

In this specimen, owing to its more purely siliceous composition, there can have been but little adsorption.

The asphalt of the Trinidad pitch lake contains, as seen above, 25 per cent. of mineral matter. Microscopic examination shows this to consist of

fine sand and clay, and its chemical analysis is given by Mr Clifford Richardson as follows:—

	Total Mineral Matter.	The finest Mineral Matter, separated from the Sand as far as possible.
	Per cent.	Per cent.
SiO <sub>2</sub> . . . . .	70·64	32·36
Al <sub>2</sub> O <sub>3</sub> . . . . .	17·04	40·38
Fe <sub>2</sub> O <sub>3</sub> . . . . .	7·62	13·14
CaO . . . . .	·70	3·65
MgO . . . . .	·90	1·83
Na <sub>2</sub> O . . . . .	1·56	...
K <sub>2</sub> O . . . . .	·35	·53
SO <sub>3</sub> . . . . .	·97	1·18
Cl . . . . .	·22	7·16
	100·00	100·23

This composition evidently indicates a certain proportion of argillaceous material, so that we may regard the 7 per cent. of non-bituminous organic matter as being not only due to inspissation, but also partly adsorbed in the inorganic 25 per cent.

A specimen from the upper beds of the La Brea oil-sand, the parent oil-rock on the outcrop of which the lake has been formed, was taken from an exposure on the seashore near the lake, and gave on analysis the following interesting results:—

Water, etc., volatile at 100° C. . .	5·24 per cent.
Bitumen . . . . .	15·10    "
Non-bituminous organic matter . . .	29·70    "
Mineral matter . . . . .	40·96    "
	<hr/> 100·00    "

Of the bitumen only 8 per cent., a little more than half, was soluble in petroleum ether, *i.e.* could be classified as malthenes. This bears striking testimony to the state of inspissation of the rock, and to the probability of a large percentage of the heavy residue having been adsorbed.

This evidence of absorption and adsorption by argillaceous rocks, and of the effects of inspissation, points very clearly to the manner in which kerogen can be formed and preserved. Our definition of kerogen is "organic matter, not petroleum or bitumen, but which will yield petroleum on distillation," and on that definition we may claim that oils which by inspissation or adsorption have become insoluble in carbon-disulphide are in effect kerogen.

That petroleum can be yielded by the distillation of these inspissated and insoluble products has been shown so often that no further proof is

necessary. Even albertite, with 90 per cent. or more insoluble in carbon-disulphide, yields as much as 112 gallons of oil and 65 lbs. of ammonium sulphate per ton.

#### IV. PROPERTIES AND ANALYSES OF CLAYS.

If it be possible, then, that an oil-shale can be formed through the absorptive and adsorptive powers of clays for inspissated petroleums, it would be natural to expect that the mineral matter of an oil-shale should possess some characteristics, chemical or mechanical, which specially favour absorption and adsorption, characteristics which distinguish it from ordinary clay-shales. It was seen in the Scottish shale-fields that the gradation from a bituminous shale not rich enough to be mined and retorted with profit to an oil-shale is frequently almost insensible, and that the richness of a shale is subject to variations. Are these variations coincident with any variation in chemical composition or mechanical constitution? It would require a great number of chemical analyses and much experimental work to prove this conclusively, but there is a certain amount of evidence upon the point which can be brought forward.

In the first place, a comparison of specific gravities gives interesting results.

The average specific gravity of Scottish oil-shales from 58 samples taken from more than 10 different mines is 2.054. This is distinctly a low figure; but as the average contents of hygroscopic moisture and volatile hydrocarbons plus fixed carbon are 1.58 and 29.20 respectively, it is evident that to arrive at the specific gravity of the mineral contents we must increase the figure slightly. As we do not know in what chemical or mechanical combination the kerogen exists, it is impossible to make an accurate allowance; but if we suppose that it has a specific gravity similar to that of an inspissated petroleum or a native bitumen, we can make a rough approximate estimate. Such an estimate is of course open to many sources of error, but an estimate, however rough, will enable us to make some comparison between the specific gravity of the mineral matter of oil-shales with other clays. Taking, then, the specific gravity of the kerogen at 1.0 to 1.1, which is certainly not erring on the low side, we find a value for the specific gravity of the mineral constituents of not more than 2.3.

Comparing this with a normal clay, we find that the average specific gravity of brick clays is approximately 2.3 to 2.5, and that for some reason the mineral contents of an oil-shale differ from normal clays by having a slightly lower specific gravity. The specific gravity of a clay, however,

may be taken in two ways: either the specific gravity of a sample of the clay can be taken, or the specific gravity of the material of which the rock is formed; the latter method takes no account of the voids or pore spaces in the rocks. For our purpose it is evident that the voids or pore spaces must be considered, as the absorptive capacity of a rock must depend chiefly upon its porosity. It is therefore possible to make a comparison with spent shale, which, though it has been somewhat affected in mineral composition during retorting by loss of alkalis, etc., must contain a considerable proportion of voids formerly occupied by the kerogen. It is found that the spent shale, as is natural, has also a low specific gravity—lower, that is, than other clays such as brick clays or fireclay (2·51) or kaolin (2·60). The average is approximately 2·20.

Turning to the question of absorptive capacity, we find that of all clays that known as fuller's earth is the most absorptive.

Fuller's earth is unfortunately a somewhat loose term, and is applied to any clay that can be used in fulling cloth. The "fulling" is the removal of grease and oils by absorption in the clay.

The composition of fuller's earth differs considerably in different formations and in different countries, but there are some characteristics that appear to be common to all deposits of this nature. Their specific gravities are low, they do not become plastic with water but crumble away, and they all contain, or can absorb, a large percentage of water, up to as much as 24 per cent.

The following table gives the chemical composition of some fuller's earths compared with brick clays and shales and spent oil-shales:—

ANALYSIS OF CLAYS, FULLER'S EARTHS, AND SPENT SHALES.

	Fireclay (Geikie).	Brick Clay (Geikie).	Average 106 American Clays.	English Fuller's Earth (Geikie).	English Fuller's Earth.	Californian Fuller's Earth.	Spent Broxburn Shale.	Spent Broxburn Shale.	Spent Broxburn Shale (Mills).	Spent Torbanite.
SiO <sub>2</sub> .	73·82	49·44	52·6	53·0	54·20	54·32	49·72	55·97	55·60	56·7
Al <sub>2</sub> O <sub>3</sub> .	15·88	34·26	21·0	10·0	14·30	18·18	18·8	31·21	22·14	36·2
Fe <sub>2</sub> O <sub>3</sub> .	2·95	7·74	14·8	9·75	6·30	6·50	16·8	2·84	12·23	3·2
CaO .	} Traces	1·48	3·58	·50	1·25	1·0	2·4	·59	} 1·55	{ 1·3
MgO .		5·14	2·98	1·25	2·72	3·22	2·2	1·87		
K <sub>2</sub> O .	} ·9	·31	·31	·	} 3·79	4·21	·	·62	*	2·2
Na <sub>2</sub> O .		1·1	1·1	·			·	·		
Water .	6·45	1·94	2·8	24·	17·44	11·86	·	1·07	·	·
Sp. gr.	2·51	·	2·4	·	·	1·85-2	2·1-2·3		·	1·2

\* 8·27 soluble in water, probably chiefly salts of K and Na.

The first point to be noted is the high percentage of water in the fuller's earth as compared with the shales or brick clays, while the spent shale naturally contains little or none.

The next point of importance is that, though the silica percentage does not differ very greatly, nor the alumina percentage, the calcium percentages are lower in the fuller's earth and the alkalis higher than in the brick clays and shales. This seems to hold good in all cases. The spent shale has lost most of the alkalis in the burning. But otherwise it is evident that in composition it approximates more nearly to fuller's earth than to brick-clays or ordinary shales.

The researches of Mr H. E. Ashley (*American Ceramic Society*, vol. xii) point to the facts that the absorptive capacity of a clay depends more upon its colloid contents than on its actual chemical composition, and that fuller's earth is very rich in colloids. The amount of colloids, however, seems to depend largely on the percentage of alkalis present, and possibly also on the percentage of alumina, so that chemical composition cannot be ignored. Addition of lime to a clay seems to destroy the colloids, and hence we find that a clay rich in lime is poor in absorptive capacity.

Much additional research would no doubt be required to establish these points beyond the possibility of doubt, but so far as the evidence is available we may conclude that the clays or shales most capable of absorbing and adsorbing inspissated petroleum must be low in specific gravity, fairly rich in alkalis and alumina, poor in lime, not very rich in magnesium and iron.

These characteristics are typical of the inorganic matter of oil-shales. In the richest variety of oil-shale, torbanite, we find that the percentage of alumina is very high, and those of iron and magnesia exceptionally low.

We may therefore accept the proposition that the colloid content of a shale is the determining factor as to whether, given the necessary conditions and the supply of inspissated petroleum, it is capable of becoming an oil-shale or not.

That a shale or clay may be absorptive and adsorptive enough to contain a sufficient percentage of hydrocarbons to become a rich oil-shale can hardly be doubted. Quite apart from the absorptive and adsorptive properties of fuller's earth in its ordinary state, the removal of hygroscopic water amounting to anything between 12 and 24 per cent. of the weight of the rock would admit of the taking up of an approximately equal weight of inspissated hydrocarbons. Where a series contains both oil and water it is usual to find them more or less completely separated, the water impregnating certain beds or bands and the oil others. When the oil is

of low specific gravity the water certainly floats it upward by hydrostatic pressure if the strata be sufficiently porous to admit of percolation; and thus it is beneath impervious bands and on the crests of gentle folds or anticlines that the petroleum becomes concentrated, and there may finally be inspissated sufficiently to reach the kerogen stage.

A clay, however absorbent, cannot contain enough light petroleum to furnish the material to form sufficient kerogen to make it a rich oil-shale. If fully impregnated with crude petroleum and subjected to weathering, without the possibility of drawing upon a further supply of petroleum as the lighter fractions are lost and dissipated, the clay will fix as kerogen a proportion of the hydrocarbons, but cannot become *fully impregnated* with kerogen. Should, however, crude petroleum be supplied as the weathering and inspissation proceed, a much greater proportion will finally be fixed as kerogen. This is a very simple experiment that can be made with any absorbent argillaceous rock.

#### V. AMMONIUM SULPHATE.

One of the principal distinctions that has often been urged to prove that there is an essential difference between oil-shales and rocks containing petroleum is that the former on distillation yield ammonium sulphate, often in large quantity, and that no yield of that salt in commercial quantity is possible from crude petroleum. This has often been held as proof that crude petroleum and kerogen must be entirely different in origin. It has even been maintained that the percentage of nitrogen in kerogen may be taken as evidence that the raw material from which it has been formed must have been to a large extent animal as distinguished from vegetable matter.

It is necessary to examine into this question thoroughly to ascertain whether or no such statements are justified.

In the first place, it may be noted that coal and peat contain percentages of nitrogen and yield ammonium sulphate when distilled. The percentage of nitrogen in some Natal coals is as high as 1·5 per cent., and in some Welsh coals as high as 2·16 per cent., and a company has been formed in South Africa to utilise this nitrogen content by means of a special process.

In Scottish oil-shales the *average* percentage of nitrogen is ·62 per cent., though running as high as ·72 per cent. In torbanite it has been estimated variously as from 1·53 per cent. to ·50 per cent.

From this it is quite evident that the percentage of nitrogen in the oil-shale need not be considered as any evidence of origin from animal matter.

An interesting passage may be quoted here from the Geological Survey memoir on *The Oil-Shales of the Lothians*, p. 165: "As already indicated, the crude oil diminishes and the ammonia increases, as a general rule, with the age of the shale. In old peat the proportion of nitrogen is sometimes, if not always, greater than in new peat of the same bog, owing to the nitrogen-free compounds decaying more rapidly than the nitrogenous compounds under the special circumstances. Recent plants subjected to destructive distillation yield a very acid distillate, peat less so, brown coal still less; Torbanehill mineral has a distillate slightly acid at the beginning and alkaline further on, while shale is very alkaline throughout. Hence the decrease of crude oil and increase of ammonia may partly be the result of age. No doubt part of the nitrogen is in combination in the kerogen, and this part may through time have increased in proportion; but the richness of the oldest shales in ammonia is more than we would expect from this cause alone."

The percentage of sulphur is another interesting point the importance of which will be seen later.

All crude petroleum contains a proportion of nitrogen, though it is often so small that it is seldom estimated. Sir Boverton Redwood, in *Petroleum and its Products*, gives the percentages of nitrogen in oils from many different fields on the authority of Beilby, Peckham, and Mabery. Thus Galician oil has been shown to contain .188 per cent. of nitrogen, and Pennsylvanian oil occasionally as little as .008 per cent. Oils from Ohio, West Virginia, and Baku have given on analysis percentages of .230 per cent., .540 per cent., and .05 per cent. respectively.

Californian oils, which as a rule are heavy, asphaltic, somewhat inspissated, and rich in unsaturated hydrocarbons, give considerably higher percentages, rising to slightly more than 1 per cent. among the heavy malphas. Thus percentages of 1.109 per cent., 1.016 per cent., and 1.08 per cent. are recorded from oils of the Heywood Petroleum Company, Pico Spring and Cañada Luca in California.

It seems to be established that the nitrogen occurs, to some extent at least, as pyridine bases.

It is noteworthy that the heavier and more inspissated the oil the larger the nitrogen and sulphur percentages become. It is evident, then, that these elements, in whatever manner they are combined with hydrocarbons, are associated with the heavier and more complex molecules. The heavy oils of the Texas field, containing sulphur percentages of 3 per cent. or over, require special refining processes.

In Trinidad asphalt, which, as has been seen, consists of highly inspissated petroleum mixed with water and inorganic matter, the average percentage of nitrogen in the bitumen is .81 per cent. (Clifford Richardson). In the less inspissated bitumen from Bermudez Lake, Venezuela, it is .75 per cent.

But if the "organic matter not bitumen" in Trinidad asphalt be examined, the nitrogen percentage is found to be 2.05 per cent., while the oxygen is as much as 27.29 per cent., indicating how thoroughly the material has been inspissated. In the malthenes, the least oxidised and inspissated portion of the asphalt, the nitrogen percentage is .6, and in the saturated hydrocarbons in the malthenes the percentage is only .07. Set out more fully, we have the following table:—

PERCENTAGES OF NITROGEN IN ORGANIC MATTER IN TRINIDAD ASPHALT.

Saturated Hydrocarbons in Malthenes.	Total Malthenes.	Total Bitumen.	Unsaturated Hydrocarbons in Malthenes.	Asphaltenes.	Organic Matter Non-bituminous.
.07	.6	.81	.92	1.2	2.05

This is direct proof that the nitrogen compounds become concentrated in the inspissated products, part of which, as we have seen, become adsorbed in the argillaceous mineral matter and insoluble in carbon disulphide.

We should expect, therefore, in a natural bitumen such as albertite, which contains some 90 per cent. of matter insoluble in carbon-disulphide, that the percentage of nitrogen will be high. It is 1.75; while in gilsonite it is .79, and in Egyptian glance pitch, a very pure bitumen containing only .2 per cent. of organic matter and .1 per cent. of mineral matter, the percentage of nitrogen is only .19.

Evidence of this nature could be given at great length, but perhaps the above is sufficient.

We should expect also that the shales associated with albertite should give a high percentage of nitrogen and give evidence of oxidation of the kerogen content. Unfortunately, I have been unable to find analyses of these shales giving the nitrogen percentage; but the fact that throughout a test lasting seventeen days, made by the Pumphreton Oil Company, the shales from Albert County gave an average of 76.94 lbs. of ammonium sulphate per ton is sufficient proof that they are richer than the average Scottish shale in nitrogen. Albertite yields 65 lbs. per ton; the average yield of ammonium sulphate is about 35 to 40 lbs. per ton for Scottish shales.



If any further proof be required that the kerogen of the Scottish shale-fields is a highly inspissated and therefore oxidised product, it is given in an analysis by Dr Mills (*Destructive Distillation*) of the kerogen of good average shale. Omitting the sulphur and nitrogen, there is a percentage of oxygen as high as 16.33, the analysis showing carbon 73.05, hydrogen 10.62, and oxygen 16.33.

Thus the evidence from nitrogen contents and yield of ammonium sulphate bears out what we have already seen to be the case with the hydrocarbons forming kerogen; if the latter be derived from inspissated petroleum, and concentrated and fixed in the shale by inspissation and adsorption, the nitrogen compounds from which the ammonia is distilled have also been concentrated in the same manner, and in favourable circumstances to an extent even greater than in the case of the majority of Scottish shales.

Thus the argument that the yield of ammonium sulphate from oil-shales shows that they are in no way connected with petroleum, not only falls to the ground, but proves to be a very strong argument in favour of the kerogen having been derived from petroleum.

In this connection it may be recalled that many bituminous shales have been called oil-shales and yet do not yield ammonium sulphate in commercial quantities. Of these the Colorado "oil-shales" and the Kimmeridge "oil-shale" may be mentioned. Both are really shales impregnated with petroleum, and have not yet reached the stage of true kerogen-bearing shales.\* Such shales are liable to be ignited spontaneously at outcrop, when they may burn and smoulder for long periods, a phenomenon never observed in the case of a true kerogen shale, with the exception of one recorded case in New South Wales. Similar burnt shales are frequent in Trinidad, where they are called "porcellanites"; they mark a transition stage between the lignitic and the petroliferous phase, but are also associated with oil-rocks. They occur frequently among argillaceous beds a short distance above lignite seams, and have in many cases been leaf-beds. On the Red Deer River in Alberta similar naturally burnt clays may be seen not far above coal-seams. In Barbados at Burnt Cliff, and near the asphalt lake in Trinidad, similar burnt clays may be seen; in the latter cases it is quite evident that the material burnt is petroleum.

The well-known burning of the Kimmeridge Clay points to the material burnt being of a similar nature, bituminous rather than kerogen.

The concentration of sulphur compounds affords another means of

\* "Oil-Shales of N.W. Colorado and N.E. Utah," by E. G. Woodruff and David T. Day, *Bulletin* 581A, U.S. Geol. Survey.

determining the state of inspissation of the material that we know as kerogen. In the Scottish oil-shales the percentage of sulphur averages about 1·4, and, though it doubtless exists partly as sulphides and sulphates apart from the kerogen, the latter must contain, according to Dr Mills, a considerable portion of the sulphur, since he found ·49 of sulphur in a total of 36·22 per cent. of organic constituents of average good Scottish oil-shale.

The concentration of sulphur in the inspissated products of petroleum is too well known to require description; but as oils vary greatly in sulphur percentage, some being almost free from it, it is obvious that the proportions of sulphur in different inspissated oils and oil-rocks will also show a considerable range of variation.

Thus analyses of the various products into which we can divide Trinidad asphalt, itself formed from a very sulphurous petroleum, show for sulphur a gradation as in the case of nitrogen compounds.

In the Malthenes.	In Total Bitumen.	In the Asphaltenes.	In Organic Matter not Bitumen.
Per cent. 2·9	Per cent. 6·16	Per cent. 10·9	Per cent. 10·32

In the solid native bitumen we find that gilsonite, the least inspissated, contains 1·79 per cent.; grahamite a percentage varying from ·93 to 1·79, the samples being taken from different districts; and albertite 1·06–1·20.

## VI. CONCLUSIONS.

To sum up the evidence brought forward is perhaps hardly necessary, but it will be as well to state the conclusions arrived at, and to apply them to the shale-fields that have been briefly described:—

- (1) Kerogen is formed by the inspissation of petroleum.
- (2) During the inspissation the nitrogen and sulphur compounds become concentrated in the most inspissated or weathered products.
- (3) At a certain stage of inspissation, which is reached gradually, the organic matter becomes insoluble in carbon-disulphide, and thus ceases to be classed as a bitumen.
- (4) *Ceteris paribus*, the state of inspissation increases with age.
- (5) An oil-shale is formed by the power of certain clays or shales of absorbing and adsorbing inspissated petroleum, particularly unsaturated hydrocarbons.
- (6) This power apparently depends on the colloid contents of the argillaceous rock.

(7) The argillaceous rock impregnated with petroleum residues is enabled to retain them long after porous sandstones in the vicinity have lost all traces of petroleum by weathering and lixiviation.

To apply these conclusions to the Scottish oil-shale fields is very simple, and it is possible to give a sketch of the geological history of the late- and post-Carboniferous period with special reference to the formation and preservation of the shale-fields.

By the close of the deposition of the Coal Measures the petroliferous stage had been reached in the Lower Carboniferous rocks, since in favourable circumstances a pressure of from 150 to 200 atmospheres is, so far as we know, ample to ensure the formation of petroleum.

The petroleum formed would be concentrated in the most porous strata, the great freestones of the Oil-Shale group, though doubtless impregnating every band of sufficient porosity.

Great earth movements set in at the close of the period and threw the strata into numerous synclines and anticlines, towards the latter of which the petroleum would naturally migrate under gas-pressure and hydrostatic pressure.

One of these great anticlines we see in the Pentland Hills, and there is distinct evidence on the map (fig. 2) of a much smaller and less important but parallel anticline running from the south of Dechmont, north of Pumpherston, and through Broxburn to Kirkliston.

Denudation of the anticlines commenced immediately, and proceeded so far that our Carboniferous strata are now found in great basins, while the intervening anticlines have been entirely removed, and with them almost all traces of the petroliferous phase. In only one region in the Carboniferous territory the lower groups remained in minor anticlines undenuded. Here the earth movement, acting in a slightly different direction, produced the secondary anticlines of Dechmont, Pumpherston, and Kirkliston. In these folds, and in the minor wrinkles of Dalmahoy, Straiton, and Carlops, the petroleum driven out by the incursion of water made its last stand.

As time went on, faulting supervened and still more complicated the structure. The faults are admittedly later than the folds, and the very fact of their formation shows that the load was becoming lighter under relentless denudation. The petroleum then came under the influence of weathering processes, which in favourable circumstances affect oil to a very great depth. Inspissation and all that it implies—loss of lighter fractions, formation of unsaturated hydrocarbons and oxidation—began and is still proceeding. Thus the petroleum would gradually become richer in unsaturated hydrocarbons, in sulphur and nitrogen compounds and complex oxidised molecules.

Those clays with a sufficient colloid content readily absorbed the inspissated products and gradually fixed them as kerogen, the porous sandstones beneath supplying the material as long as the argillaceous rock could provide accommodation for it.

In this connection two points may be noted: the occurrence of good porous sandstone beds not far beneath the most prolific oil-shales, and the occurrence of impervious shales above.

It must also be understood that the shales may have contained the material to form petroleum originally, and this may have been inspissated and turned to kerogen *in situ*, but no shale could contain *sufficient* organic matter to form its full complement of kerogen, though it doubtless contained at one time crude unweathered oil to its full capacity.

As the series became more open to weathering processes and lixiviation the sandstones would lose all traces of their former impregnation, but towards the crests of anticlines and minor flexures the process of inspissation and absorption would survive longest, so that now the richest shales occur in anticlinal areas on the crests and flanks of flexures.

Where an oil-shale passes gradually into a carbonaceous shale we have evidence of the transition stage between the petroliferous and the carbonaceous phases. Where an oil-shale insensibly passes upwards or downwards into an unworkable bituminous shale it is evidence of either a lack of colloid content or a deficiency in the supply of inspissated petroleum.

Finally, in a well-sealed anticline low in the series we find in the Broxburn district a remnant of the petroleum impregnation. That it is a true crude petroleum, though somewhat inspissated, and quite different from the oil distilled from the shales either artificially or naturally by the heat of igneous intrusions, is shown by the analysis:—

	A.	B.	C.	D.
Specific gravity . . . . .	·868	·866	·830	·950
Naphtha . . . . .	3·5	4·5	10·2	12·8
Illuminating oil . . . . .	32·0	36·1	34·1	36·0
Lubricating oil . . . . .	28·0	25·6	27·2	32·0
Solid residues . . . . .	11·0	8·8	12·5	12·3
Loss in refining . . . . .	25·5	25·0	16·0	5·7
Water . . . . .	...	...	...	1·2
	100·0	100·0	100·0	100·0

A—Crude shale-oil (Broxburn).

B—Naturally distilled oil from sill of igneous rock, Dunnet Mine (Broxburn).

C—Crude petroleum from Sandhole Pit (Broxburn).

D—Heavy inspissated oil from shallow boring near asphalt lake, Trinidad.

The residues in A, B, and C are paraffin, in D asphalt, hence the high specific gravity. D was selected on account of the percentages of the kerosene and lubricating fractions being nearly the same as those of A, B, and C. The analyses have been somewhat simplified to enable the comparison to be made easily. The high percentages of naphtha in the crude petroleums distinguish them at once from the distilled oils.

The beautiful freestones of the Oil-Shale group probably owe their economic value, their comparative lack of hard cementing material, and the absence of plant remains except as mere casts of tree trunks, to having been at one time oil-sands. No geologist who has studied petroleum can examine such freestones without comparing them mentally with oil-rocks, and thinking how admirably they are adapted for forming underground reservoirs of petroleum.

The abnormal deposit known as Torbanehill mineral or torbanite deserves special mention, as it differs in several particulars from ordinary oil-shale, and as it has been the subject of much interesting research and some famous lawsuits. It is the highest in the geological series of all the oil-shales, the lowest in specific gravity, and the richest in yield of oil.

It is described as being "of a brown or nearly black colour, having a yellow or fawn-coloured streak, without lustre, and subconchoidal fracture. It shows parallel banding by splitting, and, although homogeneous in appearance in the fresh state, shows stratification when spent in the retort. It is non-electric. It is a mass of carbonaceous matter without structure, mingled with stems or roots of trees showing structure, and the fossils were found throughout the bed and not merely on the surfaces of seams." \*

The yield of oil ran as high as 130 gallons per ton, but the yield of ammonium sulphate was small.

The deposit having been worked out, it is difficult to obtain accurate details of the field relations; but owing to the legal controversy upon the question as to whether Torbanehill mineral could be classed as a coal or not, many notes of interest are preserved. The seam mined near Bathgate in Linlithgowshire occurs just above the Sandstone group correlated with the Millstone Grit horizon in England. The thickness usually varied between 18 and 20 inches, and the deposit lies on fireclay and is overlaid by bituminous shales and occasionally blackband ironstone. The seam passes laterally into coal or black shale.

\* *Oil-Shales of the Lothians*, p. 160.

The following section of the seam shows very clearly its relations to other deposits:—

Shale . . . . .	4 inches.
Torbanite . . . . .	1 foot 4 "
Fine ironstone . . . . .	$\frac{1}{2}$ to 2 "
Bituminous shale . . . . .	2 "
Inferior coal . . . . .	7 "
Foul coal ( <i>i.e.</i> coal and shale) . . . . .	2 to 4 "

This is sufficient to prove the association with both the carbonaceous and the kerogen phases, and to suggest that we are dealing with a locality where the phenomenon of the transition stage between the carbonaceous and the petroliferous phases may be studied.

The specific gravity of the deposit varies from 1·17 to 1·316, and the ash has the very low specific gravity of 1·22 to 1·28. The percentage of mineral matter is from 21·3 to 24, and analyses of the spent material give results as follows:—

	Stenhouse.	Hofman.	Anderson.
$\text{SiO}_2$ . . . . .	58·51	56·7	56·09
$\text{Al}_2\text{O}_3$ . . . . .	33·65	36·2	40·04
$\text{Fe}_2\text{O}_3$ . . . . .	7·0	3·2	3·24
$\text{CaO}$ . . . . .	...	1·3	·34
$\text{MgO}$ . . . . .	...	0·4	·46
Alkalis . . . . .	1·21	2·2	...

The points to be noted are the very high percentage of alumina, and the low percentage of iron, lime, and magnesia. Alkalis have probably been lost to a great extent in the retorting. The analyses indicate a very pure clay, possibly rich in alkalis, as the inorganic content of the deposit. The vegetable fossils prove that the deposit was rich originally in organic matter, which no doubt has been largely converted into petroleum, inspissated, and adsorbed *in situ*; but the fact that traces of the vegetable remains still exist makes it certain that the conversion into petroleum has not been quite complete, and the richness of the deposit must be due to the absorption and adsorption of extraneous inspissated petroleum.

The position above the porous sandstone correlated with the Millstone Grit—a rock admirably adapted to act as a reservoir for petroleum—and beneath impervious shales is precisely that in which vestiges of the petroliferous phase would be expected.

Other abnormal deposits or minerals are the torbanite of New South Wales and the stellarite of Nova Scotia. The percentage of mineral matter in these cases—3·2 in the former and 6·55 in the latter—classes

them with coals or intrusive bitumen rather than shales. The field evidence proving their occurrence in regular beds makes it practically certain that they must be looked upon as very highly bituminous coals, indicating the approach to the oil-bearing stage even in beds of what is now coal; this is confirmed by the association with oil-shales in both cases.

Applying the conclusions detailed above to the shale-fields of South Africa, it is obvious that we are dealing with the overlapping of the petroliferous and carbonaceous phases. Probably the coal-bearing stage had been reached before pressure was sufficient to initiate oil-formation. No flexures exist to have had the effect of concentrating petroleum towards any particular localities, and consequently such organic matter as survived to reach the petroliferous stage is distributed very widely, the oil-shales do not show sudden changes in character and yield, and are nearly all inclining to a carbonaceous character and association with coal seams.

In New South Wales somewhat similar conditions obtain, and coals and oil-shale are very intimately associated. But the petroliferous phase has undoubtedly been reached even in some deposits which from their very low inorganic contents must be now classified as coals.

Very gentle undulations have certainly caused a concentration of what is now kerogen towards certain localities where the richest seams are worked. As a whole, however, New South Wales crystallises for us the transition stage: with greater pressure earlier applied, more pronounced earth movement, and effective sealing by impervious strata the territory would have been an oil-field. With less pressure more slowly applied the carbonaceous phase would be even more pronounced and oil-shales would be absent altogether.

It is perhaps unnecessary to add further instances which can be gleaned from many parts of the world where thick series of strata containing vegetable organic matter may be studied. The field evidence is fairly complete, and it is borne out by chemical and experimental work, so far as it has been undertaken. The idea is not new that something approaching to an oil-shale can be made artificially by the "affinity" of fuller's earth for the unsaturated hydrocarbons in natural petroleum,\* and had experiments been made with more highly inspissated material more striking and suggestive results would doubtless have been obtained. Field evidence gives the key to the conditions that must be applied in such experimental research.

\* *Oil-Shales of the Lothians*, p. 162.

It must be left to the chemist to determine what reactions, if any, take place between inspissated petroleum residues, or their unsaturated hydrocarbons, and the colloid contents of an argillaceous rock. Whether compounds insoluble in carbon-bisulphide are formed with bases, and whether the nitrogen, sulphur, and oxygen compounds play any essential role in the process, may require much elaborate research which is out of the domain of the geologist.

The practical result, as we have seen, is summed up in the formation of kerogen; and in searching for kerogen-bearing strata we must proceed even as in the search for petroleum, by giving special attention to anticlinal structures in the particular stratigraphical sequence to be prospected.

It is claimed, then, that, in light of the field and laboratory evidence set forth above, the theory of the origin of an oil-shale quoted in the earlier part of this paper is, to say the least, inadequate; that oil-shale fields and oil-fields are not separate and distinct phenomena but inextricably connected; that an oil-shale field is but the evidence of a former petroliferous phase; and incidentally that Scotland still possesses the relics of a once great and valuable oil-field, which has passed, like the presumed great coal-fields of Ireland, before man appeared on earth to benefit by it.

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V.—The "Geometria Organica" of Colin Maclaurin: A Historical and Critical Survey. By Charles Tweedie, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University.

(MS. received October 15, 1915. Read December 6, 1915.)

INTRODUCTION.

COLIN MACLAURIN, the celebrated mathematician, was born in 1698 at Kilmodan in Argyllshire, where his father was minister of the parish. In 1709 he entered Glasgow University, where his mathematical talent rapidly developed under the fostering care of Professor Robert Simson. In 1717 he successfully competed for the Chair of Mathematics in the Marischal College of Aberdeen University. In 1719 he came directly under the personal influence of Newton, when on a visit to London, bearing with him the manuscript of the *Geometria Organica*, published in quarto in 1720. The publication of this work immediately brought him into prominence in the scientific world. In 1725 he was, on the recommendation of Newton, elected to the Chair of Mathematics in Edinburgh University, which he occupied until his death in 1746.

As a lecturer Maclaurin was a conspicuous success. He took great pains to make his subject as clear and attractive as possible, so much so that he made mathematics "a fashionable study." The labour of teaching his numerous students seriously curtailed the time he could spare for original research. In quantity his works do not bulk largely, but what he did produce was, in the main, of superlative quality, presented clearly and concisely. The *Geometria Organica* and the Geometrical Appendix to his *Treatise on Algebra* give him a place in the first rank of great geometers, forming as they do the basis of the theory of the Higher Plane Curves; while his *Treatise of Fluxions* (1742) furnished an unassailable bulwark and text-book for the study of the Calculus.

In a sense he may be regarded as a founder of the Royal Society of Edinburgh, for it was at his instigation that a Medical Society in Edinburgh was encouraged to broaden its field of research and develop into the Philosophical Society, which gave rise in its turn to the Royal Society of Edinburgh in 1783.

§ 1. During comparatively recent years the study of geometrical science has been enriched by a number of publications dealing with the history of

particular curves, and the general development of the theory. Prominent among such works may be mentioned Loria's *Ebene Kurven*, Wieleitner's *Spezielle Kurven*, in German; and Teixeira's *Courbes algebriques*, in French or Spanish. These works, compiled with great care, are indispensable to the geometer in the study of his subject, but a perusal of the early rare treatise of Maclaurin on the *Geometria Organica* reveals the fact that the claims of the latter geometer have frequently been entirely overlooked.

For example, Teixeira himself, in a note on the Researches of Maclaurin on Circular Cubics (*Proc. Edinburgh Math. Soc.*, 1912), points out that many of the classic properties connected with these curves are due to Maclaurin, although his name does not even appear in the list of writings on the Strophoid published by Tortolini and Günther.

Again, the whole theory of Pedals, and more particularly the Pedals of the Conic Section, is given in the *Geometria Organica*—a theory to be rediscovered and named in the nineteenth century, more than a hundred years after the publication of Maclaurin's work. In this connection it may be pointed out that Maclaurin invented the term, the Radial Equation of a Curve (for its  $p-r$  equation), long before the term Radial came to be applied to another purpose by Tucker.

These two examples sufficiently illustrate my contention that Maclaurin's treatise has been strangely overlooked. It is the business of the present note to indicate others, to point out how fully he has in many cases anticipated writers of comparatively recent times, and to vindicate his claims to a far more careful consideration than has of late been the fashion. It may here be remarked that Poncelet in his magistral *Traité* gives full credit to the importance of Maclaurin's two geometrical treatises, the *Geometria Organica* and the *Proprietates Linearum Curvarum*, published as an appendix to his posthumous *Treatise on Algebra*. In fact, the French school generally does more justice to the Scottish geometers of the eighteenth century than do English writers in the sister kingdom.

§ 2. The *Geometria Organica* is the first great treatise of Maclaurin, and appeared in London, in 1720, under the royal imprimatur (1719) of Newton, to whom the work is dedicated. At the time the youthful Maclaurin (for he was only twenty-one years of age) held the Chair of Mathematics in the New College\* in Aberdeen. The work expands and develops two earlier memoirs published in the *Philosophical Transactions of the Royal Society*:—

(i) *Tractatus de Curvarum Constructione et Mensura*, etc., 1718, giving the Theory of Pedals: (ii) *Nova Methodus Universalis Curvas Omnes*

\* i.e. Marischal College.

*cujus-cumque Ordinis Mechanice describendi sola datorum Angulorum et Rectarum Ope*, 1719.

Maclaurin's imagination had been fired by Newton's classic *Enumeratio Linearum Curvarum Tertii Ordinis*, and by the organic description of the Conic given in the *Principia*; and in his attempt to generalise the latter so as to obtain curves of all possible degrees by a mechanical description he was led to write the *Geometria Organica*.

It will appear in the sequel how remarkably successful he was in obtaining nearly all the particular curves known in his time (which he is careful to ascribe to their inventors), besides a whole host of new curves never before discussed, and which have since been named and investigated with but scant acknowledgment of their true inventor. His method, however, does not furnish all curves, though it may furnish curves of all degrees; and it will be the business of this note to point out some of the limitations of the method applied, as well as the rare mistakes Maclaurin makes regarding the double points of the curves investigated,—a weakness more pronounced in the earlier memoirs.

In establishing his theorems he frequently employs the method of analysis furnished by the Cartesian geometry. The Cartesian geometry was then in its infancy, and Maclaurin's use of it seems to us nowadays somewhat cumbersome and certainly tedious. But when Maclaurin reasons "more veterum," he handles geometry with consummate skill; and the impression gains upon the reader that, however imperishable his reputation in analysis may be, he was greater as a geometer than as an analyst. He occasionally makes petty errors in his analytical demonstrations which somewhat mar the interest in his work, but the beauty of his synthetic reasoning is untarnished by any such blemish.

In any analysis that follows, the demonstrations he gives are frequently replaced by others that are more in touch with modern methods, but this does not apply to the geometrical reasoning proper, which is as fresh to-day as when written. The treatise is divided into two parts. In the first part the only loci admitted are straight lines along which the vertices of constant angles are made to move. In the second part the curves so found in the first part are added to the loci to obtain curves of higher order. It contains, in particular, the theory of pedals and the epicycloidal generation of curves by rolling one curve upon a congruent curve. A section is devoted to the application to mechanics; and the last section contains some general theorems in curves forming the foundation of the theory of Higher Plane Curves. It also contains what is erroneously termed Cramer's Paradox, the paradox being really Maclaurin's, for

Cramer in his *Courbes algebriques* expressly quotes the *Geometria Organica* as his authority.

For the sake of brevity I have, in what follows, restricted my attention to what would seem of modern interest, and have on this account omitted entirely the discussion of asymptotes to curves and the exhaustive enumeration of cubic curves as based on Newton's work. The nomenclature is also modern, save where the curves were already familiar to mathematicians in Maclaurin's day. Maclaurin rarely attempts to give names to the hosts of new curves generated by his methods. A remarkable feature of interest lies in the fact that many of the methods employed only require an obvious generalisation to furnish standard methods of generating unicursal cubics and quartics supposed to have been invented during the latter half of the nineteenth century. It will be my special object to indicate these at the proper time and place. In order to emphasise Maclaurin's own work I have followed the order of his Propositions, and the numbers attached to these are taken from the *Geometria Organica*.

It has been found necessary, however, to use a more convenient notation for the figures.



and to  $O'Q$  in the form

$$M_1 + tM_2 = 0 \quad (2)$$

so that the locus of  $Q$  is given by

$$L_1M_2 - L_2M_1 = 0 \quad (3)$$

and is therefore a conic through  $O$

$$(L_1 = 0; L_2 = 0),$$

and through  $O'$

$$(M_1 = 0; M_2 = 0).$$

Cor. 2. By assuming the converse theorem (proved later) Maclaurin deduces that if  $P$ , instead of lying on a straight line, moves on a conic through  $O$  and  $O'$ ,  $Q$  still generates a conic through  $O$  and  $O'$ .

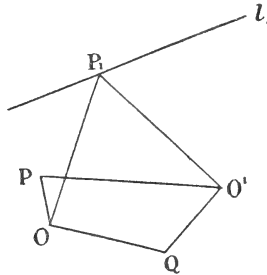


FIG. 2.

*Dem.*

For a straight line  $l_1$  can then be found, and a point  $P_1$  moving on it, so that

$$\angle P_1OP = \alpha'$$

$$\angle P_1O'P = \beta'$$

are constant angles.

Hence  $P_1OQ$  and  $P_1O'Q$  are constant angles; and so, when  $P_1$  traces out  $l_1$ ,  $Q$  generates a conic through  $O$  and  $O'$ . (There is, in fact, a 1-1 correspondence between  $OP$  and  $O'P$ , and  $\therefore$  between  $OQ$  and  $O'Q$ .  $\therefore$  etc.)

#### § 4. Prop. II

determines the species and asymptotes of the conic.

On  $OO'$  describe a segment of a circle  $OKO'$  containing an angle  $\gamma$  so that  $\alpha + \beta + \gamma =$  a multiple of two right angles. Let it cut  $l$  in  $A$  and  $B$ . When  $P$  coincides with either  $A$  or  $B$  the angle at  $Q$  in  $POQO'$  is zero, *i.e.*  $Q$  is at infinity on the curve, and  $OQ$  (or  $O'Q$ ) is parallel to an asymptote. The angle  $AOB$  ( $=AO'B$ ) measures the angle between the asymptotes.

The conic is a hyperbola, a parabola, or an ellipse, according as  $A$  and  $B$  are real and distinct, coincident, or imaginary.

Cor. 4. The curve cannot be a circle when  $l$  is not the line at infinity.

Cor. 6. When  $\alpha + \beta = \pi$  the curve is a hyperbola in general, but a parabola when  $l$  is parallel to  $OO'$ .

Cor. 7. If  $l$  passes through the centre of the circle  $OKO'$ , the angle  $AOB$  is a right angle and the hyperbola is equilateral.

### § 5. Prop. III.

But if for any position of  $P$  on  $l$ ,  $OQ$  and  $O'Q$  coincide simultaneously with  $OO'$ , the conic degenerates into a straight line (along with  $OO'$ ).

The proof given is analytical.

Cor. 2. This may happen, for example, when  $\alpha + \beta = \pi$ , and  $l$  is inclined at angle  $\alpha'$  to  $OO'$  (viz. when  $P$  is at infinity on  $l$ ).

Cor. 3. In particular this is so when  $\alpha = \beta = \pi/2$ , and  $l$  perpendicular to  $OO'$ , when  $Q$  is on another line perpendicular to  $OO'$ , which is the image of  $l$  in the mid-point of  $OO'$ .

Cor. 4 contains an important statement.

Find  $I$  on  $l$  such that  $\angle OO'I = \beta$ , and let  $\angle IOO' = \alpha'$ .

Let  $P$  trace out  $l$ , and let  $Q'$  be taken in quadrilateral  $POQ'O'$  for angles  $\alpha'$  and  $\beta$ . Then  $Q'$  traces out a straight line. The angle  $QQQ' = \alpha' - \alpha$  and is therefore constant, and  $O', Q, Q'$  are collinear. Hence we

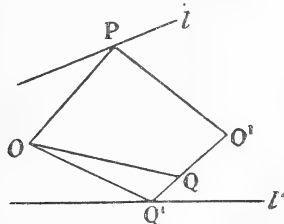


FIG. 3.

may trace the conic locus of  $Q$  by making  $Q'$  lie in  $l'$  and taking  $QQQ' = \alpha' - \alpha$ ; i.e. in the preceding constructions we may replace an angle by a straight line rotating round one of the fixed points.

### § 6. Prop. IV

proves the converse of Prop. I by solving the problem:—*To describe a conic through five given points.*

Let the points be  $A, B, C, D, E$ . Form  $\triangle CAB$ , and let  $\angle CAB = \alpha$ ,  $\angle CBA = \beta$ . Rotate angles  $\alpha$  and  $\beta$  round  $A$  and  $B$  respectively, and let the intersection of two arms be in  $D$  and then in  $E$ , while the intersection of the other arms comes to be at  $D'$  and  $E'$ .

Let the line  $D'E'$  be taken for  $l$ ; then if  $P$  traces out  $l$ ,  $Q$  generates a

conic which passes through D and E and also through C, A, and B, *i.e.* Q generates the conic through the five given points.

If four points only are given, an infinity of conics can be described through them. Thus there are two parabolas through the four points, or two hyperbolas whose asymptotes intersect at a given angle. For example, let the parabolas through A, B, C, D be sought. Proceed as before and find D'. On AB describe a segment of a circle containing an angle  $\gamma$  such that

$$\alpha + \beta + \gamma = \pi \text{ (or } 2\pi\text{)}.$$

Either tangent from D' to this circle will furnish the line  $l$  for the parabola.

"The method employed will furnish the complete system of conic sections which were the objects of research of the older geometers. Newton was the first to attack the problem to enumerate and classify Curves of the Third Order, and thereby added a fresh triumph to his genius. We now proceed to delineate curves of this order."

#### § 7. *Newton's Organic Description as a Cremona Transformation.*

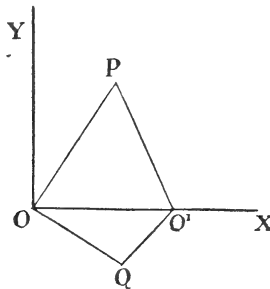


FIG. 4.

Let O be the origin, O' the point  $(\alpha, 0)$ , P any point  $(\xi, \eta)$ . Then O'P is given by

$$y = \frac{\eta}{\xi - \alpha}(x - \alpha) = \mu(x - \alpha), \text{ say} \quad . \quad . \quad . \quad (1)$$

and O'Q is given by

$$y = m(x - \alpha)$$

where

$$\frac{m - \mu}{1 - m\mu} = \tan \beta,$$

*i.e.*

$$\begin{aligned} m &= (\mu + \tan \beta)/(1 - \mu \tan \beta) \\ &= \frac{\eta + (\xi - \alpha) \tan \beta}{\xi - \alpha - \eta \tan \beta} \quad . \quad . \quad . \quad (2) \end{aligned}$$



so that O'Q has the equation

$$y(\xi - \alpha - \eta \tan \beta) = (x - \alpha)(\eta + \overline{\xi - \alpha} \tan \beta)$$

or

$$\xi(y - \overline{x - \alpha} \tan \beta) - \eta(y \tan \beta + x - \alpha) - \alpha(y - \overline{x - \alpha} \tan \beta) = 0 \quad . \quad . \quad (3)$$

Also OQ has the equation

$$\xi(y - x \tan \alpha) - \eta(y \tan \alpha + x) = 0 \quad . \quad . \quad . \quad (4)$$

So that if P traces out the line

$$A\xi + B\eta + C = 0 \quad . \quad . \quad . \quad (5)$$

Q traces out the conic

$$\begin{vmatrix} A & -B & C \\ y - (x - \alpha) \tan \beta & y \tan \beta + x - \alpha & -\alpha(y - \overline{x - \alpha} \tan \beta) \\ y - x \tan \alpha & y \tan \alpha + x & 0 \end{vmatrix} = 0 \quad . \quad (6)$$

passing through the fixed points (0, 0), ( $\alpha$ , 0); and

$$\left( -\frac{\alpha \tan \beta}{\tan \alpha - \tan \beta}, -\frac{\alpha \tan \alpha \tan \beta}{\tan \alpha - \tan \beta} \right).$$

Denote the last point by O''.

The three points are the singular points of the transformation, and are as in the figure.

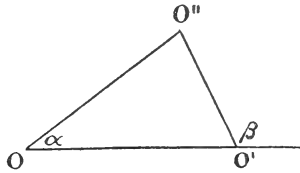


FIG. 5.

When  $\alpha = \beta$ , O'' is at infinity. The curve cannot be an ellipse, and is a parabola when  $l$  is parallel to OO'. When  $l$  passes through one of the points O, O', O'', the conic reduces to a straight line.

## SECTION II.

### DESCRIPTION OF LINES OF THE THIRD ORDER HAVING A DOUBLE POINT.

§ 8. Maclaurin's researches on these curves will well stand comparison with the modern theory of these curves, which he may fairly be described as anticipating.



magnitude  $\beta$ . If  $O_1P$  and  $O_2R$  intersect in  $R$  on a line  $l'$ , then the point  $Q$  traces out a cubic having a double point at  $O_2$ .

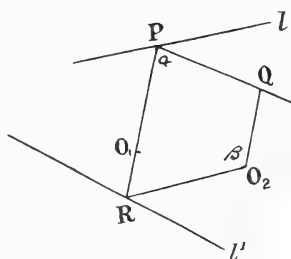


FIG. 6.

Let  $O_1$  be chosen as origin, and let  $RP$  have the equation

$$y = tx \quad (1)$$

Then  $PQ$  has an equation of the form

$$L_0 + tL_1 + t^2L_2 = 0 \quad (2)$$

while  $O_2R$  and therefore  $O_2Q$  has an equation of the form

$$M_0 + tM_1 = 0 \quad (3)$$

The elimination of  $t$  from (2) and (3) leads to a cubic with a double point at  $O_2$ . Also  $O_2Q$  and  $O_1P$  cut in a conic. Cf. § 18.

A geometrical construction for the tangents at the double point is also given.

[It is at once obvious that Maclaurin's generation of a singular cubic is the simplest case of the standard generation of these curves, which may be stated geometrically as follows:—

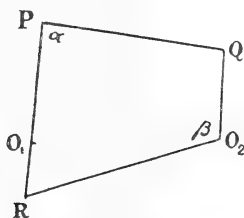


FIG. 7.

In the quadrilateral  $RPQO_2$  the angles at  $P$  and  $O_2$  are constant, and  $O_1$  and  $O_2$  are fixed points. Let  $R$  be on a conic that passes through  $O_1$  and  $O_2$ , or on a straight line. Let  $PQ$  be constantly tangent to a conic whose focus is at  $O_1$ . Then  $P$  lies upon a circle (or a straight line, if the conic is a parabola). See the Theory of Pedals in Part II.

There is thus a projective correspondence between the ray  $O_2Q$  and the tangent  $PQ$  to the conic, and  $Q$  generates the singular cubic. For the present he is restricted to the use of straight lines as loci, and of these he uses two.]

### § 11. *Prop. VI*

shows how to determine the asymptotes, and also the species of the cubic, according to Newton's classification of cubics.

The next theorem is Lemma I.

*If  $O$  is a fixed point,  $P$  any point on a given straight line, and  $OPQ$  a triangle of given species, then the locus of  $Q$  is a straight line.*

We need not add the proof.

### § 12. *Prop. VII.*

*All the cubics of Prop. V may be obtained by taking  $\angle QO_2R = \pi$ .*

Find  $K$  on  $l'$  such that  $O_1O_2K = \beta$ . Let  $KO_1O_2 = \gamma$ . Draw  $O_1T$  so that  $PO_1T = \pi - \gamma$ , and let it meet  $QP$  in  $T$  and  $QO_2$  in  $S$ . Then, by the lemma, when  $P$  moves on  $l$ ,  $T$  generates a straight line, while  $S$  also generates a straight line  $l''$  (by Prop. III).

We may thus obtain the locus of  $Q$  from the constant angle  $STQ$  and the intersection of  $O_2S$  with  $QT$ .

Cor. 1. Either  $\alpha$  or  $\beta$  may be replaced by a right angle or by an angle of any given magnitude.

This is easily deduced by starting from  $TQS$ .

The remaining cor. discuss the asymptotes and a variety of particular cases.

*E.g.* Maclaurin notes that when  $O_2$  goes to infinity, the pencil of lines becomes a system of parallel lines. Special cases arise when  $l$  and  $l'$  are parallel, or when the rays are inclined to  $l'$  at angle  $\alpha$ .

### § 13. *Prop. VIII*

considers the reduction of the equation of the cubic to a standard Newtonian form.

Some particular sub-cases are given.

*Ex.* 1.

Let  $l$  and  $l'$  be parallel and perpendicular to  $O_1O_2$ , and let  $O_1PQ = \frac{\pi}{2}$ .

Choose the origin at  $O_2$ , and let  $A, B, O_1$  be the points  $(a, o), (b, o), (d, o)$ .

If the equation to  $O_2R$  is

$$y - tx = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$O_1RP$  is given by

$$y = \frac{bt}{b-d}(x-d) ;$$

and PQ by

$$y - bt \frac{a-d}{b-d} + \frac{b-d}{bt}(x-a) = 0. \quad (2)$$

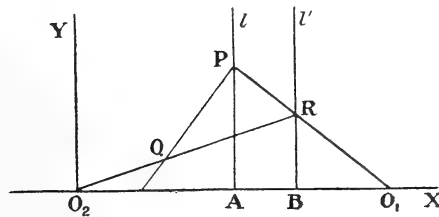


FIG. 8.

The locus of  $Q$  is therefore given by

$$xy^2 - y^2b \frac{a-d}{b-d} + \frac{b-d}{b}(x^3 - ax^2) = 0 \quad (3)$$

In particular :

If  $l$  passes through  $O_2$ , so that  $a=0$ , (3) reduces to the form

$$y^2 = \frac{Kx^3}{(x - d')}.$$

If  $O_2$  is the foot of the perpendicular from  $O_1$  on  $l$ , and  $l'$  is the line at infinity, the curve is the Cissoïd of Diocles.

[The Trisectrix of Maclaurin is also the particular case when  $l'$  is the line at infinity,  $l$  perpendicular to  $O_1O_2$ , and cutting  $O_1O_2$  in A, so that

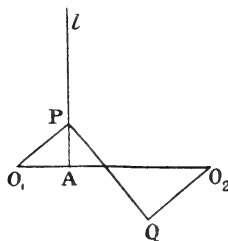


FIG. 9.

$\text{O}_1\text{A} = \frac{1}{3}\text{AO}_2$ ; but is not here explicitly quoted, occurring with another definition in the *Fluxions*.]

Case X.

Let  $l$  be parallel to  $O_1O_2$ ,  $l'$  perpendicular to  $O_1O_2$ ;  $O_1PQ = \frac{\pi}{2}$ .

Let  $O_2A = a$ ;  $O_2B = b$ ;  $O_2O_1 = d$ .

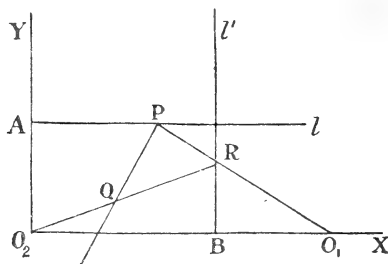


FIG. 10.

Then the equation to the locus of  $Q$  is

$$y^3 - ay^2 + \frac{b-d}{b}(x^2y - dxy - a\frac{b-d}{b}x^2) = 0.$$

Case XVIII.

Let  $O_1PQ = QO_2R = \frac{\pi}{2}$  (in Prop. V);  $l$  parallel to  $O_1O_2$ ,  $l'$  perpendicular to  $O_1O_2$ .

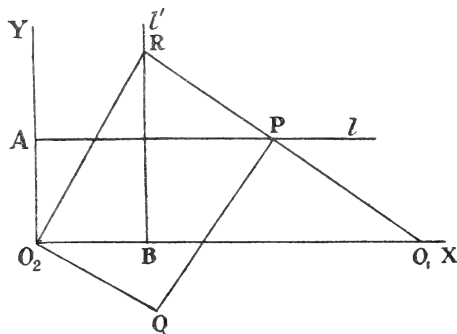


FIG. 11.

If  $y = tx$  is the equation to  $O_2R$ ,  $R$  is the point  $(b, tb)$ ,  $O_1PR$  has the equation

$$y = bt(x - d)/(b - d) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and  $P$  is the point

$$\left(d + \frac{a(b-d)}{bt}, a\right).$$

Hence  $PQ$  has the equation

$$y - a = \frac{d-b}{bt}\left(x - d - \frac{a(b-d)}{bt}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

while  $O_2Q$  is given by

$$ty + x = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The equation to the locus of Q is therefore

$$dx^2y - abx^2 + d(b-d)xy = a\frac{(b-d)^2}{b}y^2. \quad (4)$$

Case XXI.

$l$  and  $l'$  both perpendicular to  $O_1O_2$ .

Locus of Q,

$$\frac{d}{b}xy^2 + \frac{b(a-d)}{b-d}x^2 + a\frac{b-d}{b}y^2 = 0.$$

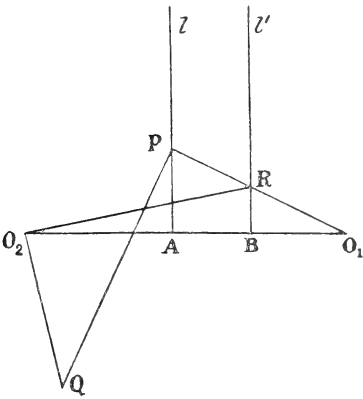


FIG. 12.

Case XXII.

$l$  and  $l'$  parallel to  $O_1O_2$ ;  $l$  midway between  $l'$  and  $O_1O_2$ .

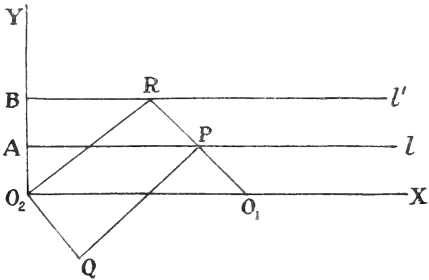


FIG. 13.

Then

$$\begin{aligned} O_2A &= a, \\ O_2B &= 2a. \end{aligned}$$

Let the equation to  $O_2R$  be

$$ty = x. \quad (1)$$

Then R is the point  $(2at, 2a)$ .

The equation to  $O_1R$  is

$$\frac{y}{2a} = \frac{x-d}{2at-d} \quad (2)$$

so that P is the point

$$\left(\frac{2at+d}{2}, a\right).$$

∴ the equation to PQ is

$$y - a = \frac{d - 2at}{2a} \left(x - \frac{2at+d}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the equation to the locus of Q is

$$4a^2y^2 = (d^2 - 4a^2)x^2 - 2dx^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In particular, when  $d = \pm 2a$  (4) becomes

$$y^2 = \mp \frac{1}{a} x^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which is Neil's parabola.

§ 14. In XVII the remark occurs : "*Curvas Omnes pure Hyperbolicas tertii Ordinis quæ punctum duplex habent ad distantiam finitam descripsimus. Restant Curvæ Hyperbolo-Parabolicae et pure Parabolicae quarum Descriptiones facillimæ ex methodo ipsius Prop. V deduci possunt.*"

[We proceed to discuss Maclaurin's claim to have found a method for generating all rational cubics, by showing that his method is the simplest for obtaining the standard generation of these curves as given by

$$L_0 + tL_1 + t^2L_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$M_0 + tM_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Maclaurin proves in Part II that the pedal of a conic when the pole is at the focus is a circle for the central conic, and a straight line for the parabola.

The converse also holds, and the analysis shows that the pencil of perpendiculars through the focus is in projective correspondence with the tangents to the conic, *i.e.* corresponding ray and tangent have equations of the form

$$\left. \begin{aligned} M_0 + tM_1 &= 0 \\ L_0 + tL_1 + t^2L_2 &= 0 \end{aligned} \right\}$$

though not the most general of this kind.

Let  $O_1$  be the focus, and let  $O_2$  be another point the rays through which are in 1-1 correspondence with the rays through  $O_1$ , so that corresponding rays intersect in R on a conic passing through  $O_1$  and  $O_2$ . This latter conic may be replaced by a straight line without loss of generalisation. For let T be any point on it. Let

$$\angle TO_1O_2 = \alpha'$$

$$\angle TO_2O_1 = \beta'.$$



Make

$$\begin{aligned}\angle RO_1S &= \alpha' \\ \angle RO_2S &= \beta'\end{aligned}$$

and let R move on the conic. Then S generates a straight line.

Let  $O_1S$  cut  $PQ$  in  $P'$ . Since  $PO_1P'$  is constant, and P lies on a straight line, the locus of  $P'$  is, by the lemma, likewise a straight line; and the angle  $SO_2Q$  is constant. Hence we obtain a reduction to Prop. V as for the quadrilateral  $P'SO_2Q$ ; and thence to Prop. VII. We may therefore assume that R lies on a straight line, and P on a straight line or circle. It remains to prove that the locus of P may without loss of generality be taken to be a straight line in general, so that we obtain a reduction to Maclaurin's generation of the singular cubic.

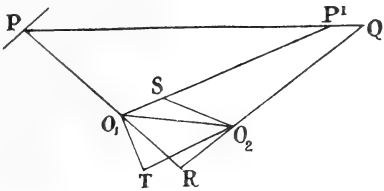


FIG. 14.

Let the cubic be given by the intersection of

$$a_0x + b_0y + c_0 + t(a_1x + b_1y + c_1) + t^2(a_2x + b_2y + c_2) = 0$$

or

$$L_0 + tL_1 + t^2L_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$m_0x + n_0y + p_0 + t(m_1x + n_1y + p_1) = 0$$

or

$$M_0 + tM_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These may be replaced by

$$L_0 + tL_1 + t^2L_2 + (At + B)(M_0 + tM_1) = 0 \quad . \quad . \quad . \quad . \quad (3)$$

and

$$M_0 + tM_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which A and B are arbitrary.

The equation (3) will envelop a parabola provided a value of  $t$  can be found for which (3) is the line at infinity, *i.e.* so that

$$a_0 + ta_1 + t^2a_2 + (At + B)(m_0 + tm_1) = 0 \quad . \quad . \quad . \quad . \quad (5)$$

$$b_0 + tb_1 + t^2b_2 + (At + B)(n_0 + tn_1) = 0 \quad . \quad . \quad . \quad . \quad (6)$$

Hence  $t$  must be such that

$$\frac{a_0 + ta_1 + t^2a_2}{b_0 + tb_1 + t^2b_2} = \frac{m_0 + tm_1}{n_0 + tn_1} \quad . \quad . \quad . \quad . \quad (7)$$





Hence we may replace P by T and  $l$  by  $l''$ .

Cor. 5. If  $O_1EB = O_1PQ$  the locus is a conic and not a cubic. For in such a case  $O_1PEQ$  is a cyclic quadrilateral and  $QO_1P$  is constant, being the supplement of  $PEQ$ , so that the locus is that of Prop. I.

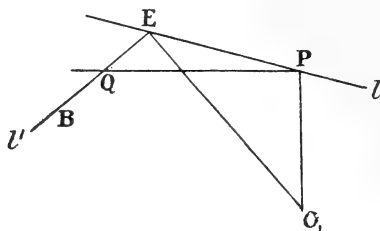


FIG. 17.

§ 17. *Prop. XI.*

If, in *Prop. X*,  $O_1P$  and  $O_2R$  simultaneously coincide with  $O_1O_2$ , then the curve degenerates into a conic.

Cor. 1. Thus, if

$$O_2AB = \alpha$$

$$AO_2B = \beta,$$

where B is on  $l'$ , the locus is a conic.

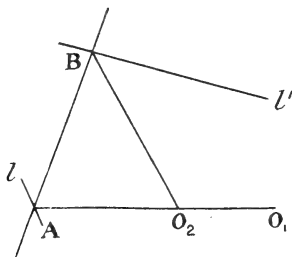


FIG. 18.

Cor. 2. In particular, if  $\alpha + \beta = \pi$ , and  $l'$  parallel to AB, the curve is a conic, e.g. when  $\alpha = \beta = \frac{\pi}{2}$ , and  $l' \perp O_1O_2$ .

THE CIRCULAR CUBIC WITH A DOUBLE POINT.

§ 18. *Lemma II.*

This lemma, along with the corollaries attached to it by Maclaurin, contains a variety of ways of tracing an important species of cubics to which Teixeira has recently drawn attention (*Proc. Ed. Math. Society*, 1912).

$O_1$  and  $O_2$  are two fixed points,  $P$  any point on a fixed line  $l$ . If  $O_1PN = a$  is constant and  $Q$  is taken on  $PN$  so that  $O_2QN$  is constant,  $= \beta$ , then the locus of  $Q$  is a cubic.

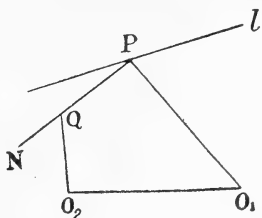


FIG. 19.

[We may note that, if  $O_1P$  and  $O_2Q$  cut in  $R$ , the locus of  $R$  is a segment of a circle on  $O_1O_2$ . Hence another method of generating the curve.

Of course, Maclaurin is restricted to the use of linear loci only.]

Maclaurin first shows that we may, without loss of generality, suppose  $\alpha = \beta = \frac{\pi}{2}$ , so that  $O_1P$  and  $O_2Q$  cut on the line at infinity, and the lemma is a particular case of Prop. VII.

For draw  $O_1B \perp O_1O_2$  as in fig. 20, and make  $O_1O_2B = \frac{\pi}{2} - \beta$ , so that  $B$  is a fixed point.

Draw  $O_1R$  parallel to  $O_2Q$ , meeting  $PQ$  in  $R$ ; so that, by Lemma I,  $R$  generates a straight line. Draw  $RS$  parallel to  $O_1B$ , and  $QS$  perpendicular

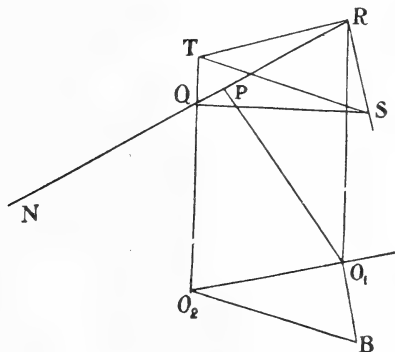


FIG. 20.

to  $O_2Q$ ; also  $RT$  parallel to  $O_1O_2$ , cutting  $O_2Q$  in  $T$ . Then  $SRTQ$  is cyclic, and  $RTS = RQS = \frac{\pi}{2} - \beta = O_1O_2B$ .

But  $RT = O_1O_2$ . Hence  $\triangle RTS \simeq \triangle O_1O_2B$  and  $RS = O_1B$  is constant.

Therefore S generates a straight line  $l'$ , and BS, parallel to  $O_2T$ , is perpendicular to SQ.

But B is fixed, S lies on  $l'$ ,  $BSQ = O_2QS = \frac{\pi}{2}$ ;  $\therefore$  etc.

[Since PN in the original construction is always tangent to a parabola, the constant angle  $\beta$  shows that the locus of Q is simply the oblique pedal of a parabola and falls to be discussed in Part II as a pedal.]

We now assume

$$\alpha = \beta = \frac{\pi}{2}.$$

Maclaurin notes when  $O_2$  is a node, a conjugate point, or a cusp. When  $l$  passes through  $O_2$  and is  $\perp^r O_1O_2$  the curve is the cissoid.

### § 19. Equation to the Curve.

Choose the origin at  $O_2$ .

Let  $O_1$  be the point  $(a, b)$ ; and let the equation to  $l$  be

$$y = mx + n \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let P be the point

$$(\xi, m\xi + n),$$

so that the gradient of  $O_1P$  is

$$\frac{m\xi + n - b}{\xi - a},$$

and PQ has the equation

$$y - m\xi - n = \frac{a - \xi}{m\xi + n - b}(x - \xi) \quad . \quad . \quad . \quad . \quad (2)$$

while  $O_2Q$  has the equation

$$y = \frac{m\xi + n - b}{\xi - a}x \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To obtain the locus of Q, eliminate  $\xi$  between (2) and (3).

$$\therefore (y - mx)(x^2 + y^2) + x^2(b - n) + xy(bm - a) - y^2(am + n) = 0 \quad . \quad . \quad (4)$$

Now any circular cubic with double point at O may be written as

$$(y - \lambda x)(x^2 + y^2) + Ax^2 + Bxy + Cy^2 = 0 \quad . \quad . \quad . \quad . \quad (5)$$

If (4) and (5) represent the same curve, we must have

$$\left. \begin{array}{l} m = \lambda \\ b - n = A \\ bm - a = B \\ -am - n = C \end{array} \right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

These determine  $m, n, a, b$  uniquely, corresponding to any equation (5).

Maclaurin's generation therefore furnishes all the circular cubics.

To the lemma Maclaurin attaches several corollaries of special interest.

Cor. 1. Cissoidal Generation of the Curve.

Draw  $PT$  parallel to  $O_1O_2$  cutting  $O_2Q$  in  $T$ . Describe the semicircle  $O_1RO_2$ . Then  $PO_1RQ$  is a rectangle, and  $PO_1O_2T$  is a parallelogram.

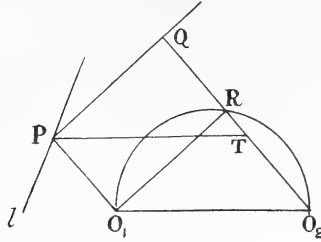


FIG. 21.

Hence

$$QR = O_1P = O_2T,$$

so that

$$TQ = O_2R.$$

Now  $T$  traces out a line  $l'$ .

Thus, to get the locus of  $Q$ , take  $T$  any point on  $l'$  and let the circle determine the chord  $O_2R$  on  $O_2T$ . Produce  $O_2T$  to  $Q$  so that  $TQ = O_2R$ .

Cor. 2. The same results obtain if on  $O_1O_2$  is described a segment of a circle instead of a semicircle.

Cor. 3 gives another method of generating the curve.

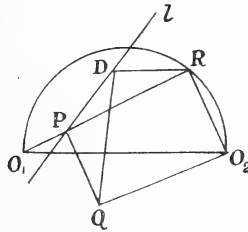


FIG. 22.

On  $O_1O_2$  describe the semicircle  $O_1RO_2$ . Draw  $O_2D$  at right angles to  $l$ .

Then  $O_2RDPQ$  lie on a circle whose diameter is  $O_2P$ ; and  $PRO_2Q$  is a rectangle. Also  $RDQ = RO_2Q = \frac{\pi}{2}$ .

Hence rotate two right angles  $RDQ$  and  $RO_2Q$  round the two fixed points  $D$  and  $O_2$ , and let  $R$  trace out the semicircle, when  $Q$  generates the cubic. Cf. § 39.

Cor. 4 contains a generalisation of Cor. 3, as Cor. 2 is of Cor. 1.

Let

$$O_1PQ = \alpha, \quad O_2QP = \beta.$$

Describe a circle round  $O_2QP$  cutting  $l$  in  $D$ , and  $O_1P$  again in  $R$ .  $D$  is  $\therefore$  a fixed point, and  $O_1RO_2 = \pi - \beta$ , so that  $R$  generates a segment of a circle on  $O_1O_2$ .

Also  $RDQ$  is constant  $=\pi-\alpha$ , and  $RO_2Q=\alpha$ . Hence in Cor. 3 replace the right angle  $RDQ$  by  $\pi-\alpha$ , and  $RO_2Q$  by  $\alpha$ .

Cor. 5. (The Strophoid.)

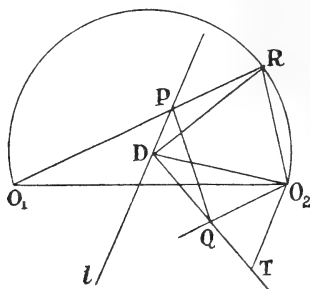


FIG. 23.

Let  $\alpha=\beta$ , and let  $D$  coincide with the centre of the circle  $O_1RO_2$ . Draw  $O_2T$  parallel to  $l$  cutting  $DQ$  in  $T$ .

Then

$$O_2QT = O_2RD = RO_2D = O_1PD = QO_2T.$$

Hence

$$TQ = TO_2.$$

Hence (Barrow's) generation of the curve:—

$D$  is a fixed point in the plane, and  $T$  any point on a fixed line  $O_2T$ .

If  $Q$  is taken on  $DT$  so that  $TQ(=TQ')=O_2T$ , the locus of  $Q$  is the strophoid (oblique, or right when  $DO_2$  is perpendicular to  $O_2T$ ).

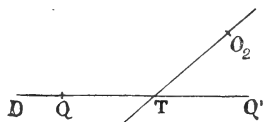


FIG. 24.

Cor. 7. In this corollary Maclaurin generalises the construction of Cor. 5 by taking for the point  $O_2$  any point in the plane,  $T$  being still on the line  $l$ , while  $TQ=Tq=O_2T$ .

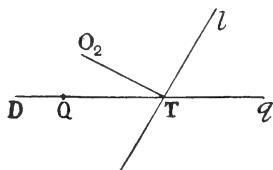


FIG. 25.

When the origin is taken at  $D$ , with the  $y$ -axis parallel to  $l$ , Teixeira gives the equation to the locus of  $Q$  as



$$x(x^2 + y^2) - 2a(x^2 + y^2) + (2aa - \alpha^2 - \beta^2)x + 2\alpha\beta y = 0.$$

( $O_2$  is the point  $(\alpha, \beta)$  and  $l$  is the line  $x - \alpha = 0$ .)

Teixeira points out that the identical locus is discussed by Lagrange (*Nouvelles Annales*, 1900), and that the equation represents part of the curves known as Van Rees' Focals, for which the equation may be reduced to

$$x(x^2 + y^2) = A(x^2 + y^2) + Bx + Cy.$$

Maclaurin shows that the curve has a closed oval and a serpentine branch save (Cor. 8) when  $O_2$  is on the line  $l$ , when there is a node.

Cor. 10. If  $O_2$  is on  $l$ , and  $DO_2$  perpendicular to  $l$ , the curve is that described by De Moivre in No. 345 of the *Philosophical Transactions*.

Cor. 11. The strophoid may also be thus generated:—

$D$  and  $O_2$  are fixed points, and  $O_2P$  a fixed line  $l$ .

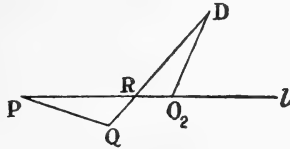


FIG. 26.

$PQD$  is a constant angle  $= PO_2D$ , and  $PQ = DO_2$ . Then, as  $P$  slides on  $l$ , point  $Q$  generates the strophoid ( $RO_2 = RQ$ ).

Also the mid-point of  $PQ$  generates the cissoid of Diocles when  $PQD = \frac{\pi}{2}$  (Newton).

[The description of the strophoid as the intersection of two rays rotating round two fixed centres with angular velocities in the ratio 1: 2 is ascribed to Plateau (1828) by Kohn and Loria in their article on Special Plane Curves in the *Encyk. der. Math. Wiss.* This is historically inaccurate, for Maclaurin gave this generation three-quarters of a century earlier in his *Fluxions* (p. 262).]

## § 20. Prop. XII

discusses the asymptotes and also the subvarieties of the curves of Prop. X.

Ex. 1. Let  $\alpha = \beta = \frac{\pi}{2}$ ;  $l \perp O_1O_2$ ;  $l' \parallel O_1O_2$ .

If  $O_1$  is the origin,  $O_2$  the point  $(d, 0)$ ; equation to  $l$ ,  $x = a$ ; equation to  $l'$ ,  $y = b$ ; the locus of  $R$  is given by

$$ay^2(x - d) + bdx y = (d - a)x^2(x - d).$$

The case  $l$  and  $l'$  both parallel to  $O_1O_2$  is discussed in Ex. 4.

*Ex. 5.*  $O_1PQ = \frac{\pi}{2}$ ;  $l'$  and  $l \perp O_1O_2$ ;  $QO_2R$  three collinear points.

$$\text{Equation } xy^2(b-d) = (x-d)(ay^2 + \overline{a-bx^2}).$$

§ 21. *Prop. XIII.*

When  $Q$  and  $R$  move on fixed straight lines  $l'$  and  $l''$ , then the locus of  $P$  is in general a cubic with a double point at  $O_1$ .

Maclaurin's proof is analytic.

The geometrical method he would employ later runs thus:—

Leave  $Q$  free, and restrict  $P$  to lie on a straight line  $m$ . Then  $Q$  lies on a cubic cutting  $l'$  in three points  $Q_1, Q_2, Q_3$ , to which correspond  $P_1, P_2, P_3$  on  $m$ . Hence, when  $Q$  lies on  $l'$ ,  $P$  traces out a curve cut by  $m$  in three points  $P_1, P_2, P_3$ . The curve is therefore in general a cubic.

But it may degenerate.

### SECTION III.

#### ON THE DESCRIPTION OF LINES OF THE FOURTH ORDER, AND THOSE OF THE THIRD ORDER WHICH HAVE NO DOUBLE POINT.

§ 22. “We have described Lines of the Second Order by the rotation of two constant angles round two fixed points; also Lines of the Third Order by the use of as many angles, of which we have supposed one to be rotated round a fixed point, while the other is conducted along a fixed straight line.

“We now proceed to the description of Lines of the Fourth Order by conducting each angle along a straight line.” (The quartics obtained have either two or three double points.)

*Prop. XIV.*

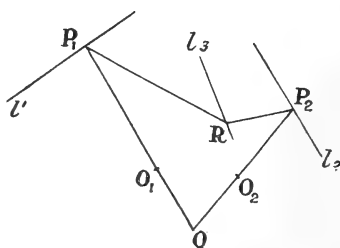


FIG. 27.

Given  $O_1$  and  $O_2$  two fixed points;  $\angle O_1P_1R = \alpha$ ;  $\angle O_2P_2R = \beta$ , constant angles, where  $P_1$  and  $P_2$  lies on fixed lines  $l_1$  and  $l_2$  respectively.

If  $R$  is restricted to lie on a straight line  $l_3$ , the intersection  $Q$  of  $O_1P_1$  and  $O_2P_2$  in general generates a quartic having double points at  $O_1$  and  $O_2$ .

*Dem.*

Let  $O_1P_1$  have equation

$$L_1 + \lambda L_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and  $O_2P_2$  have equation

$$M_1 + \mu M_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Then  $P_1R$  has an equation of the form

$$\lambda^2 N_1 + \lambda N_2 + N_3 = 0,$$

or

$$xA_2(\lambda) + yB_2(\lambda) + C_2(\lambda) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Similarly,  $P_2R$  has an equation of the form

$$xf_2(\mu) + y\phi_2(\mu) + \psi_2(\mu) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The condition that  $R$  lies on the line  $l_3$ , viz. on

$$lx + my + n = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

gives rise to the condition

$$\begin{vmatrix} l & m & n \\ A_2(\lambda) & B_2(\lambda) & C_2(\lambda) \\ f_2(\mu) & \phi_2(\mu) & \psi_2(\mu) \end{vmatrix} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In (6) substitute  $-L_1/L_2$  for  $\lambda$ , and  $-M_1/M_2$  for  $\mu$ , when we obtain a quartic equation for the locus of  $Q$  representing a quartic curve having double points at  $O_1$  and  $O_2$ .

The biquadratic relation (6) at once indicates the *genre* of the curve.

The existence of the double points is deduced analytically in Cor. 1, geometrically in Cor. 2; and the six possible varieties of these are enumerated in Cor. 4.

### § 23. Prop. XV.

If  $P_1Q$  and  $P_2Q$  coincide simultaneously with  $O_1O_2$ , the quartic degenerates into the straight line  $O_1O_2$  and a cubic curve through  $O_1O_2$  devoid of double points.

Cor. 2. This can happen when  $l_1$  and  $l_2$  cut on  $O_1O_2$  and  $\alpha + \beta = \pi$ .

Cor. 3. Also when  $\alpha + \beta = \pi$  and  $l_3$  makes an angle  $\alpha$  with  $O_1O_2$ .

For, when  $P_1$  comes to lie on  $O_1O_2$ ,  $R$  goes to infinity on  $l_3$ , while  $O_1Q$  and  $O_2Q$  coincide simultaneously with  $O_1O_2$ .

Cor. 4. In particular this will happen when  $\alpha = \beta = \frac{\pi}{2}$ , and either  $l_1$  and  $l_2$  intersect on  $O_1O_2$ , or  $l_3 \perp O_1O_2$ .

Cor. 10. It can also happen when  $l_1, l_2, l_3$  are parallel, and  $\alpha$  and  $\beta$  are the angles at which they cut  $O_1O_2$ .

§ 24. *Prop. XVI.*

Let  $l_1$  and  $l_3$  cut in  $A$ . If  $O_1Al_3 = \alpha$  the curve is a cubic (with a double point).

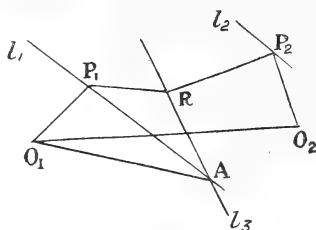


FIG. 28.

For  $O_1ARP$  is a cyclic quadrilateral.

Hence  $P_1O_1R = P_1AR$  is constant, so that there is a reduction to Prop. V.

Similarly, if  $l_2$  and  $l_3$  cut in  $A_2$  and  $O_2Al_3 = \beta$  the curve is a cubic.

When both hypotheses hold the curve is a conic, as in Prop. I.

§ 25. *Prop. XVII.*

When in the quadrilateral  $P_1RP_2Q$  it is  $Q$  and not  $R$  that is restricted to lie on a straight line, the locus of  $R$  is a quartic curve.

*Dem.*

Let as before  $O_1P_1$  and  $O_2P_2$  be given by

$$L_1 + \lambda L_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$M_1 + \mu M_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Then the condition that  $Q$  lies on  $l_3$  leads to a relation

$$\mu = (a\lambda + b)/(c\lambda + d).$$

Hence the equations to  $P_1R$  and  $P_2R$  may be written in the form

$$\lambda^2 L_1 + \lambda M_1 + N_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\lambda^2 L_2 + \lambda M_2 + N_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the elimination of  $\lambda$  from (3) and (4) leads to a quartic equation in  $x$  and  $y$ .

Cor. 1. The curve does not pass through  $O_1$  or  $O_2$ .

[This description of a quartic is of especial interest. Maclaurin does not observe that the curve must possess three double points; for in virtue





or

$$\frac{\xi^2}{c^2}d + \xi\left(\frac{a}{c} - \frac{x}{c} - \frac{2ad}{c^2}\right) - y + d + \frac{a}{c}\left(x - a + \frac{ad}{c}\right) = 0 \quad (2)$$

Similarly  $P_2R$  has the equation

$$\frac{\xi^2}{c^2}\delta + \xi\left(\frac{b}{c} - \frac{x}{c} - \frac{2b\delta}{c^2}\right) - y + \delta + \frac{b}{c}\left(x - b + \frac{b\delta}{c}\right) = 0 \quad (3)$$

On solving (2) and (3) for  $\xi^2$  and  $\xi$  we obtain

$$\xi^2 = \frac{ax^2 + \beta xy + \gamma x + \delta y + \epsilon}{Ax + B}$$

$$\xi = \frac{\lambda x + \mu y + \nu}{Ax + B}.$$

$\therefore$  the equation to the locus of  $R$  is

$$(\lambda x + \mu y + \nu)^2 = (Ax + B)(ax^2 + \dots + \epsilon) \quad (4)$$

Cor. 7. If, as before,  $\alpha = \beta = \frac{\pi}{2}$ , and  $l_1$  and  $l_2$  coincide, the curve is a cubic.

For, let  $l_1$  and  $l_3$  cut  $O_1O_2$  in  $D_1$  and  $D_3$  respectively.

Then  $D_1E \perp O_1O_2$  forms part of the locus.

#### § 27. Prop. XIX.

If in the figure of Prop. XIV  $Q$ ,  $P_2$ , and  $R$  are restricted to lie on straight lines, the point  $P_1$  generates a quartic with a triple point at  $O_1$ .

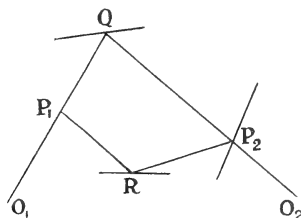


FIG. 31.

Take  $O_1$  as origin.

Let  $O_1Q$  have the equation

$$y - \mu x = 0 \quad (1)$$

and  $O_2Q$  have the equation

$$y = m(x - a) \quad (2)$$

There is  $\therefore$  a 1-1 correspondence between  $m$  and  $\mu$ .

$P_2R$  has an equation of the form

$$xf_2(\mu) + y\phi_2(\mu) + \psi_2(\mu) = 0 \quad (3)$$





## SECTION IV.

WHEREIN ARE DEMONSTRATED GENERAL THEOREMS REGARDING THE DESCRIPTION OF CURVES OF ANY ORDER BY THE USE ONLY OF LINEAR LOCI AND CONSTANT ANGLES.

§ 29. This section takes up the discussion from a more general point of view, and, while Maclaurin's theorems are adhered to in their order, their demonstrations, when analytical, are frequently altered. Before we proceed to these it will be convenient, just as Maclaurin does, to pave the way by some preliminary theorems.

Instead of an ordinary angle he makes use of what may be termed a *serrate angle* consisting of a broken line  $OP_1P_2 \dots P_nP$ , in which the component angles at the teeth are of constant magnitude, while the

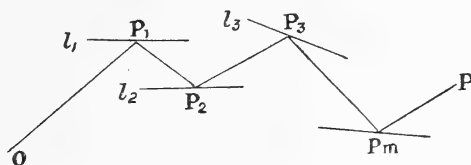


FIG. 32.

segments of the line are freely variable. The vertices  $P_1P_2 \dots P_n$  lie on linear loci  $l_1, l_2, \dots l_n$ , and  $O$  is a fixed point.

Let the equation to  $OP_1$  depend on a parameter  $t$ , and be given by

$$L_0 + tL_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Then the equation to  $P_1P_2$  is of the form

$$M_0 + tM_1 + t^2M_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Similarly, for  $P_2P_3$  we in general find an equation of the form

$$N_0 + tN_1 + t^2N_2 + t^3N_3 = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

etc., etc.

The lines  $P_1P_2, P_2P_3$ , etc., envelop unicursal curves of class 2 (parabola), 3, etc., having a special relation to the line at infinity.

Also the co-ordinates of  $P_n$  are rational functions of  $t$  of degree  $n$ .

## § 30. Prop. XX.

Let  $O_1P_1P_2 \dots P$  be a serrate angle ( $n$  lines  $l_1, l_2, \dots l_n$ ),  $O_2$  a second fixed point through which  $O_2Q$  is drawn such that  $O_2QP_n$  is a constant angle, then the locus of  $Q$  is a curve of degree  $n+2$ .

For  $P_nQ$  has an equation of the form

$$L_0 + tL_1 + \dots + t^{n+1}L_{n+1} = 0 \quad (4)$$

and  $O_2Q$ , which really makes a constant angle with  $O_1P_1$ , has an equation of the form

$$M_0 + tM_1 = 0 \quad (5)$$

The elimination of  $t$  between (4) and (5) leads to an  $(n+2)$ -ic having an  $(n+1)$ -ple point at  $O_2$ .

[We might state the theorem thus. Given  $O_1$  and  $O_2$  fixed points, and the serrate angle

$$O_1P_1P_2 \dots P_nQO_2,$$

in which  $P_1 \dots P_n$  lie on fixed straight lines, the locus of  $Q$  is an  $(n+2)$ -ic with an  $(n+1)$ -ple point at  $O_2$ . Or, again, the locus of  $Q$  is simply a pedal of the envelope of  $P_nQ$ .]

### § 31. Prop. XXI.

Given the serrate angle  $O_1P_1 \dots P_{n-1}Q$  ( $n-1$  lines  $l_1, l_2, \dots, l_{n-1}$ ) and the constant angle  $R_1O_2R_2$  which is rotated round  $O_2$ . If the intersection  $R_1$  of  $O_1P_1$  and  $O_2R_1$  lies on a fixed line  $l_n$ , then the intersection  $Q$  of  $P_{n-1}Q$  and  $O_2R_2$  generates a curve of degree  $n+1$ .

For the equation to  $P_{n-1}Q$  is of the form

$$L_0 + tL_1 + \dots - t^nL_n = 0 \quad (1)$$

In virtue of  $l_n$  the parameter of  $O_2R_1$  and  $\therefore$  of  $O_2R_2$  is in 1-1 correspondence with  $t$ , so that the equation to  $O_2R_2$  is of the form

$$M_0 + tM_1 = 0 \quad (2)$$

The elimination of  $t$  between (1) and (2) gives rise to an  $(n+1)$ -ic with an  $n$ -ple point at  $O_2$ .

Cor. The curve may, of course, degenerate and be of lower order in its component curves.

Cor. 6. The angle  $R_1O_2R_2$  may be a straight angle, so that  $R_1O_2R_2$  is a straight line rotating round  $O_2$ .

Cor. 7. When  $n=3$ , the curve is a quartic with a triple point at  $O_2$ .

### § 32. Prop. XXII.

If all the points but one of the  $n+1$  points

$$R_1P_1 \dots P_{n-1}Q$$

are restricted to lie on straight lines, the remaining point generates a curve of degree  $n+1$ .

The proof is exactly on the lines of Prop. XIII.

§ 33. *Prop. XXIII.*

Let the intersection of  $R_1O_2$  and  $P_{r-1}P_r$  lie on the straight line

$$lx + m'y + n' = 0 \quad (1)$$

and  $Q$  will generate a curve of degree  $n+r$ .

Let  $R_1O_2$  be given by the equation

$$M_0 + \mu M_1 = 0$$

or

$$xA_1(\mu) + yB_1(\mu) + C_1(\mu) = 0 \quad (2)$$

Then  $P_{r-1}P_r$  has an equation of the form

$$xf_r(t) + y\phi_r(t) + \psi_r(t) = 0 \quad (3)$$

The condition that (1), (2), (3) be concurrent is

$$\begin{vmatrix} l' & m' & n' \\ A_1(\mu) & B_1(\mu) & C_1(\mu) \\ f_r(t) & \phi_r(t) & \psi_r(t) \end{vmatrix} = 0 \quad (4)$$

Hence  $\mu$  = a rational function of  $t$  of degree  $r$  in numerator and denominator, and the equation to  $O_2R_2$  may be written in the form

$$N_0 + tN_1 + \dots + t^rN_r = 0 \quad (5)$$

But  $P_{n-1}Q$  has an equation of the form

$$M_0 + tM_1 + \dots + t^nM_n = 0 \quad (6)$$

The equations (5) and (6) therefore give for the locus of  $Q$  a unicursal curve of degree  $n+r$ .

Cor. 1. The line (6) envelops a curve of class  $n$ . Hence  $n$  lines of the system pass through  $O_2$ , so that  $O_2$  is an  $n$ -ple point on the locus of  $Q$ .

Cor. 3. When of the points  $R_1QP_1 \dots P_{n-1}$  all but one lie on fixed straight lines, the remaining point generates a curve of degree  $n+r$ .

Cor. 5. By variation of  $n$  and  $r$  subject to the condition  $n+r = \text{constant}$ , we may deduce a variety of ways of drawing curves of degree  $n+r$ .

Cor. 6 is not correct.

Maclaurin states the following generation of a quartic:—

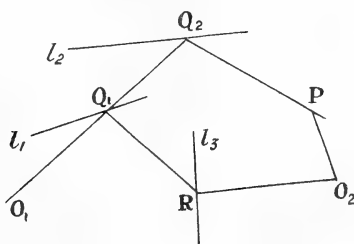


FIG. 33.



Then  $R_2(t)$  and

$$\begin{vmatrix} f_2(t) & -\phi_2(t) \\ P_2(t) & Q_2(t) \end{vmatrix}$$

vanish when  $t = \alpha$ .

Hence on solving (5) and (6) for  $x$  and  $y$  we find

$$\left. \begin{aligned} x &= f_3(t)(t-\alpha)/\phi_3(t)(t-\alpha) \\ y &= \psi_3(t)(t-\alpha)/\phi_3(t)(t-\alpha) \end{aligned} \right\} \quad \therefore \text{etc.} \quad (8)$$

§ 34. Prop. XXIV.

Consider two serrate angles

$$O_1P_1P_2 \dots P_mP$$

and

$$O_2Q_1Q_2 \dots Q_nQ$$

in which  $P_1P_2 \dots P_m$  lies on  $m$  fixed lines, and  $Q_1Q_2 \dots Q_n$  on  $n$  fixed lines.

If the intersection of  $P_mP$  and  $Q_nQ$  also lies on a given straight line, the intersection of  $O_1P_1$  and  $O_2Q_1$  in general generates a curve of degree  $n+m+2$  possessing an  $(m+1)$ -ple point at  $O_1$  and an  $(n+1)$ -ple point at  $O_2$ .

Let  $O_1P_1$  have equation

$$y - \lambda x = 0 \quad (1)$$

Then  $P_mP$  has an equation of the form

$$xA_{m+1}(\lambda) + yB_{m+1}(\lambda) + C_{m+1}(\lambda) = 0 \quad (2)$$

If  $O_2Q_1$  has an equation of the form

$$L_1 + tL_2 = 0 \quad (3)$$

$Q_nQ$  has an equation of the form

$$xA_{n+1}(t) + yB_{n+1}(t) + C_{n+1}(t) = 0 \quad (4)$$

Let  $P_mP$  and  $Q_nQ$  intersect on

$$ax + by + c = 0 \quad (5)$$

Hence

$$\begin{vmatrix} a & b & c \\ A_{m+1}(\lambda) & B_{m+1}(\lambda) & C_{m+1}(\lambda) \\ A_{n+1}(t) & B_{n+1}(t) & C_{n+1}(t) \end{vmatrix} = 0 \quad (6)$$

Substitute  $y/x$  for  $\lambda$ , and  $-L_1/L_2$  for  $t$  in (6), when the result follows at once.

Cor. 2. If of the points  $P_1P_2 \dots P_mQ_1Q_2 \dots Q_nRT$  ( $T$  being the intersection of  $O_1P_1$  and  $O_2Q_1$ , and  $R$  of  $P_mP$  and  $Q_nQ$ ), all but one lie on straight lines, the remaining point generates a curve of degree  $n+m+2$ .

Cor. 4. There is no change in the nature of the curve if, instead of the intersection of  $O_1P_1$  and  $O_2Q_1$ , we take the intersection of two lines through  $O_1$  and  $O_2$  making given angles with  $O_1P_1$  and  $O_2Q_1$  (in virtue of the 1-1 correspondence).

Cor. 5. The number  $n+m+2$  for the degree is a maximum, and may not always be attained.

### § 35. Prop. XXV.

If the intersection of  $P_{s-1}P_s$  and  $O_2Q_1$  is restricted to lie on a straight line, the point of intersection of  $P_mP$  and  $Q_nQ$  is on a curve of degree  $ns+s+m+1$

For  $P_{s-1}P_s$  has an equation of the form

$$xA_s(\lambda) + yB_s(\lambda) + C_s(\lambda) = 0 \quad (1)$$

and  $O_2Q_1$  of the form

$$L_1 + tL_2 = 0 \quad (2)$$

and  $\therefore$

$$t = f_s(\lambda)/\phi_s(\lambda), \text{ say} \quad (3)$$

But  $P_mP$  and  $Q_nQ$  have equations

$$xA_{m+1}(\lambda) + yB_{m+1}(\lambda) + C_{m+1}(\lambda) = 0 \quad (4)$$

and

$$xA_{n+1}(t) + yB_{n+1}(t) + C_{n+1}(t) = 0$$

or

$$xA_{ns+s}(\lambda) + yB_{ns+s}(\lambda) + C_{ns+s}(\lambda) = 0 \quad (5)$$

and the desired result follows from (4) and (5).

### § 36. Prop. XXVI

If the intersection of  $P_mP$  and  $Q_nQ$  is on the line

$$ax + by + c = 0 \quad (1)$$

then the intersection of  $P_{r-1}P_r$  and  $Q_{s-1}Q_s$  generates a curve of degree

$$r(n+1) + s(m+1).$$

We have the relation

$$\begin{vmatrix} a & b & c \\ A_{m+1}(\lambda) & B_{m+1}(\lambda) & C_{m+1}(\lambda) \\ A_{n+1}(t) & B_{n+1}(t) & C_{n+1}(t) \end{vmatrix} = 0 \quad (2)$$

while  $P_{r-1}P_r$  and  $Q_{s-1}Q_s$  have equations of the form

$$XA_r(\lambda) + yB_r(\lambda) + C_r(\lambda) = 0 \quad (3)$$

and

$$XA_s(t) + yB_s(t) + C_s(t) = 0 \quad (4)$$

In how many points can the curve given by the intersection of (3) and (4) be cut by the straight line

$$Ax + By + C = 0? \quad (5)$$

At such points we must have

$$\begin{vmatrix} A & B & C \\ A_r(\lambda) & B_r(\lambda) & C_r(\lambda) \\ A_s(t) & B_s(t) & C_s(t) \end{vmatrix} = 0 \quad (6)$$

taken along with (2).

By the theory of equations the  $t$ -eliminant of (2) and (6) is of degree

$$(m+1)s + (n+1)r,$$

and this number therefore represents the number of intersections of the curve with a straight line, and so the degree of the curve.

Cor. 2. The theorem may be extended as in Cor. 2 of Prop. XXIV.

§ 37. Prop. XXVII.

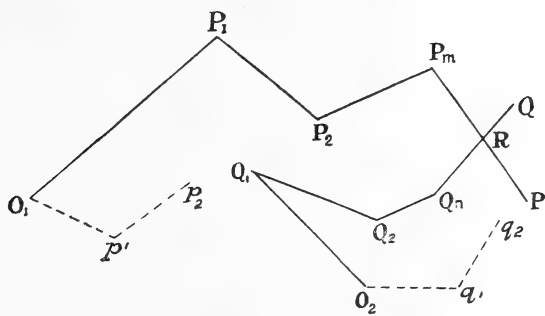


FIG. 34.

Suppose that, in addition to the data of Prop. XXIV, there are given the serrate angles

$$\begin{matrix} P_1O_1p_1 & \dots & p_{r-1}p \\ Q_1O_2q_1 & & q_{s-1}q, \end{matrix}$$

then the intersection of  $p_{r-1}p$  and  $q_{s-1}q$  is on a curve of degree  $ms + nr + s + r$ .

For the datum that  $P_mP$  and  $Q_nQ$  intersect on a straight line leads to (2) of preceding.

There is a 1-1 correspondence between  $P_1O_1$  and  $O_1p_1$  so that  $p_{r-1}p$  has an equation like (3). Similarly,  $q_{s-1}q$  has an equation like (4).  $\therefore$  etc.

SCHOLIUM.

§ 38. In the scholium Maclaurin gives credit to Fermat, Varignon, De la Hire, Nicole for special curves: and to Newton's great work on Cubic

Curves. He points out the desirability of having a general method of generating curves of all degrees. The method employed does not give all curves, but it may serve to pave the way for future perfection of the theory.

In the part just completed only straight lines and constant angles have been employed. In Part II other curve loci are utilised from which to obtain more complicated curves of higher degree.

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## Part II.

### Wherein Curves of all Higher Orders are described by the Use of Curves of Lower Order.

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#### SECTION I.

##### § 39. NEWTON'S ORGANIC DESCRIPTION OF CURVES.

*Prop. I.*

*Round the fixed points  $O_1$  and  $O_2$  are rotated the constant angles  $PO_1Q = \alpha$ ,  $PO_2Q = \beta$ .*

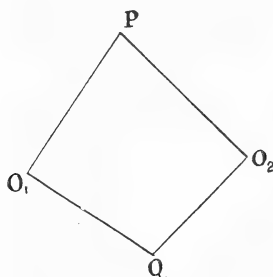


FIG. 35.

*If  $P$  traces out a conic through  $O_1$ ,  $Q$  generates a cubic having a double point at  $O_1$  and an ordinary point at  $O_2$ .*

Maclaurin's proof runs thus. Find in how many points a straight line  $l$  can cut the curve, *i.e.* how many points  $Q$  can lie on  $l$ .

Let  $Q$  trace out the line  $l$ ,  $P$  being left free.  $P$  will generate a conic through  $O_1$  and  $O_2$  cutting the given conic in four points  $O_1, P_1, P_2, P_3$ . To  $P_1, P_2, P_3$  correspond three points  $Q_1, Q_2, Q_3$  on  $l$ : so that the locus cuts  $l$  in three points and is therefore a cubic.



Let  $O_1O_2R=\beta$  and let  $O_2R$  cut the given conic in  $R$  and  $R'$ . Then, when  $P$  comes to  $R$  or to  $R'$ ,  $Q$  comes to  $O_1$ , which is thus a double point. Similarly it passes once through  $O_2$ .

Cor. 6. If  $O_1Q$  and  $O_2Q$  coincide simultaneously with  $O_1O_2$ , the curve reduces to a conic.

Cor. 8. Particular cases of Newton's organic description as a Cremona transformation when  $\alpha=\beta=\frac{\pi}{2}$ .

Choose  $O_1$  as origin, and  $O_2$  as  $(a, 0)$ .

Then, if  $P$  is  $(\xi, \eta)$  and  $Q$   $(\xi', \eta')$ ,

$$\xi' = a - \xi \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\eta' = \xi(\xi - a)/\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus :

(I.) To

$$lx + my + n = 0$$

corresponds

$$l(a - x)y - mx(a - x) + ny = 0,$$

a conic through

$$(0, 0); (a, 0); (a, \infty).$$

(II.) To the parabola

$$y^2 - mx = 0$$

corresponds

$$x^2(x - a)^2 + m(x - a)y^2 = 0,$$

or

$$my^2 = x^2(a - x).$$

(III.) To the rectangular hyperbola

$$x^2 - y^2 = mx$$

corresponds

$$y^2(a - m - x) = x^2(a - x).$$

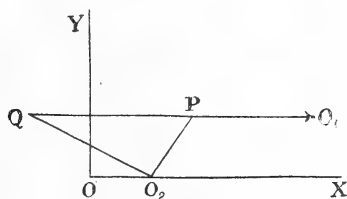


FIG. 36.

(IV.)  $O_1$  at infinity on the  $x$ -axis;  $O_2$ , as before,  $(a, 0)$ ;  $PO_2Q = \frac{\pi}{2}$ ;  $O_1PQ$  collinear points;  $xy=p$  the locus of  $P$ .

Let  $P$  be the point  $(p/\eta, \eta)$ .  $\therefore O_2P$  has equation

$$y/\eta = \eta(x - a)/(p - a\eta) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The equation to  $O_2Q$  is

$$y = \frac{a\eta - p}{\eta^2}(x - a) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the locus of  $Q$  is given by

$$y^3 = (ay - p)(x - a) \quad \dots \quad (3)$$

(V.) Let  $P$  lie on  $xy = p$ .

If  $O_1$  is the point at infinity on the  $y$ -axis,  $O_1PQ$  parallel to  $OY$ , and  $\angle PO_2Q = \pi/2$ , the locus of  $Q$  is given by

$$y = -x(a = x)^2/p.$$

Cor. 9. In this corollary Maclaurin proves the generality of this construction for a singular cubic by using it to describe a cubic having a double point at  $O_1$ , and passing through other six points  $O_2P_1P_2 \dots P_5$ .

Construct  $\Delta P_1O_1O_2$ , and let  $\hat{O}_1 = \alpha$ ,  $\hat{O}_2 = \beta$ . Take a quadrilateral  $PO_1QO_2$ , in which  $\hat{O}_1 = \alpha$ ,  $\hat{O}_2 = \beta$ , and place  $P$  in coincidence with  $P_2, P_3, P_4, P_5$ , when  $Q$  will take up positions  $Q_2, Q_3, Q_4, Q_5$ . Construct the conic through  $O_1, Q_2, Q_3, Q_4, Q_5$ , and restrict the vertex  $Q$  to lie on this conic, when  $P$  traces out a cubic having a double point at  $O_1$  and passing through  $O_2P_1P_2 \dots P_5$ . There cannot be two such cubics; for, if there were, they would require to be considered as intersecting in ten points, whereas they cannot cut in more than nine points.

#### § 40. Prop. II.

*When the locus conic, as in Prop. I, passes through neither  $O_1$  nor  $O_2$ , then the same method of proof shows that the curve traced by  $Q$  is a curve of the fourth order, having double points at  $O_1$  and  $O_2$ , and also at a third point.*

For let  $O_1O_2$  cut the conic in  $A_1$  and  $A_2$ . To  $A_1$  and  $A_2$  corresponds a common point  $B$  on the quartic, which is thus a third double point.

Cor. 4. If  $O_1Q$  and  $O_2Q$  coincide simultaneously with  $O_1O_2$ , the locus is only of the third degree, with ordinary points at  $O_1$  and  $O_2$  and a double point at  $B$ .

[Maclaurin might have shown how to use Prop. II to construct a quartic having three double points  $O_1, O_2, O_3$ , and through five other points  $P_1 \dots P_5$ .

Construct  $\Delta O_1O_2O_3$ , and let  $\hat{O}_1 = \alpha$ ,  $\hat{O}_2 = \beta$ ,  $\hat{O}_3 = \gamma$ . Use the quadrilateral  $PO_1QO_2$  in which  $\hat{O}_1 = \alpha$  and  $\hat{O}_2 = \beta$ , and place  $P$  in coincidence with  $P_1, P_2, \dots P_5$ , when  $Q$  takes up the positions  $Q_1, Q_2, \dots Q_5$ . Let the five points  $Q$  determine the conic  $C$ . Now restrict  $Q$  to lie on  $C$ , and  $P$  will trace out a quartic having double points at  $O_1O_2O_3$  and through  $P_1P_2 \dots P_5$ .

There cannot be two such quartics, for, if so, they would require to be considered as intersecting in seventeen points, which is impossible.]

§ 41. *Prop. III.*

If  $P$  lies on a curve of degree  $n$ ,  $Q$  traces out a curve of degree  $2n$ .

For let  $Q$  lie on a straight line  $l$ ; then  $P$  will generate a conic which cuts the  $n$ -ic in  $2n$  points  $P_1, P_2, \dots, P_{2n}$ , to which correspond on  $l$   $2n$  points,  $Q_1, Q_2, \dots, Q_{2n}$ .  $\therefore$  etc.

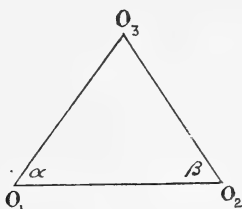


FIG. 37.

Cor. 1. Construct  $\Delta O_1 O_2 O_3$ , and let  $\hat{O}_1 = \alpha$ ,  $\hat{O}_2 = \beta$ , as in figure.

Then each side of  $O_1 O_2 O_3$  cuts the  $n$ -ic in  $n$  points, to which corresponds the unique point the opposite vertex. The curve therefore has  $n$ -ple points at  $O_1, O_2, O_3$ .

Cor. 2. There cannot be four  $n$ -ple points on the new curve; for through these and a fifth point on the curve we could describe a conic cutting the curve in  $4n+1$  points, which is impossible.

Cor. 4. It has been assumed that  $O_1$  and  $O_2$  do not lie on the given curve of degree  $n$ . If  $O_1$  is an ordinary point on the latter the curve obtained is of degree  $2n-1$ ; and if  $O_1$  is an  $r$ -ple point the curve is of degree  $2n-r$ .

[We might, of course, have established this proposition by showing that the co-ordinates of  $P$  and  $Q$  are connected by an ordinary Cremona quadratic transformation. We therefore have before us established, for the first time, the fundamental features of a Cremona transformation more than a century before it was to become the property of all mathematicians through Cremona's researches.]

## REMARK.

"Newton has given Props. I and II and indicated their generalisation.

"This generalisation we have attempted to effect in Prop. III. We have to this end made use of a given curve and two constant angles. In the following we shall attempt to generalise all the propositions of Part I, just as we have generalised the Proposition I of Part I."

## SECTION II.

WHEREIN CURVES ARE INVESTIGATED SUCH AS MAY BE OBTAINED  
FROM CERTAIN OTHERS BY THE USE OF GIVEN ANGLES.

§ 42. *Prop. IV.*

With data similar to those in *Prop. V* of *Part I*, viz.  $O_1, O_2$  fixed points, angles at  $P$  and  $O_2$  in quadrilateral  $PRO_2Q$  constant. Let  $P$  lie

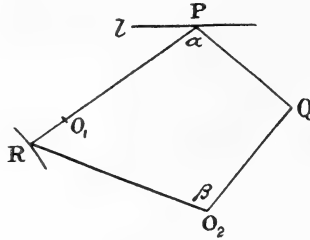


FIG. 38.

on a straight line  $l$ . But let  $R$  now lie on a curve  $C_n$  of degree  $n$ . Then  $Q$  will generate a curve of degree  $3n$ .

*Dem.*

Let  $Q$  lie on a straight line  $l_1$ , and  $P$  on its locus  $l$ , then  $R$  will generate a cubic\* cutting  $C_n$  in  $3n$  points  $R_1, R_2, \dots, R_{3n}$ , to which correspond on  $l_1$  the  $3n$  points  $Q_1, Q_2, \dots, Q_{3n}$ . Hence  $l_1$  cuts the locus in these  $3n$  points.  $\therefore$  etc. *Q.E.D.*

*Cor. 1.* Construct on  $O_1O_2$  a circle containing an angle equal to  $O_1PQ$  and cut by  $l$  in two points  $A$  and  $B$ . To each of the  $n$  points in which  $O_1A$  cuts  $C_n$  corresponds the point  $O_2$ , and similarly for  $O_1B$ . Hence  $O_2$  is a  $2n$ -ple point on the curve. But  $O_1$  is not in general a point on the new curve.

*Cor. 4.* If  $O_1$  is on  $C_n$  the degree of the new curve is less by 2, if  $O_2$  is on the curve less by unity. If both points are on  $C_n$  the new curve is of degree  $3n-3$ .

*Cor. 5.* If of the three vertices  $P, Q, R$  of  $O_2RPQ$  one is restricted to lie on a straight line, a second on a curve  $C_n$ , the remaining vertex generates a curve  $C_{3n}$  of degree  $3n$ .

§ 43. *Prop. V.*

If  $R$  in the preceding is restricted to lie on a curve  $C_n$ , and  $P$  on a curve  $C_m$ , then  $Q$  generates a curve  $C_{3mn}$  of degree  $3mn$ .

\* This cubic has a double point at  $O_1$  and passes through  $O_2$ .

*Dem.*

Let  $Q$  lie on a line  $l$ , and  $R$  on  $C_n$ , then, by Cor. 5 of Prop. IV,  $P$  generates a curve  $C_{3n}$  which cuts  $C_m$  in  $3mn$  points  $P_1, P_2, \dots, P_{3mn}$ , to which correspond  $Q_1, Q_2, \dots, Q_{3mn}$  on  $l$ .

The new curve is  $\therefore$  cut by  $l$  in  $3mn$  points.  $\therefore$  etc.

Cor. 2.  $O_2$  is a multiple point on the locus of order  $2mn$ .

For on  $O_1O_2$  describe a segment of a circle containing an angle equal to  $O_1PQ$ . It cuts  $C_m$  in  $2n$  points  $A_1, A_2, \dots, A_{2n}$ . The lines  $O_1A$  cut  $C_n$  in  $2mn$  points, to each of which corresponds the point  $O_2$ .  $\therefore$  etc.

Cor. 3. If of the vertices  $P, Q, R$  one lies on a curve  $C_m$ , and a second on a curve  $C_n$ , the third generates a curve  $C_{3mn}$ .

#### § 44. Prop. VI.

Generalisation of Prop. XIV in Part I.

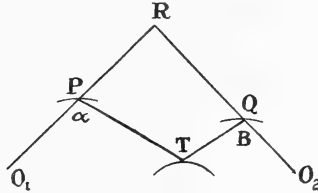


FIG. 39.

*In the figure  $O_1PT$  and  $O_2QT$  are constant angles. If  $P$  lies on  $C_n$ ,  $Q$  on  $C_n$ ,  $T$  on  $C_r$ , then  $R$  generates a curve  $C_{4mnr}$ .*

*Dem.*

Let  $P$  and  $Q$  lie on straight lines, and let  $R$  lie on a line  $l$ : then  $T$  would generate a quartic  $C_4$  cutting  $C_r$  in  $4r$  points, to which would correspond  $4r$  points  $R$  on  $l$ ; *i.e.*  $R$  would generate a curve  $C_{4r}$ .

Next, let  $P$  lie on a straight line,  $Q$  on  $C_n$ , and  $T$  on  $C_r$ ; then  $R$  lies on a curve  $C_{4nr}$ . For let  $R$  lie on a line  $l'$ ,  $P$  on  $l$ , and  $T$  on  $C_r$ : then  $Q$  would generate a curve cutting  $C_{rL}$  in  $4nr$  points, to which correspond  $4nr$  points on  $l'$ .

Hence the locus of  $R$  would be a  $C'_{4nr}$ .

Finally, let  $P$  lie on  $C_m$ . Let  $R$  lie on a line  $l''$ ,  $Q$  on  $C_n$ , and  $T$  on  $C_r$ ; when  $P$  would generate a curve  $C_{4mr}$  cutting  $C_m$  in  $4mnr$  points: and to these correspond  $4mnr$  points on  $l''$ .  $\therefore$  etc.

Cor. 1. Each of the points  $O_1, O_2$  is multiple of order  $2mnr$ .

#### § 45. Prop. VII.

Generalisation of Prop. XXI of Part I.

*Let  $O_1P_1P_2 \dots P_{n-1}Q$  be a serrate angle,  $QO_2R$  a constant angle*

rotating round  $O_2$ , with  $R$  on  $O_1P_1$ . If  $R, P_1, P_2, \dots, P_{n-1}$  lie on curves  $C_r, C_{1p}, \dots, C_{pn-1}$ , the locus of  $Q$  is a curve of order  $rp_1p_2 \dots p_{n-1}(n+1)$ .

The demonstration is similar to that of Prop. VI.

Cor. 1. The point  $O_2$  is multiple on the curve of order

$$rp_1p_2 \dots p_{n-1}n.$$

§ 46. Prop. VIII.

Generalisation of Prop. XXIV of Part I.

Consider two serrate angles

$$O_1P_1P_2 \dots P_mP$$

and

$$O_2Q_1Q_2 \dots Q_nQ$$

in which  $P_1P_2 \dots P_m$  lie on curves of orders  $p_1, p_2, \dots, p_m$ , and  $Q_1Q_2 \dots Q_n$  on curves of orders  $q_1, q_2, \dots, q_n$  respectively. If  $P_mP$  and  $Q_nQ$  intersect on a curve  $C_r$  the intersection of  $O_1P_1$  and  $O_2Q_1$  generates a curve of order

$$r(n+m+2)\Pi p_1\Pi q_1.$$

### SECTION III.

#### MACLAURIN'S THEORY OF PEDALS.

§ 47. An intelligent perusal of the preceding shows that Maclaurin would inevitably have been led to the pedal transformation of curves,\* which he now discusses very thoroughly in general terms, along with its application to conic sections and other familiar curves.

He gives no name to the transformation, and the term pedal (*podaire*) was introduced by the geometers of the nineteenth century.

Almost the only nomenclature he introduces, the "Radial Equation" for the  $p-r$  equation to a curve (p. 96), has been quite overlooked and adapted to another purpose by Tucker.

*Definition.*

The definition is the usual one.

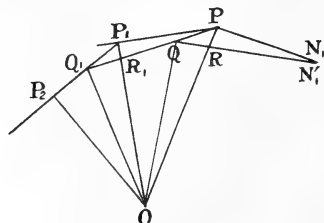


FIG. 40.

\* But compare § 2 of Part I.

Let  $O$  be a fixed point in the plane of a curve  $C$ , on which  $P$  and  $Q$  are two infinitely near points.  $PP_1$  is the tangent at  $P$  to the curve, and  $OP_1$  is drawn perpendicular to  $P_1P$ . Then as  $P$  moves on its locus  $P_1$  generates a curve, the pedal of the given curve for the pole  $O$ .

§ 48. *Prop. IX.*

Draw  $PN_1 \perp OP$ , and  $QN_1 \perp OQ$ ;  $OQ_1 \perp QQ_1$ ;  $OP_2 \perp P_1Q_1$ .

Then the following pairs of similar triangles arise:—

$$\triangle OPP_1 \approx \triangle Q_1P_1R_1,$$

$$\triangle OR_1Q_1 \approx \triangle PR_1P_1,$$

$$\triangle OR_1P \approx \triangle Q_1R_1P_1.$$

Also  $P_1Q_1R_1$ ,  $PQR$ ,  $P_1OP_2$ ,  $POP_1$  are similar; and

$$OP/OP_1 = OP_1/OP_2.$$

Denote  $OP$  by  $r$ , and  $OP_1$  by  $p$ . If the curve  $C$  is given by the equation  $f(x, y) = 0$  (1) referred to axes through  $O$ ,

$$OP = \sqrt{(x^2 + y^2)},$$

$PP_1$  has for equation

$$Y - y = (X - x)y' \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$p = (y - xy')/\sqrt{(1 + y'^2)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The elimination of  $x$ ,  $y$ , and  $y'$  leads to a single relation

$$\phi(p, r) = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is sufficient to characterise the curve, and it is this equation which Maclaurin calls the Radial Equation of the curve.

Cor. 1. From the locus of  $P_1$  may be similarly described its pedal, the locus of  $P_2$ . We may thus derive an infinite series of curves (the positive pedals of  $C$ ).

From the radial equation of  $C$  can be easily deduced the radial equation of the locus of  $P_1$ .

Let  $p_1$  and  $r_1$  correspond to the locus of  $P_1$ .

Then

$$\left. \begin{array}{l} p_1/r_1 = p/r \\ r_1 = p \end{array} \right\} \quad \therefore p = r_1; \quad r = r_1^2/p_1 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Cor. 2. The series of curves may be continued in the opposite sense, viz. by drawing  $PN_1$  and  $QN_1$  perpendicular to  $OP$  and  $OQ$ , and finding the locus of  $N_1$  (the negative pedal); or  $N_1$  may be found by drawing  $ON_1$ , so that  $PON_1$  is the complement of  $OPP_1$ .







Then

$$CO_1O_3 = R'O_3O_1 = R''O_3O_1,$$

and

$$O_1C = O_3R' = O_3R'',$$

Hence

$$O_3R'' \parallel O_1C \text{ and } = O_1C;$$

$$\therefore \parallel OO_1 \text{ and } = OO_1;$$

so that

$$OR'' \parallel O_1C \text{ and } = O_1O_3 = r.$$

Also

$$CR' \parallel O_1O_3.$$

Hence

$$OR'R' = CR'R'' = \pi/2,$$

and the locus traced by  $R'$  is the pedal of the circle of centre  $O$  and radius  $r$  for a pole at  $C$ .

Cor. 7. The limaçon cannot be rectified (*vide* Nicole in *Actis Academiæ Parisiensis*, 1707).

Cor. 8. It is a conchoid of the circle whose diameter is  $OC$ . For let  $OP$  cut this circle in  $G$ . Hence  $PG = PO + OG = r - d \cos \theta + d \cos \theta = r$ , and the curve is a conchoid for the pole at  $O$  and the constant  $r$ .

"From this it is obvious that the curve is the conchoid of circular base described by De la Hire in 1708, and which Roberval and Pascal have also discussed. None of these, however, as far as I know, observed that it is an Epicycloid ('Eorum tamen nemo quantum novi eam Epicycloidem esse observavit')."

Loria (*Ebene Kurven*) ascribes the discovery of this property to Cramer, but clearly Maclaurin has a prior claim. It is one of the few occasions on which he *does* lay claim to a discovery, and he should certainly be credited with it.

Cor. 9. The mid-points of the chords through  $O$  lie on a circle.

#### Generalisation.

Cor. 10. Take two fixed points  $O$  and  $O'$ ,  $P$  any point on the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let  $\angle OPT$  be a constant angle, which, without loss of generalisation, Maclaurin takes to be a right angle. Draw  $OT \perp PT$ .

If the origin is chosen at  $O$  and  $O'$  is the point  $(d, o)$ , the equation to the locus of  $T$  is

$$(x^2 + y^2)^2 + (2gx + 2fy)(x^2 + y^2) + cx^2 + y^2(d^2 + c + 2gd) - 2fdxy = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)^*$$

\* The pedals of the central conics give rise to the rational bicircular quartics (Khon Loria, *Encykl. d. Math. Wiss.*).

It is obtained from the circle in the same way as the rational circular cubics are obtained from the straight line (and is likewise a rational or unicursal curve). It appears to be the simplest of the curves of the fourth order, the conchoid excepted, just as the circular cubics with a double point are the simplest of the third order.

[Teixeira has shown that just as the rational circular cubic is the cissoidal of a circle and a straight line, so these bicircular quartics are the cissoidals of a circle and a circle.]

§ 50. *Prop. XI.*

*Pedals of the Conic Sections.*

(I.) For the Parabola.

The pedal of the focus is the tangent at the vertex. Hence by Cor. 8 of Prop. IX the pedal of any other point  $O'$  is the rational circular cubic already discussed in Lemma II of Part I. The curve has a double point at the pole with real, coincident, or imaginary tangents according as the pole is outside, on, or inside the parabola. It has a line of symmetry when the pole is on the axis of the parabola, and is the cissoid of Diocles when the pole is the vertex of the parabola.

(II.) For the Ellipse.

The pedal of the focus is the major auxiliary circle (Maclaurin's theorem). Hence the pedal of any other point is the bicircular quartic of Cor. 10 of Prop. X.

(III.) For the Hyperbola.

The pedal of the focus is again a circle, and we have a conclusion similar to that of (II).

Cor. If  $O$  is a fixed point,  $P$  any point on a circle, and  $OPT$  a constant angle, then  $PT$  envelops a conic section.

§ 51. *Prop. XII.*

*When a curve rolls on a congruent curve, corresponding points being points of contact, the roulette of any carried point can be obtained easily as a pedal.*

The usual proof is given.

Cor. 1. The curves described by this method coincide with the epicycloids of Nicole generated by a curve rolling on a congruent curve.

Cor. 2. Thus the epicycloids generated by a parabola rolling upon a congruent parabola are (1) a straight line for the focus, (2) a cissoid of Diocles for the vertex, (3) a rational circular cubic for any other point.

Cor. 3. If the generating curves are ellipses, the focus of the moving ellipse generates a circle, and any other point a bicircular quartic.

Similar conclusions hold for the hyperbola.

§ 52. *The curves whose radial equation can be represented in the form*

$$p = \mathbf{A}r^{n+1}$$

or

$$p/r = (r/a)^n.$$

These curves have the property that their pedals for the same pole have a similar radial equation.

Let  $p_1$  and  $r_1$  be the elements of the first positive pedal corresponding to  $p$  and  $r$  of the given curve.

Then

$$p_1/r_1 = p/r \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$r_1 = p; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\therefore p = r_1, \quad r = r_1^2/p_1.$$

But

$$p/r = (r/a)^n,$$

$$\therefore p_1/r_1 = (r_1^2/ap_1)^n;$$

and finally

$$\rho_1/r_1 = (r_1/a)^{\frac{n}{n+1}}, \quad (4)$$

Similarly the second positive pedal is given by

$$p_2/r_2 = (r_2/a)^{n/(2n+1)}$$

and the  $m$ th pedal by

$$p_m/r_m = (r_m/a)^{\frac{n}{mn+1}}. \quad (5)$$

Similarly, the  $m$ th negative pedal is given by

$$\pi_m/\rho_m = (\rho_m/a)^{\frac{n}{-m n + 1}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Particular examples of  $p-r$  equations are:—

(I.) Circle of radius  $a$ , the pole being on the circumference,

$$p/r = r/2a \quad (n = 1).$$

(II.) The straight line at a distance  $a$  from the pole,

$$p/r \equiv a/r \quad (n \equiv -1).$$

(III.) The Parabola (first negative pedal of the straight line),

$$p/r = (r/a)^{-\frac{1}{2}} \quad (n = -\frac{1}{2}).$$

(IV.) The Rectangular Hyperbola,

$$\begin{aligned} x^2 - y^2 - a^2 \\ \therefore p/r = a^2/r^2 \quad (n = -2), \end{aligned}$$

the pole being at the centre.

(V.) The Cardioid (first positive pedal of the circle),

$$\begin{aligned} p/r &= (r/2a)^{\frac{1}{2}} \\ (i.e. \quad p^2 &= r^3/2a). \end{aligned}$$

(VI.) The Lemniscate (first positive pedal of the rectangular hyperbola of IV),

$$p/r = r^2/a^2$$

or

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$

in Cartesian co-ordinates.

(VII.) Maclaurin also gives later the logarithmic spiral\* whose  $p-r$  equation is

$$p/r = C; \quad n = 0,$$

(*vide* Section IV), but this is not an algebraic curve.

### § 53. Prop. XIV.

Property of the curve  $p/r = (r/a)^n$ .

Let B be the point  $p=r=a$ ; this point is a vertex on the curve and its pedals.

The following relation holds (*vide* fig. 40):—

$$\angle P_1 O Q_1 = (n+1) \angle P O Q.$$

*Dem.*

Let the polar co-ordinates of P be  $(r, \theta)$  and of  $P_1$   $(p, \phi)$ . Since  $p/r = (r/a)^n$ , therefore

$$\frac{dp}{p} = (n+1) \frac{dr}{r}.$$

But by the pedal transformation

$$r \frac{d\theta}{dr} = p \frac{d\phi}{dp},$$

and therefore

$$d\phi = (n+1)d\theta; \quad \therefore \text{etc.}$$

Cor. 1. In particular, if  $\theta$  and  $\phi$  are measured from the initial position OB, then

$$\phi = (n+1)\theta.$$

### § 54. Prop. XV.

*Maclaurin's theorem regarding the rectification of such curves.*

If P traces out the curve  $p/r = (r/a)^n$ , starting from the vertex B, while

\* Since  $p/r = p_1/r_1 = \text{etc.}$ , a logarithmic spiral can be described to pass through the points P,  $P_1$ ,  $P_2$ , etc.

$P_1$  and  $N_1$  are the points corresponding to  $P$  on the first positive and negative pedals, then

$$\text{arc } BP_1 = (n+1)(\text{arc } BN_1 + \text{straight line } N_1P).$$

*Dem.*

By Prop. XIV (fig. 40)

$$\frac{Q_1R_1}{p} = (n+1)\frac{QR}{r}, \text{ or } \frac{r}{p}Q_1R_1 = (n+1)QR,$$

that is

$$P_1Q_1 = (n+1)QR.$$

Now

$$P_1Q_1 = ds_1 \text{ if } s_1 = \text{arc } BP_1,$$

and

$$QR = QN_1' - RN_1' = QN_1' - PN_1' = QN_1' - PN_1 + N_1'N_1.$$

Hence if

$$\begin{aligned} \sigma_1 &= \text{arc } BN_1, \\ QR &= d \cdot PN_1 + d\sigma_1. \end{aligned}$$

Thus

$$ds_1 = (n+1)(d \cdot PN_1 + d\sigma_1)$$

and

$$\text{arc } BP_1 = (n+1)(\text{arc } BN_1 + PN_1).$$

[The following analytical proof\* may be given.

Let  $(r, \theta)$  be the polar co-ordinates of any point  $P$  on the curve, and let  $P_1$  and  $N_1$  correspond to  $P$  on the first positive and first negative pedals respectively. Let the positive direction of the arc  $s$  be such that  $s$  increases as  $\theta$  increases, and let the positive direction of the tangent  $PP_1$  be that of the curve at  $P$ . Also let  $\psi$  be the angle from the positive direction of  $OP$  to that of  $PP_1$ . Then

$$\overline{OP} = r; \quad ds = dr \sec \psi.$$

Let

$$p = \overline{OP}_1 = r \sin \psi$$

and

$$t = \overline{P}_1P = -\overline{PP}_1 = r \cos \psi,$$

so that

$$t^2 + p^2 = r^2.$$

Then

$$\begin{aligned} dt &= \frac{r}{t} dr - \frac{p}{t} dp \\ &= dr \sec \psi - \frac{p}{r} dp \sec \psi \\ &= ds - \frac{p}{r} ds_1, \end{aligned}$$

where  $ds_1$  is the element of arc at  $P_1$ .

Similarly, if  $\varpi, \rho, \sigma, \tau$  correspond on the first negative pedal to  $p, r, s, t$ ,

\* Suggested by Professor G. A. Gibson.

$$d\tau = d\sigma - \frac{\overline{\omega}}{\rho} ds = d\sigma - \frac{p}{r} ds \quad (1)$$

But

$$ds_1 = dp \sec \psi = (n+1) \frac{p}{r} ds \quad (2)$$

where

$$p/r = (r/a)^n.$$

Hence

$$ds_1 = (n+1)(d\sigma - d\tau),$$

so that

$$\begin{aligned} \text{arc } BP_1 &= (n+1)(\text{arc } BN_1 - \overline{PN_1}) \\ &= (n+1)(\text{arc } BN_1 + \overline{N_1P}). \end{aligned}$$

The signs of  $BP_1$ ,  $BN_1$ , and  $N_1P$  must be attended to. Thus when  $n > 1$  the angle  $BON_1$ , which is equal to  $(1-n)\theta$ , is of opposite sign to that of  $BOP$  and of  $BOP_1$ ; the arcs  $BP_1$  and  $BN_1$  are therefore of opposite sign, and  $N_1P$  has the same sign as arc  $BP_1$ . Numerically, the arc  $BP_1$  is less than  $(n+1)$  times the length of the line  $PN_1$  by  $(n+1)$  times the arc  $BN_1$ .]

This interesting theorem Maclaurin proceeds to apply to deduce various conclusions regarding the rectification of the series of curves formed by a given curve,  $p/r = (r/a)^n$ , along with its positive and negative pedals.

Cor. 1. If two consecutive curves of the series admit of rectification, so do all.

Cor. 2. If one of the curves admits of rectification, but not the next in the series, then half only of the curves of the series admit of rectification.

Cor. 5. When the first negative pedal passes through the pole, the theorem for the total lengths from B may be written

$$s_1 = (n+1)\sigma_1.$$

For in such a case  $PN_1$  vanishes.

# § 55. Prop. XVII.

When the given curve is a circle of radius  $a/2$ , and the radial equation is  $p/r = r/a$ , the first negative pedal reduces to a point B.

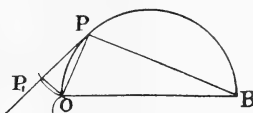


FIG. 43.

The first positive pedal is the cardioid  $BP_1$ , and for it

$$\text{arc } BP_1 = 2 \text{ chord } BP \quad (1)$$

Thus half the complete cardioid  $= 2a$ , and the whole length  $= 4a$ .

The pedal of the cardioid is given by

$$p/r = (r/a)^{\frac{1}{2}}$$

and

$$\text{arc BP}_2 = \frac{3}{2} (\text{arc BP} + \text{P}_1\text{P}).$$

Hence it cannot be measured by a straight line alone, nor by an arc of a circle alone save for the complete curve up to O, when

$$\text{BP}_2\text{O} = \frac{3}{2} \text{BPO} = 3\pi a/4.$$

The whole curve cannot be cut in any given ratio, for otherwise the quadrature of the circle would follow.

The third positive pedal

$$\frac{p}{r} = \left(\frac{r}{a}\right)^{\frac{1}{2}}$$

has an arc

$$\begin{aligned} \text{BP}_3 &= \frac{4}{3} (\text{BP}_1 + \text{P}_1\text{P}_2) \\ &= \frac{4}{3} (2 \text{ chord BP} + \text{line P}_1\text{P}_2) \end{aligned}$$

and

$$\text{BP}_3\text{O} = \frac{8}{3} \text{BO} = \frac{8a}{3}.$$

In general the  $n$ th pedal is given by

$$p/r = (r/a)^{1/(n+1)}.$$

If  $S_n$  denote the arc of the pedal from B to O,

$$\begin{aligned} S_n &= \frac{n+1}{n} S_{n-2} \\ &= \frac{n+1}{n} \cdot \frac{n-1}{n-2} S_{n-4} \\ &= \frac{(n+1)(n-1) \dots 2}{n(n-2) \dots 1} \text{OB} \end{aligned}$$

when  $n$  is odd,

$$= \frac{(n+1)(n-1) \dots 3}{n(n-2) \dots 2} \frac{\pi a}{2}$$

when  $n$  is even.

The areas of the pedals are also discussed.

§ 56. *Prop. XVIII.*

*The pedals of the straight line*

$$\frac{p}{r} = \left(\frac{r}{a}\right)^{-1}.$$

The first positive pedal reduces to a point.



The negative pedals are given by

$$\frac{p}{r} = \left(\frac{r}{a}\right)^{\frac{-1}{1+m}} = \left(\frac{a}{r}\right)^{\frac{1}{m+1}}.$$

The first negative pedal is the parabola

$$p/r = \left(\frac{r}{a}\right)^{-\frac{1}{2}}.$$

The second negative pedal is given by

$$p/r = \left(\frac{r}{a}\right)^{-\frac{3}{2}},$$

whose arcs can be expressed by straight lines. Only these may increase beyond all limit, as the curve goes to infinity with the parabola.

We thus form two sets of curves: in one set the arcs can be expressed by parabolic arcs and straight lines, and in the other set by straight lines only.

§ 57. *Prop. XIX.*

*The pedals of the equilateral hyperbola*

$$x^2 - y^2 = a^2,$$

or

$$p/r = a^2/r^2.$$

The first positive pedal is the lemniscate

$$(x^2 + y^2)^2 = a^2(x^2 - y^2),$$

or

$$p/r = (r/a)^2.$$

Two series of curves are obtained, in one of which arcs are expressible by hyperbolic arcs and straight lines, and in the other by arcs of lemniscates and straight lines.

§ 58. *Prop. XXI.*

The radius of curvature of the curve

$$p/r = (r/a)^n \text{ is } \frac{a^n}{n+1} \frac{1}{r^{n-1}}.$$

Maclaurin proves the more general formula  $\rho = r \, dr/dp$ , from which the formula is easily deduced.

[§ 59. *Remarks on Pedals.*

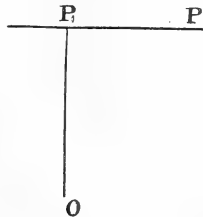


FIG. 44.

If the line  $PP_1$  is given by

$$lx + my - 1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and  $OP_1$  is  $\perp$   $PP_1$  through the origin  $O$ , the co-ordinates of  $P_1$  are

$$\left. \begin{aligned} \xi &= l/(l^2 + m^2) \\ \eta &= m/(l^2 + m^2) \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also

$$\left. \begin{aligned} l &= \xi/(\xi^2 + \eta^2) \\ m &= \eta/(\xi^2 + \eta^2) \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence if the equation in line co-ordinates to a curve is

$$\phi(l, m) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

its first positive pedal has the equation in point co-ordinates

$$\phi\left(\frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence we may, in general, say that *the degree of the first pedal is twice the class of the original curve.*

If

$$f(x, y) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

is the equation in point co-ordinates to the first positive pedal of a curve, the curve itself has the equation in line co-ordinates

$$f\left(\frac{l}{l^2 + m^2}, \frac{m}{l^2 + m^2}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and we should therefore say that *the class of the latter is twice the degree of its first pedal.*

The paradox arising from the application of these theorems is explained in the same way as for the degrees of a curve and its inverse, and both theorems are subject to important modifications. But the analysis given indicates the importance of line co-ordinates in the theory of pedals.

*Ex.* A conic, being of class 2, has a pedal in general of degree 4, having a special relation to the circular points at infinity.

But if the pole  $O$  is at the focus the line equation to the conic is

$$l^2 + m^2 + al + bm + c = 0$$

and its pedal is

$$c(x^2 + y^2) + ax + by + 1 = 0,$$

i.e. a circle.

If the curve is a parabola

$$al^2 + 2hlm + bm^2 + 2gl + 2fm = 0$$

the pedal is a circular cubic; and if the focus is at  $O$  the pedal is the straight line

$$2gx + 2fy + a = 0.$$

§ 60. *Differential Geometry of Pedals and Maclaurin's Theorem for the Curves*  $p/r = (r/a)^n$ .

*Use of the  $p-r$  Equation.*

Let, as usual,

$$p, r; p_1, r_1; p_2, r_2; \text{ etc.},$$

denote corresponding elements of the curve and its pedals.

Then

$$p/r = p_1/r_1 = p_2/r_2 = \text{etc.} \quad (1)$$

and

$$r_m = p^m/r^{m-1}; p_m = p^{m+1}/r^m \quad (2)$$

Let  $S, S_1, \dots, S_m$  be corresponding arcs of the curve and its pedals, and let  $\phi$  be the angle between a radius vector and the corresponding tangent.

Then, up to sign

$$\left. \begin{aligned} ds &= dr \sec \phi = r dr / \sqrt{r^2 - p^2} \\ ds_1 &= dr_1 \sec \phi = r dp / \sqrt{r^2 - p^2} \\ ds_2 &= dr_2 \sec \phi = 2 \frac{p}{r} ds_1 - \frac{p^2}{r^2} ds \\ ds_m &= 2 \frac{p}{r} ds_{m-1} - \frac{p^2}{r^2} ds_{m-2} \end{aligned} \right\} \quad (3)$$

Cor. 1.

$$ds_1/ds = dp/dr.$$

Cor. 2. The elimination of  $p/r$  from (3) gives rise to a variety of results, e.g. from the equivalents of  $ds_2$  and  $ds_3$  in (3) we deduce

$$4 \begin{vmatrix} ds_1 & ds_2 \\ ds_2 & ds_3 \end{vmatrix} \times \begin{vmatrix} ds & ds_1 \\ ds_1 & ds_2 \end{vmatrix} = \begin{vmatrix} ds & ds_2 \\ ds_1 & ds_3 \end{vmatrix}^2 \quad (4)$$

Also

$$\begin{vmatrix} ds_k & ds_{k+1} & ds_{k+2} \\ ds_l & ds_{l+1} & ds_{l+2} \\ ds_m & ds_{m+1} & ds_{m+2} \end{vmatrix} = 0 \quad (5)$$

Let the tangent

$$\begin{aligned} PP_1 &= t \\ \dots P_1 P_2 &= t_1 \\ \dots P_2 P_3 &= t_2, \text{ etc.} \end{aligned}$$

Then

$$\left. \begin{aligned} t &= \sqrt{r^2 - p^2} \\ t_1 &= \frac{p}{r} \sqrt{r^2 - p^2} \\ t_2 &= \frac{p^2}{r^2} \sqrt{r^2 - p^2} \\ &\text{etc.} \end{aligned} \right\} \quad (6)$$





## SECTION V.

## ON THE DESCRIPTION OF GEOMETRICAL CURVES THROUGH GIVEN POINTS

§ 64. *Lemma III.*

A curve  $C_n$  meets a conic in  $2n$  points and a cubic in  $3n$  points.

The proof is analytical.

From it is suggested:

Cor. 1. Two curves  $C_m$  and  $C_n$  seem to cut in  $mn$  points.

This is easily proved when one of the curves is  $y = x^m$ ; but the general demonstration is beyond Maclaurin's powers. The truth of the statement is assumed in what follows.

Cor. 2. Two curves of degree  $n$  cut in  $n^2$  points. Thus we may find two curves of degree  $n$  through the same  $n^2$  points. Now the equation to  $C_n$  involves  $\frac{1}{2}(n^2 + 3n)$  conditions, and  $\therefore \frac{1}{2}(n^2 + 3n)$  points may not be sufficient to determine a curve uniquely when  $\frac{1}{2}(n^2 + 3n)$  is not greater than  $n^2$ .

Thus nine points may not uniquely determine a cubic, and yet ten points are too many.

[This is the source of the so-called Cramer's Paradox. Cramer, who simply repeats what Maclaurin gives with the additional application to quartics, quotes Maclaurin as his authority (*vide* Cramer, *Courbes algebriques*).

The paradox is therefore Maclaurin's and not Cramer's.]

Cor. 3. If, of the points given to determine a  $C_n$ ,  $nr + 1$  lie on  $C_r$ , where  $n > r$ , then either the problem is impossible or the  $C_n$  degenerates into  $C$  along with  $C_{n-r}$ .

Cor. 4. A curve  $C$  cannot have more than  $\frac{1}{2}(n-1)(n-2)$  double points.

Cor. 5. If, on a curve  $C_m$ , three points are multiple of order  $m/2$  and one of order  $\frac{m}{2} - 1$ , all the other points will be simple.

§ 65. *Prop. XXV*

shows how to draw a curve  $C_n$  through  $2n+1$  given points one of which is an  $(n-1)$ -ple point.

*Prop. XXVI*

shows how to draw a  $C_{2n}$  through as many points as suffice to determine a  $C_n$ , and other three points each of which is an  $n$ -ple point.

*Prop. XXVII*

shows how to draw a  $C_{2n}$  through  $2n+4$  given points, of which three are  $n$ -ple points, while a fourth is an  $(n-1)$ -ple point.

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APPENDIX.

In the light of the account just given, the student will find it interesting to examine the following references to Loria's *Ebene Kurven*. The pages refer to the first edition of Loria's treatise.

*Page 39.*

The locus of the image of the vertex of a parabola in the tangent is a cissoid of Diocles.

Loria refers to Mirman: "Sur la Cissoide de Diokles," *Nouvelles Annales*, 1885.

When a parabola rolls externally on a congruent parabola its vertex describes a cissoid.

Reference to Hendrick's "Demonstration of a Proposition" (*Analyst*, 1877).

*Page 48.*

*The Ophiuride.*

Given a right angle OBC on whose sides O and C are fixed points.

Through C is drawn CD cutting OB in D; DM is  $\perp$  CD, and OM  $\perp$  DM. The locus of M as CD varies is the ophiuride.

Reference to Uhlhorn: *Entwickelungen in der höheren Geometrie*, 1809.

*Page 49.*

The pedal of a parabola for a pole on the tangent at the vertex is an ophiuride, and a cissoid for the vertex.

*Page 60.*

*The Strophoid.*

This name was given by Montucci (*Nouvelles Annales*, 1846).

It is the logocyclic curve of Booth (1877).

*Page 69.*

The generalised strophoidal curve, given by Maclaurin, is ascribed to Lagrange (*Nouv. Ann.*, 1900).

*Page 86.*

The trisectrix of Catalan is the first negative pedal of the parabola when the pole is at the focus.

( $\therefore$  a sine spiral admitting of rectification.)

## Page 89.

*The Cubic Duplicatrix of G. de Longchamps.*

A is a fixed point, P any point on the  $y$ -axis. PQ  $\perp$  AP meets the  $x$ -axis in Q. If QR is drawn parallel to the  $y$ -axis and cuts AP in R, R traces the curve in question.

## Page 90.

*The Parabolic Leaf of De Longchamps, 1890.*

A is a fixed point, P any point on the  $y$ -axis. PQRS is a rectangle, Q being on OX, R on OY, and S on AP between A and P. The locus of S is the parabolic leaf.

## Page 223.

Given a circle of centre O and radius R, the locus of the vertices of the parabolas which touch the circle and have a fixed point on the circumference as focus is a curve whose equation is given by Barisien (*Intermediaire des Math.*, 1896), which Retali (*J. de Math. Spec.*, 1897) observed to be the pedal of the cardioid, when the pole is at the cusp.

## Page 498.

The cardioid as a special epicycloid is ascribed to Cramer, and not to Maclaurin.

These extracts may serve to show the importance of Maclaurin's methods in the invention of curves.

The *Geometria Organica* is, in fact, remarkable for the great number and variety of the curves invented by the young Maclaurin, and had he never written another page of mathematics, Maclaurin's name would have been entitled to a conspicuous place in the annals of mathematicians.

If I have succeeded in pointing this out in the foregoing summary of his work, my object in writing it has been attained.



## VI.—The Theory of Circulants from 1880 to 1900.

By Sir Thomas Muir, LL.D., F.R.S.

(MS. received November 8, 1915. Read December 20, 1915.)

My last paper on the "Theory of Circulants" appeared four years ago (*Proc. Roy. Soc. Edin.*, xxxii, pp. 136-149), and brought up the history of the subject to 1880. I have now completed the research up to the close of the nineteenth century—the goal originally aimed at. It is considered that from the beginning of the present century the indexing of mathematical writings is sufficiently full and detailed to make it comparatively easy for a worker to follow up for himself the recent lines of development.

STUDNIČKA, F. J. (1880).

[Ueber eine neue Determinanteneigenschaft. *Sitzungsb. . . Ges. d. Wiss.* (Prag), pp. 50-54.]

The circulant dealt with, namely,  $C(-1, 1, 1, 1, \dots, 1)$  is a case of Sylvester's of 1855 (*Hist.*, ii, p. 407).

SCOTT, R. F. (1880).

[Note on a determinant theorem of Mr Glaisher's. *Quart. Journ. of Math.*, xvii, pp. 129-132.]

Taking  $C(a, b, c, d, e, f)$  as an example of an even-ordered circulant, and performing the operations

$$\begin{array}{lll} \text{col}_1 + \text{col}_4, & \text{col}_2 + \text{col}_5, & \text{col}_3 + \text{col}_6, \\ \text{row}_3 - \text{row}_1, & \text{row}_4 - \text{row}_2, & \text{row}_5 - \text{row}_3, \end{array}$$

Scott finds it equal to

$$\begin{vmatrix} a+d & b+e & c+f \\ c+f & a+d & b+e \\ b+e & c+f & a+d \end{vmatrix} \cdot \begin{vmatrix} a-d & b-e & c-f \\ f-c & a-d & b-e \\ e-b & f-c & a-d \end{vmatrix}.$$

The form of the second factor leads him to draw attention to what he calls a cyclically skew determinant, namely,

$$\begin{vmatrix} a_1 & a_2 & a_3 & \dots & a_n \\ -a_n & a_1 & a_2 & \dots & a_{n-1} \\ -a_{n-1} & -a_n & a_1 & \dots & a_{n-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -a_2 & -a_3 & -a_4 & \dots & a_1 \end{vmatrix},$$

of which an equivalent is

$$\prod_{r=1}^{r=n} (a_1 + a_2 \omega_r + a_3 \omega_r^2 + \dots + a_n \omega_r^{n-1})$$

where  $\omega_r$  is any root of the equation

$$x^n + 1 = 0.$$

His theorem is thus to the effect that a circulant of the  $(2m)^{th}$  order is expressible as the product of a circulant and a skew circulant each of the  $m^{th}$  order; and he gives an alternative proof of it dependent on the ordinary factorial expressions for the determinants in question. It is noted also that on account of the relation between the roots of the equations

$$x^{2n+1} + 1 = 0, \quad x^{2n+1} - 1 = 0,$$

the circulant of

$$a_1, -a_2, a_3, -a_4, \dots, -a_{2n}, a_{2n+1}$$

is equal to the skew circulant of

$$a_1, a_2, a_3, a_4, \dots, a_{2n}, a_{2n+1},$$

or, say, to

$$C'(a_1, a_2, \dots, a_{2n+1}).$$

GLAISHER, J. W. L. (1880).

[On some algebraical expressions which are unaltered by certain substitutions. *Messenger of Math.*, x, pp. 60-63.]

In effect the main result of the paper is that if  $s$  stands for  $a_1 + a_2 + \dots + a_n$ , then

$$C(s - a_1, s - a_2, \dots, s - a_n) = (-1)^{n-1} (n-1) C(a_1, a_2, \dots, a_n),$$

and consequently that

$$\frac{C(s - a_1, s - a_2, \dots, s - a_n)}{(s - a_1) + (s - a_2) + \dots + (s - a_n)} = (-1)^{n-1} \frac{C(a_1, a_2, \dots, a_n)}{a_1 + a_2 + \dots + a_n},$$

a simple example of the latter being

$$\begin{vmatrix} 1 & c+a & a+b \\ 1 & b+c & c+a \\ 1 & a+b & b+c \end{vmatrix} = (-1)^2 \begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix},$$

or, in non-determinant rotation,

$$(b+c)^2 + (c+a)^2 + (a+b)^2 - (c+a)(a+b) - (a+b)(b+c) - (b+c)(c+a) \\ = a^2 + b^2 + c^2 - bc - ca - ab.$$

The result of making one or two other readily suggested substitutions for  $a_1, a_2, \dots, a_n$  in  $C(a_1, a_2, \dots, a_n)$  is also noted.

WEIHRAUCH, K. (1880).

[Ueber doppelt-orthosymmetrische Determinanten. *Zeitschrift f. Math. u. Phys.*, xxvi, pp. 64-70.]

Weihrauch establishes Spottiswoode's property by using Fürstenau's theorem of 1879; that is to say, he multiplies each element in the  $(r, s)^{\text{th}}$  place by  $\omega^{r-s}$ , where  $\omega$  is any one of the  $n^{\text{th}}$  roots of unity, and then takes the sum of the rows.

More interesting is the fresh proposition that *the circulant of  $a_1, a_2, \dots, a_n$ , is equal to the sum of the coefficients of the equation whose roots are the  $n^{\text{th}}$  powers of the roots of the equation*

$$a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0.$$

By way of proof it is noted that in the case where  $n$  is 4 we have

$$C(a, b, c, d)$$

$$= (a + b\omega_1 + c\omega_1^2 + d\omega_1^3)(a + b\omega_2 + c\omega_2^2 + d\omega_2^3)(a + b\omega_3 + c\omega_3^2 + d\omega_3^3)(a + b\omega_4 + c\omega_4^2 + d\omega_4^3),$$

so that, if  $\xi_1, \xi_2, \xi_3$  be the roots of the equation

$$ax^3 + bx^2 + cx + d = 0,$$

there results

$$C(a, b, c, d)$$

$$= a(1 - \omega_1\xi_1)(1 - \omega_1\xi_2)(1 - \omega_1\xi_3)$$

$$\cdot a(1 - \omega_2\xi_1)(1 - \omega_2\xi_2)(1 - \omega_2\xi_3)$$

$$\cdot a(1 - \omega_3\xi_1)(1 - \omega_3\xi_2)(1 - \omega_3\xi_3)$$

$$\cdot a(1 - \omega_4\xi_1)(1 - \omega_4\xi_2)(1 - \omega_4\xi_3)$$

$$= a^4 \cdot (1 - \omega_1\xi_1)(1 - \omega_2\xi_1)(1 - \omega_3\xi_1)(1 - \omega_4\xi_1)$$

$$\cdot (1 - \omega_1\xi_2)(1 - \omega_2\xi_2)(1 - \omega_3\xi_2)(1 - \omega_4\xi_2)$$

$$\cdot (1 - \omega_1\xi_3)(1 - \omega_2\xi_3)(1 - \omega_3\xi_3)(1 - \omega_4\xi_3)$$

$$= a^4(1 - \xi_1^4)(1 - \xi_2^4)(1 - \xi_3^4),$$

as desired.

Instead of this, however, Weihrauch in effect proposes to substitute a wider proposition, namely, that *the equation whose roots are the 4<sup>th</sup> powers of the roots of the equation  $ax^3 + bx^2 + cx + d = 0$  is*

$$- \begin{vmatrix} a & b & c & d \\ b & c & d & ay \\ c & d & ay & by \\ d & ay & by & cy \end{vmatrix} = 0,$$

the previous result being thence obtained on putting  $y=1$ . To establish this  $y^{\frac{1}{4}}$  is substituted for  $x$  in the given equation and rationalisation is

effected in Cayley's manner of 1852 (*Hist.*, ii, pp. 124-126), the full set of equations being

$$\left. \begin{aligned} ay^{\frac{2}{3}} + by^{\frac{1}{3}} + cy^{\frac{1}{3}} + d &= 0 \\ ay + by^{\frac{2}{3}} + cy^{\frac{2}{3}} + dy^{\frac{1}{3}} &= 0 \\ ay \cdot y^{\frac{1}{3}} + by + cy^{\frac{2}{3}} + dy^{\frac{2}{3}} &= 0 \\ ay \cdot y^{\frac{2}{3}} + by \cdot y^{\frac{1}{3}} + cy + dy^{\frac{2}{3}} &= 0 \end{aligned} \right\}$$

and the eliminants  $y^{\frac{1}{3}}, y^{\frac{2}{3}}, y^{\frac{1}{3}}$ .

It is also noted that the same procedure is effective when the roots of the new equation are to be the  $p^{\text{th}}$  powers of the roots of the given equation. Thus,  $p$  being 3, we substitute  $y^{\frac{1}{3}}$  for  $x$ , and multiply twice by  $y^{\frac{1}{3}}$ , the set of equations now being

$$\left. \begin{aligned} ay + by^{\frac{2}{3}} + cy^{\frac{1}{3}} + d &= 0 \\ ay \cdot y^{\frac{1}{3}} + by + cy^{\frac{2}{3}} + dy^{\frac{1}{3}} &= 0 \\ ay \cdot y^{\frac{2}{3}} + by \cdot y^{\frac{1}{3}} + cy + dy^{\frac{2}{3}} &= 0 \end{aligned} \right\}$$

and the desired equation

$$- \begin{vmatrix} b & c & d+ay \\ c & d+ay & by \\ d+ay & by & cy \end{vmatrix} = 0.$$

Here, it may be remarked, the sum of the coefficients is still a circulant, namely,  $C(d+a, b, c)$ .

WEIHRAUCH, K. (1880).

[Werth einiger doppelt-orthosymmetrischer Determinanten. *Zeitschrift f. Math. u. Phys.*, xxvi, pp. 132-133.]

Three of the four results are Scott's of the year 1878, and the remaining one, namely,

$$C(1, 3, 6, \dots, \tfrac{1}{2}n(n+1)) = (-1)^{n-1} \frac{n^{n-2}(n+1)(n+2)}{3 \cdot 2^{n+1}} \left\{ (n+3)^n - (n+1)^n \right\},$$

resembles Scott's third.

MUIR, T. (1881).

[On new and recently discovered properties of certain symmetric determinants. *Quart. Journ. of Math.*, xviii, pp. 166-177.]

To start with, Glaisher's theorem regarding a circulant of the  $(2n)^{\text{th}}$  order is shown to be readily obtained by multiplying together two slightly altered forms of the circulant. The first form is got by advancing the odd-numbered rows in order to the first  $n$  places, and then altering the signs in the even-numbered columns, the result being

$$(-1)^{\frac{1}{2}(3+1)^3} \cdot C(a, b, c, d, e, f) = \begin{vmatrix} a & -b & c & -d & e & -f \\ e & -f & a & -b & c & -d \\ c & -d & e & -f & a & -b \\ f & -a & b & -c & d & -e \\ d & -e & f & -a & b & -c \\ b & -c & d & -e & f & -a \end{vmatrix}.$$

The second form is got from the first by deleting the negative signs, reversing the order of the rows and then the order of the columns, the result being

$$(-1)^{\frac{1}{2}(3-1)^3} \cdot C(a, b, c, d, e, f) = \begin{vmatrix} a & f & e & d & c & b \\ c & b & a & f & e & d \\ e & d & c & b & a & f \\ b & a & f & e & d & c \\ d & c & b & a & f & e \\ f & e & d & c & b & a \end{vmatrix}.$$

From these two results by multiplication there is obtained

$$(-1)^3 \left\{ C(a, b, c, d, e, f) \right\}^2 = \begin{vmatrix} P & R & Q & . & . & . \\ Q & P & R & . & . & . \\ R & Q & P & . & . & . \\ . & . & . & -P & -R & -Q \\ . & . & . & -Q & -P & -R \\ . & . & . & -R & -Q & -P \end{vmatrix},$$

and therefore

$$C(a, b, c, d, e, f) = C(P, Q, R),$$

where

$$P = a^2 - bf + ce - d^2 + ec - fb = a^2 + 2ce - d^2 - 2bf,$$

$$Q = ae - bd + c^2 - db + ea - f^2 = c^2 + 2ea - f^2 - 2db,$$

$$R = ac - b^2 + ca - df + e^2 - fd = e^2 + 2ac - b^2 - 2fd.$$

In the second place, Scott's theorem regarding a circulant of the  $(2n)^{\text{th}}$  order is shown to be immediately deducible from Zehfuss' theorem of 1862, the reason being that by reversing the order of the last  $n$  rows and thereafter the order of the last  $n$  columns the circulant becomes centrosymmetric. For example,

$$C(a, b, c, d, e, f) = \begin{vmatrix} a & b & c & f & e & d \\ f & a & b & e & d & c \\ e & f & a & d & c & b \\ b & c & d & a & f & e \\ c & d & e & b & a & f \\ d & e & f & c & b & a \end{vmatrix}$$

$$\begin{aligned}
&= \begin{vmatrix} a+d & b+e & c+f \\ c+f & a+d & b+e \\ b+e & c+f & a+d \end{vmatrix} \cdot \begin{vmatrix} a-d & b-e & c-f \\ f-c & a-d & b-e \\ e-b & f-c & a-d \end{vmatrix} \\
&= C(a+d, b+e, c+f) \cdot C'(a-d, b-e, c-f).
\end{aligned}$$

It is next found that when  $n$  is odd there is no difficulty in deducing Glaisher's theorem from Scott's. We have only to change the columns of  $C(a+d, b+e, c+f)$  into rows, after the signs of the second row and second column of  $C'(a-d, b-e, c-f)$ , and then perform row-by-row multiplication, there being a series of identities like

$$(a+d, c+f, b+e) \times (a-d, e-b, c-f) = a^2 + 2ce - d^2 - 2bf$$

to be relied on.

Lastly, attention is drawn to the fact that *there is a second way of expressing a circulant of the  $(4n+2)^{th}$  order as a circulant of the  $(2n+1)^{th}$  order*, namely, by resolving it in accordance with Scott's theorem, substituting for the skew circulant an ordinary circulant, and then performing row-by-row multiplication. For example,

$$\begin{aligned}
C(a, b, c, d, e, f) &= \begin{vmatrix} a+f & b+e & c+d \\ c+d & a+f & b+e \\ b+e & c+d & a+f \end{vmatrix} \cdot \begin{vmatrix} a-f & e-b & c-d \\ c-d & a-f & e-b \\ e-b & c-d & a-f \end{vmatrix}, \\
&= \begin{vmatrix} L & M & N \\ N & L & M \\ M & N & L \end{vmatrix},
\end{aligned}$$

where

$$L = a^2 + e^2 + c^2 - b^2 - d^2 - f^2 = \sum a^2 - \sum b^2 \text{ say}$$

$$M = \sum ae - \sum bd + \sum ab - \sum ad$$

$$N = \sum ae - \sum bd - \sum ab + \sum ad,$$

the  $\sum$  implying summation of the terms got by changing

$$a, e, c \text{ into } e, c, a,$$

and

$$b, d, f \text{ into } d, f, b.$$

As companion results to Scott's are given (§§ 9, 10) the identities

$$\begin{vmatrix} a & b & c & d & e & f \\ c & a & b & e & f & d \\ b & c & a & f & d & e \\ e & d & f & a & c & b \\ d & f & e & b & a & c \\ f & e & d & c & b & a \end{vmatrix} = C(a+f, b+e, c+d) \cdot C(a-f, b-e, c-d),$$

$$\begin{vmatrix} a & b & c & d & e & f \\ -c & a & b & e & f & -d \\ -b & -c & a & f & -d & -e \\ -e & -d & f & a & -c & -b \\ -d & f & e & b & a & -c \\ f & e & d & c & b & a \end{vmatrix} = C'(a+f, b+e, c+d) \cdot C'(a-f, b-e, c-d),$$

where the determinants on the left are no longer circulants but resemble circulants in having the elements of the first row repeated in each of the other rows. Both are centro-symmetric; and in addition the former of the two has the array of its first three rows made up of the array of  $C(a, b, c)$  and the array of  $(-1)^{\frac{1}{2}(3-1)^3}C(d, e, f)$ , while in the other arrays of  $C'(a, b, c)$  and  $(-1)^{\frac{1}{2}(3-1)^3}C'(d, e, f)$  are similarly used.

MUIR, T. (1881).

[On the resolution of a certain determinant into quadratic factors.  
*Messenger of Math.*, xi, pp. 105-108.]

The theorem here established is that *the determinant whose every element is the sum of the corresponding elements of the circulants*

$$C(a_1, a_2, \dots, a_n), \quad (-1)^{\frac{1}{2}n(n-1)} \cdot C(b_1, b_2, \dots, b_n)$$

*is divisible by*

$$(a_1 + \omega a_2 + \dots + \omega^{n-1} a_n)(a_1 + \omega^{-1} a_2 + \dots + \omega^{-n+1} a_n) \\ - (b_1 + \omega b_2 + \dots + \omega^{n-1} b_n)(b_1 + \omega^{-1} b_2 + \dots + \omega^{-n+1} b_n),$$

$\omega$  being one of the imaginary  $n^{\text{th}}$  roots of unity.

When  $n$  is 5 the determinant in question is

$$\begin{vmatrix} a_1 + b_1 & a_2 + b_2 & a_3 + b_3 & a_4 + b_4 & a_5 + b_5 \\ a_5 + b_2 & a_1 + b_3 & a_2 + b_4 & a_3 + b_5 & a_4 + b_1 \\ a_4 + b_3 & a_5 + b_4 & a_1 + b_5 & a_2 + b_1 & a_3 + b_2 \\ a_3 + b_4 & a_4 + b_5 & a_5 + b_1 & a_1 + b_2 & a_2 + b_3 \\ a_2 + b_5 & a_3 + b_1 & a_4 + b_2 & a_5 + b_3 & a_1 + b_4 \end{vmatrix},$$

and may be viewed as the eliminant of a set of linear homogeneous equations in  $x_1, x_2, x_3, x_4, x_5$ . Using the multipliers  $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4$  with these equations and performing addition we obtain

$$\frac{a_1 + \omega^{-1} a_2 + \dots + \omega^{-4} a_5}{b_1 + \omega b_2 + \dots + \omega^4 b_5} = - \frac{x_1 + \omega x_5 + \omega^2 x_4 + \omega^3 x_3 + \omega^4 x_2}{x_1 + \omega x_2 + \omega^2 x_3 + \omega^3 x_4 + \omega^4 x_5},$$

and therefore on putting  $\omega^{-1}$  for  $\omega$

$$\frac{a_1 + \omega a_2 + \dots + \omega^4 a_5}{b_1 + \omega^{-1} b_2 + \dots + \omega^{-4} b_5} = - \frac{x_1 + \omega^4 x_5 + \omega^3 x_4 + \omega^2 x_3 + \omega x_2}{x_1 + \omega^4 x_2 + \omega^3 x_3 + \omega^2 x_4 + \omega x_5},$$

from which two equalities there results by multiplication

$$\frac{a_1 + \omega^{-1}a_2 + \dots + \omega^{-4}a_5}{b_1 + \omega b_2 + \dots + \omega^4b_5} \cdot \frac{a_1 + \omega a_2 + \dots + \omega^4a_5}{b_1 + \omega^{-1}b_2 + \dots + \omega^{-4}b_5} = 1.$$

This being independent of the  $\omega$ 's furnishes a factor of the eliminant, and the factor thus reached is seen to be the quadratic factor specified in the enunciation of the theorem.

Note is taken that when  $n$  is odd there is the linear factor

$$a_1 + a_2 + \dots + a_n + b_1 + b_2 + \dots + b_n,$$

and that when  $n$  is even there is in addition to this factor

$$a_1 - a_2 + \dots - a_n + b_1 - b_2 + \dots - b_n.$$

Further, the multiplications indicated in the quadratic factor are actually performed, and  $\omega$  made to disappear through substitutions like  $2 \cos \frac{2}{5}\pi$  for  $\omega + \omega^{-1}$ .

MUIR, T. (1881).

[On circulants of odd order. *Quart. Journ. of Math.*, xviii, pp. 261-265.]

The main theorem here established is that *the cofactor of  $a_1 + a_2 + \dots + a_{2n+1}$  in  $C(a_1, a_2, \dots, a_{2n+1})$  is expressible as a symmetric determinant of the  $n^{\text{th}}$  order whose distinct elements are the  $\frac{1}{2}n(n+1)$  differences of the cyclic sums  $\dot{\Sigma}a_1^2, \dot{\Sigma}a_1a_2, \dot{\Sigma}a_1a_3, \dots, \dot{\Sigma}a_1a_n$ .*

Calling these cyclic sums  $A_1, A_2, A_3, A_4$ , in the case where  $n$  is 3, and multiplying  $C(a_1, a_2, \dots, a_7)$  by itself in the form obtained by diminishing each row by the immediately preceding row, we find

$$(C_7)^2 = \begin{vmatrix} A_1 & A_2 & A_3 & A_4 & A_4 & A_3 & A_2 \\ A_2 - A_1 & A_1 - A_2 & A_2 - A_3 & A_3 - A_4 & . & A_4 - A_3 & A_3 - A_2 \\ A_3 - A_2 & A_2 - A_1 & A_1 - A_2 & A_2 - A_3 & A_3 - A_4 & . & A_4 - A_3 \\ A_4 - A_3 & A_3 - A_2 & A_2 - A_1 & A_1 - A_2 & A_2 - A_3 & A_3 - A_4 & . \\ . & A_4 - A_3 & A_3 - A_2 & A_2 - A_1 & A_1 - A_2 & A_2 - A_3 & A_3 - A_4 \\ A_3 - A_4 & . & A_4 - A_3 & A_3 - A_2 & A_2 - A_1 & A_1 - A_2 & A_2 - A_3 \\ A_2 - A_3 & A_3 - A_4 & . & A_4 - A_3 & A_3 - A_2 & A_2 - A_1 & A_1 - A_2 \end{vmatrix}.$$

Here the increasing of the first column by all the others shows that

$$A_1 + 2A_2 + 2A_3 + 2A_4, \quad \text{i.e.} \quad (a_1 + a_2 + \dots + a_7)^2$$

is a factor, and the full result takes the form

$$(C_7)^2 = (a_1 + a_2 + \dots + a_7)^2 \begin{vmatrix} a & \beta & \gamma & . & -\gamma & -\beta \\ -a & a & \beta & \gamma & . & -\gamma \\ -\beta & -a & a & \beta & \gamma & . \\ -\gamma & -\beta & -a & a & \beta & \gamma \\ . & -\gamma & -\beta & -a & a & \beta \\ \gamma & . & -\gamma & -\beta & -a & a \end{vmatrix}.$$



if for shortness' sake we write

$$\alpha, \beta, \gamma \text{ for } A_1 - A_2, A_2 - A_3, A_3 - A_4.$$

Multiplying this new determinant by

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ . & 1 & 1 & 1 & 1 & . \\ . & . & 1 & 1 & . & . \\ . & . & . & 1 & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & 1 \end{vmatrix}, \begin{vmatrix} 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & 1 & . & . \\ . & . & 1 & 1 & 1 & . \\ . & 1 & 1 & 1 & 1 & 1 \end{vmatrix}$$

in succession we obtain

$$(C_7)^2 = (\Sigma \alpha)^2 \begin{vmatrix} \alpha & \beta & \gamma & . & . & . \\ \beta & \alpha + \beta + \gamma & \beta + \gamma & . & . & . \\ \gamma & \beta + \gamma & \alpha + \beta & . & . & . \\ . & \gamma & \beta & \alpha & \beta & \gamma \\ -\gamma & . & \gamma & \beta & \alpha + \beta + \gamma & \beta + \gamma \\ -\beta & -\gamma & . & \gamma & \beta + \gamma & \alpha + \beta \end{vmatrix},$$

and thence

$$C_7 = \Sigma \alpha \cdot \begin{vmatrix} A_1 - A_2 & A_2 - A_3 & A_3 - A_4 \\ A_2 - A_3 & A_1 - A_4 & A_2 - A_4 \\ A_3 - A_4 & A_2 - A_4 & A_1 - A_3 \end{vmatrix},$$

as desired.

The latter part of the proof is seen to turn on the transformability of a certain six-line persymmetric determinant into the square of a three-line axisymmetric determinant, and this property is stated to hold for a persymmetric determinant of the  $(2n)^{\text{th}}$  order whose distinct elements are

$$-a_2, \dots, -a_n, 0, a_n, \dots, a_1, -a_1, \dots, -a_n, 0, a_n, \dots, a_3,$$

the elements of the  $n$ -line axisymmetric determinant being of the form

$$a_h + a_{h+1} + \dots + a_{h+k-1}.$$

TORELLI, G. (1882).

[Sui determinanti circolanti. *Rendic. . . . Accad. delle Sci. Fis. e Mat.* (Napoli), pp. 3-11.]

This paper, following avowedly on those of Glaisher and Scott and Muir, contains important generalisations.

The first theorem is to the effect that *any circulant of the  $(rs)^{\text{th}}$  order is expressible as a circulant of the  $s^{\text{th}}$  order, and each of the elements of the latter is an aggregate of  $r$ -line minors,  $r^{s-1}$  in number, of the former.* Torelli's proof, which is the natural extension of Glaisher's, is stated in

perfectly general terms. We shall, for the sake of greater clearness, apply it only to the case where  $rs=15$ , the elements of the circulant being

$$a_1, a_2, a_3, \dots, a_{15}.$$

The fifteenth roots of 1 are the three third roots of each of the five fifth roots of 1, so that if  $\gamma_1, \gamma_2, \gamma_3$  be the third roots of 1 and  $\epsilon_1, \epsilon_2, \dots, \epsilon_5$  the fifth roots of 1, the fifteenth roots of 1 may be represented by

$$\gamma_1\epsilon_1^{\frac{1}{3}}, \gamma_2\epsilon_1^{\frac{1}{3}}, \gamma_3\epsilon_1^{\frac{1}{3}}; \gamma_1\epsilon_2^{\frac{1}{3}}, \gamma_2\epsilon_2^{\frac{1}{3}}, \gamma_3\epsilon_2^{\frac{1}{3}}; \dots; \gamma_1\epsilon_5^{\frac{1}{3}}, \gamma_2\epsilon_5^{\frac{1}{3}}, \gamma_3\epsilon_5^{\frac{1}{3}},$$

and the first of the fifteen linear factors of the given circulant by

$$a_1 + a_2(\gamma_1\epsilon_1^{\frac{1}{3}}) + a_3(\gamma_1\epsilon_1^{\frac{1}{3}})^2 + a_4(\gamma_1\epsilon_1^{\frac{1}{3}})^3 + \dots + a_{15}(\gamma_1\epsilon_1^{\frac{1}{3}})^{14}.$$

But this, by attending to the powers of  $\gamma_1\epsilon_2^{\frac{1}{3}}$  and rearranging accordingly is equal to

$$(a_1 + a_4\epsilon_1 + a_7\epsilon_1^2 + a_{10}\epsilon_1^3 + a_{13}\epsilon_1^4) + (a_2 + a_5\epsilon_1 + \dots + a_{14}\epsilon_1^4)\gamma_1\epsilon_1^{\frac{1}{3}} + (a_3 + a_6\epsilon_1 + \dots + a_{15}\epsilon_1^4)^2\gamma_1\epsilon_1^{\frac{2}{3}},$$

and as the second and third factors differ from it merely in having  $\gamma_2, \gamma_3$  respectively for  $\gamma_1$ , the product of the three is the circulant

$$\begin{vmatrix} a_1 + a_4\epsilon_1 + \dots & (a_2 + a_5\epsilon_1 + \dots)\epsilon_1^{\frac{1}{3}} & (a_3 + a_6\epsilon_1 + \dots)\epsilon_1^{\frac{2}{3}} \\ (a_3 + a_6\epsilon_1 + \dots)\epsilon_1^{\frac{2}{3}} & a_1 + a_4\epsilon_1 + \dots & (a_2 + a_5\epsilon_1 + \dots)\epsilon_1^{\frac{1}{3}} \\ (a_2 + a_5\epsilon_1 + \dots)\epsilon_1^{\frac{1}{3}} & (a_3 + a_6\epsilon_1 + \dots)\epsilon_1^{\frac{2}{3}} & a_1 + a_4\epsilon_1 + \dots \end{vmatrix},$$

which, by multiplying the rows in order by  $\epsilon_1^{\frac{1}{3}}, \epsilon_1^{\frac{2}{3}}, \epsilon_1^{\frac{1}{3}}$ , and thereafter dividing the columns in order by the same, becomes

$$\begin{vmatrix} a_1 + a_4\epsilon_1 + \dots & a_2 + a_5\epsilon_1 + \dots & a_3 + a_6\epsilon_1 + \dots \\ (a_3 + a_6\epsilon_1 + \dots)\epsilon_1 & a_1 + a_4\epsilon_1 + \dots & a_2 + a_5\epsilon_1 + \dots \\ (a_2 + a_5\epsilon_1 + \dots)\epsilon_1 & (a_3 + a_6\epsilon_1 + \dots)\epsilon_1 & a_1 + a_4\epsilon_1 + \dots \end{vmatrix}.$$

Performing the multiplications by  $\epsilon_1$ , we see this to be equal to

$$\begin{vmatrix} a_1 + a_4\epsilon_1 + \dots + a_{13}\epsilon_1^4 & a_2 + a_5\epsilon_1 + \dots + a_{14}\epsilon_1^4 & a_3 + a_6\epsilon_1 + \dots + a_{15}\epsilon_1^4 \\ a_{15} + a_3\epsilon_1 + \dots + a_{12}\epsilon_1^4 & a_1 + a_4\epsilon_1 + \dots + a_{13}\epsilon_1^4 & a_2 + a_5\epsilon_1 + \dots + a_{14}\epsilon_1^4 \\ a_{14} + a_2\epsilon_1 + \dots + a_{11}\epsilon_1^4 & a_{15} + a_3\epsilon_1 + \dots + a_{12}\epsilon_1^4 & a_1 + a_4\epsilon_1 + \dots + a_{13}\epsilon_1^4 \end{vmatrix},$$

and therefore to be expressible as a sum of  $5^3$  determinants with monomial elements which can be grouped so as to take the form

$$P + Q\epsilon_1 + R\epsilon_1^2 + S\epsilon_1^3 + T\epsilon_1^4,$$

where each of the coefficients of the powers of  $\epsilon_1$  is a sum of  $5^{3-1}$  such determinants. It is next observed that the fourth, fifth, sixth linear factors of the given circulant differ from the first, second, third merely in having  $\epsilon_2$  in place of  $\epsilon_1$ : their product therefore is

$$P + Q\epsilon_2 + R\epsilon_2^2 + S\epsilon_2^3 + T\epsilon_2^4:$$

and hence finally the product of the five triads is equal to

$$C(P, Q, R, S, T),$$

as predicated. Here  $r$  is 3 and  $s$  is 5.

Had we written the first linear factor of the given circulant in the form

$$a_1 + a_2(\epsilon_1\gamma_1^{\frac{1}{5}}) + a_3(\epsilon_1\gamma_1^{\frac{1}{5}})^2 + \dots + a_{15}(\epsilon_1\gamma_1^{\frac{1}{5}})^{14},$$

we should have changed it at the outset into

$$(a_1 + a_6\gamma_1 + a_{11}\gamma_1^2) + (a_2 + a_7\gamma_1 + a_{12}\gamma_1^2)\epsilon_1\gamma_1^{\frac{1}{5}} + \dots + (a_5 + a_{10}\gamma_1 + a_{15}\gamma_1^2)\epsilon_1^4\gamma_1^{\frac{4}{5}},$$

and so have obtained for the product of the first five linear factors the circulant

$$C \left\{ (a_1 + a_6\gamma_1 + a_{11}\gamma_1^2), (a_2 + a_7\gamma_1 + a_{12}\gamma_1^2)\gamma_1^{\frac{1}{5}}, \dots, (a_5 + a_{10}\gamma_1 + a_{15}\gamma_1^2)\gamma_1^{\frac{4}{5}} \right\}.$$

This by multiplication of rows and division of columns we should have changed into

$$\begin{vmatrix} a_1 + a_6\gamma_1 + a_{11}\gamma_1^2 & a_2 + a_7\gamma_1 + a_{12}\gamma_1^2 & \dots & a_5 + a_{10}\gamma_1 + a_{15}\gamma_1^2 \\ a_{15} + a_5\gamma_1 + a_{10}\gamma_1^2 & a_1 + a_6\gamma_1 + a_{11}\gamma_1^2 & \dots & a_4 + a_9\gamma_1 + a_{14}\gamma_1^2 \\ a_{14} + a_4\gamma_1 + a_9\gamma_1^2 & a_{15} + a_5\gamma_1 + a_{10}\gamma_1^2 & \dots & a_3 + a_8\gamma_1 + a_{13}\gamma_1^2 \\ a_{13} + a_3\gamma_1 + a_8\gamma_1^2 & a_{14} + a_4\gamma_1 + a_9\gamma_1^2 & \dots & a_2 + a_7\gamma_1 + a_{12}\gamma_1^2 \\ a_{12} + a_2\gamma_1 + a_7\gamma_1^2 & a_{13} + a_3\gamma_1 + a_8\gamma_1^2 & \dots & a_1 + a_6\gamma_1 + a_{11}\gamma_1^2 \end{vmatrix},$$

from which we should have passed on to the form

$$X + Y\gamma_1 + Z\gamma_1^2.$$

Thereupon we should have seen the product of the next five linear factors to be

$$X + Y\gamma_2 + Z\gamma_2^2,$$

and the product of the last five to be

$$X + Y\gamma_3 + Z\gamma_3^2;$$

and therefore the product of the whole to be

$$C(X, Y, Z).$$

Here  $r$  is 5 and  $s$  is 3.

From this transformation of a circulant of composite order into one of lower order Torelli proceeds to deduce a circulant factor of the original. Thus, having reached  $C(P, Q, R, S, T)$  as an equivalent of the fifteen-line circulant, we know that

$$P + Q + R + S + T$$

must be a factor of it, and, looking back at the three-line determinant whence  $P, Q, \dots$  originated, we see that their sum equals

$$C(a_1 + a_4 + \dots + a_{13}, \quad a_2 + a_5 + \dots + a_{15}, \quad a_3 + a_6 + \dots + a_{15}),$$

which is the factor desired. In like manner from the existence of the form  $C(X, Y, Z)$  there is obtained the factor

$$C(a_1 + a_6 + a_{11}, a_2 + a_7 + a_{12}, \dots, a_5 + a_{10} + a_{15}).$$

The same result is established by operating directly on the original circulant; and this procedure has the advantage of giving at the same time a convenient expression for the cofactor. Thus by performing on  $C(a_1, a_2, \dots, a_{15})$  the operations

$$\begin{aligned} &\text{row}_1 + \text{row}_4 + \text{row}_7 + \text{row}_{10} + \text{row}_{15} \\ &\text{row}_2 + \text{row}_5 + \dots \\ &\text{row}_3 + \text{row}_6 + \dots, \end{aligned}$$

and writing

$$\xi, \eta, \zeta \text{ for } a_1 + a_4 + \dots, a_2 + a_5 + \dots, a_3 + a_6 + \dots,$$

we obtain

$$\begin{vmatrix} \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta \\ \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta \\ \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi & \eta & \zeta & \xi \\ a_{13} & a_{14} & a_{15} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_1 \end{vmatrix}.$$

By column-subtraction twelve corresponding elements in the first three rows can then be made 0, and the determinant resolved into two; for example, if we choose the last twelve elements there is obtained

$$C(\xi, \eta, \zeta) \cdot \begin{vmatrix} a_1 - a_{13} & a_2 - a_{14} & a_3 - a_{15} & a_4 - a_1 & \dots & a_{12} - a_9 \\ a_{15} - a_{12} & a_1 - a_{13} & a_2 - a_{14} & a_3 - a_{15} & \dots & a_{11} - a_8 \\ a_{14} - a_{11} & a_{15} - a_{12} & a_1 - a_{13} & a_2 - a_{14} & \dots & a_{10} - a_7 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_5 - a_2 & a_6 - a_3 & a_7 - a_4 & a_8 - a_5 & \dots & a_1 - a_{13} \end{vmatrix}.$$

Since 3 is prime to 5, the pair of circulant factors can themselves have no factor in common save  $a_1 + a_2 + \dots + a_{15}$ , and therefore

$$(a_1 + a_2 + \dots + a_{15})C(a_1, a_2, \dots, a_{15})$$

must be divisible by the product of the said pair. As an expression for the resulting quotient, which must be of the eighth degree in the original elements, Torelli obtains a determinant of the eighth order. He also extends the reasoning to a circulant having its order-number resolvable into more than two mutually prime integers: and finally makes application (§§ 8, 9) of his results to generalise Stern's cofactor theorem of 1871 and a theorem of Kummer's of 1851 on complex numbers.

It will suffice to add that throughout his paper Torelli makes due

reference to the corresponding results regarding *skew* circulants ("gobbi circolanti").

MUIR, T. (1882).

[A proposed general method for the solutions of equations. *Proceed. Philos. Soc. Glasgow*, xiii, p. 616: *Educ. Times*, xlv, p. 442: *Math. from Educ. Times* (2), v, pp. 106-107.]

The property proposed to be utilised is the resolvability of  $C(x, a, b, c, \dots)$  into linear factors and the consequent solubility of the equation

$$C(x, a, b, c, \dots) = 0.$$

The application to the quartic  $x^4 + px^2 + qx + r = 0$  is carried out by E. Nesbitt in the *Educ. Times*, and, in effect, to a specialised quintic by Cayley in the *Quart. Journ. of Math.*, xviii, pp. 154-167.

BHUT, A. B. (1882).

[Question 7072. *Educ. Times*, xxxv, p. 155: *Math. from Educ. Times*, xl, p. 48.]

The problem is to find  $x, y, z, u$  from the equations

$$\begin{vmatrix} x & y & z \\ u & x & y \\ z & u & x \end{vmatrix} = a^3, \quad \begin{vmatrix} y & z & u \\ x & y & z \\ u & x & y \end{vmatrix} = b^3, \quad \begin{vmatrix} z & u & x \\ y & z & u \\ x & y & z \end{vmatrix} = c^3, \quad \begin{vmatrix} u & x & y \\ z & u & x \\ y & z & u \end{vmatrix} = d^3,$$

and it is not observed that this is the same as to express  $x, y, z, u$  in terms of their complementary minors in  $C(x, y, z, u)$ . Calling these when signed X, Y, Z, U, we have from the theorem regarding a minor of the adjugate

$$\begin{vmatrix} X & Y & Z \\ U & X & Y \\ Z & U & X \end{vmatrix} = x \begin{vmatrix} x & y & z & u \\ u & x & y & z \\ z & u & x & y \\ y & z & u & x \end{vmatrix}^2,$$

and therefore

$$x = \begin{vmatrix} X & Y & Z \\ U & X & Y \\ Z & U & X \end{vmatrix} \div \begin{vmatrix} X & Y & Z & U \\ U & X & Y & Z \\ Z & U & X & Y \\ Y & Z & U & X \end{vmatrix}.$$

CESÁRO, E. (1883): NEUBERG, J. (1883).

[Question 245. *Mathesis*, iii, pp. 118-119; vi, pp. 60-62.]

[Question 273. *Mathesis*, iii, p. 192; viii, p. 215; x, pp. 117-119.]

These concern special circulants which have already been fully dealt with.

FORSYTH, A. R. (1884).

[On certain symmetric products involving prime roots of unity. *Messenger of Math.*, xiv, pp. 40-56.]

Forsyth begins in effect by equating two forms of the eliminant of

$$\left. \begin{aligned} x^n - p_1 x^{n-1} + p_2 x^{n-2} - \dots &\equiv (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_n) = 0 \\ x^s - 1 &\equiv (x - \omega_1)(x - \omega_2) \dots (x - \omega_s) = 0 \end{aligned} \right\}$$

obtaining naturally

$$\prod_{x=\omega_1}^{x=\omega_s} (x^n - p_1 x^{n-1} + \dots) = (-1)^{n(s+1)} \cdot (1 - \alpha_1^s)(1 - \alpha_2^s) \dots (1 - \alpha_n^s),$$

and when  $s=3$

$$C(\sigma, \tau, v) = (1 - \alpha_1^3)(1 - \alpha_2^3) \dots (1 - \alpha_n^3)$$

where

$$\begin{aligned} \sigma &= 1 - p_3 + p_6 - p_9 + \dots \\ \tau &= -p_1 + p_4 - p_7 + p_{10} - \dots \\ v &= p_2 - p_5 + p_8 - p_{11} + \dots \end{aligned}$$

Three paragraphs (pp. 43-46) are then devoted to the evaluation of circulants. The procedure adopted is essentially the same as Minozzi's of 1878. As illustrative examples of its effectiveness the five-line and seven-line circulants are taken, the result in the former case agreeing with Glaisher's, and in the latter being new.

MUIR, T. (1884).

[Note on the final expansion of circulants. *Messenger of Math.*, xiv, pp. 169-175.]

The mode of expansion referred to is that used by Minozzi and Forsyth, the main object being to correct, if necessary, the latter's expansion of  $C(a_0, a_1, a_2, \dots, a_6)$ , or, say,  $C(0, 1, 2, 3, 4, 5, 6)$ . It is first pointed out that the procedure is considerably shortened by using an additional property, namely, that when

$$A\alpha_0^a\alpha_1^b\alpha_2^c\alpha_3^d\alpha_4^e\alpha_5^f\alpha_6^g$$

is a term of the expansion, then also is

$$A\alpha_0^a\alpha_m^b\alpha_n^c\alpha_p^d\alpha_q^e\alpha_r^f\alpha_s^g$$

a term, where  $m, n, p, q, r, s$  are numbers less than 7, and such that

$$m, 2m, 3m, 4m, 5m, 6m \equiv m, n, p, q, r, s \pmod{7}.$$

Thus, having got the term  $-27a_0^3a_1^2a_2a_3$  in the expansion of  $C(0, 1, 2, 3, 4, 5, 6)$ , we operate on the suffixes 1, 2, 3 by multiplying in succession by the numbers 2, 3, 4, 5, 6 and casting out the sevens, the result being

the new sets of suffixes 2, 4, 6; 3, 6, 2; 4, 1, 5; 5, 3, 1; 6, 5, 4: and so finally the complete set of terms

$$- 27a_0^3(a_1^2a_2a_3 + a_2^2a_4a_6 + a_3^2a_6a_2 + a_4^2a_1a_5 + a_5^2a_3a_1 + a_6^2a_5a_4).$$

The "reason of the rule" is that

$$\begin{aligned} C(0, 1, 2, 3, 4, 5, 6) &= C(0, 2, 4, 6, 1, 3, 5), \\ &= C(0, 3, 6, 2, 5, 1, 4), \\ &= C(0, 4, 1, 5, 2, 6, 3), \\ &= C(0, 5, 3, 1, 6, 4, 2), \\ &= C(0, 6, 5, 4, 3, 2, 1); \end{aligned}$$

as we can show either by transposition of rows and columns, or by considering the circulant in its form as a product of seven linear factors and using the fact that, if  $\omega$  be one of the imaginary seventh roots of unity,  $\omega^2, \omega^3, \omega^4, \omega^5, \omega^6$  are the others.

A caution is then given that *two terms having apparently the same form have not necessarily the same coefficient*; and as an illustration it is shown that in Forsyth's expression half of the terms with the coefficient 35 should have the coefficient -14 and that the coefficient of his final term should be changed from -448 to -105, the whole expansion then being

$$\begin{aligned} &\sum a_0^7 \\ &- 7 \sum a_0^5 (a_1a_6 + a_2a_5 + a_3a_4) \\ &+ 7 \sum a_0^4 (a_1^2a_5 + a_2^2a_3 + a_3^2a_1 + a_4^2a_6 + a_5^2a_4 + a_6^2a_2) \\ &+ 14 \sum a_0^4 (a_1a_2a_4 + a_3a_5a_6) \\ &- 7 \sum a_0^3 (a_1^3a_6 + a_2^3a_5 + a_3^3a_4) \\ &- 21 \sum a_0^3 (a_1^2a_2a_3 + a_2^2a_4a_6 + a_3^2a_6a_2 + a_4^2a_1a_5 + a_5^2a_3a_1 + a_6^2a_5a_4) \\ &+ 14 \sum a_0^3 (a_1^2a_6^2 + a_2^2a_5^2 + a_3^2a_4^2) \\ &+ 7 \sum a_0^3 (a_1a_2a_5a_6 + a_2a_4a_3a_5 + a_3a_6a_1a_4) \\ &- 7 \sum a_0^2 (a_1^2a_2^2a_4^2 + a_3^2a_5^2a_6^2) \\ &+ 35 \sum a_0^2 (a_1^2a_3a_4a_5 + a_2^2a_6a_1a_3 + a_3^2a_2a_5a_1) \\ &- 14 \sum a_0^2 (a_1^2a_2a_4a_6 + a_2^2a_4a_1a_5 + a_3^2a_6a_5a_4) \\ &- 105a_0a_1a_2a_3a_4a_5a_6; \end{aligned}$$

or, with the help of the fresh property, in the still more condensed form

$$\sum_{\mu=0}^{\mu=6} \sum_{\nu=1}^{\nu=6} \left( a_{\mu}^7 - 7a_{\mu}^5 a_{\mu+\nu} a_{\mu+6\nu} + 7a_{\mu}^4 a_{\mu+\nu}^2 a_{\mu+5\nu} + \dots \right),$$

where every suffix greater than 6 is to be made less than 7 by subtracting a multiple of 7 from it and duplicates are to be neglected.

An entirely different mode of saving labour in the computation rests on the fact that all the terms which contain an element in the first power can be got by evaluating a zero-axial determinant of the next lower order. For example, since every term of  $C(a_0, a_1, a_2, a_3, a_4)$  is of this kind except  $\sum a_0^5$ , we evaluate

$$5 \begin{vmatrix} . & a_1 & a_2 & a_3 \\ a_4 & . & a_1 & a_2 \\ a_3 & a_4 & . & a_1 \\ a_2 & a_3 & a_4 & . \end{vmatrix},$$

and thus finding the cofactor of  $a_0$  to be

$$-5(a_1^2 a_2 + a_2^2 a_4 + a_3^2 a_1 + a_4^2 a_2) + 5(a_1^2 a_4^2 + a_2^2 a_3^2) - 5a_1 a_2 a_3 a_4$$

we obtain the full expansion

$$\sum a_0^5 - 5 \sum a_0^3 (a_1 a_4 + a_2 a_3) + 5 \sum a_0^2 (a_1^2 a_3 + a_2^2 a_1) - 5a_0 a_1 a_2 a_3 a_4.$$

In the case of the seven-line continuant the saving is almost quite as great, since in addition to  $\sum a_0^7$  the term  $14 \sum a_0^5 a_1^2 a_6^2$  is the only one that does not involve a first power of an element.

MUIR, T. (1885).

[Detached theorems on circulants. *Trans. Roy. Soc. Edin.*, xxxii, pp. 639-643.]

The first of the theorems is that *if in a circulant the element in the place (p, q) be by the nature of the circulant the same as the element in the place (r, s), the complementary minor of the former element is to that of the latter as  $(-1)^{r+s} : (-1)^{p+q}$* . But more than this is intended to be brought out, namely, that by cyclical transposition of rows and columns the element in the place (r, s) may be made to take the place (p, q) and the determinant be in outward form exactly the same as before. The second theorem is Stern's of 1871, although not so credited. The mode of establishing it is to alter the first column of  $C(a, b, c, d, e)$  so as to make it possible to remove  $a + b\omega + c\omega^2 + d\omega^3 + e\omega^4$  as a factor, and leave

$$1, \omega, \omega^2, \omega^3, \omega^4$$



for the elements of the column. The third theorem is that *if the linear factors of a circulant, say of the fifth order, be  $\alpha, \beta, \gamma, \dots$ , and the complementary minors of the elements of the first row be  $A, -B, C, -\dots$  then*

$$C(\alpha\beta\gamma\delta, \alpha\beta\gamma\epsilon, \alpha\beta\delta\epsilon, \alpha\gamma\delta\epsilon, \beta\gamma\delta\epsilon) = 5^5 ABCDE.$$

This is proved by using the preceding theorem five times over in the form

$$A + \omega E + \omega^2 D + \omega^3 C + \omega^4 B = \beta\gamma\delta\epsilon,$$

$$A + \omega^2 E + \omega^4 D + \omega C + \omega^3 B = \gamma\delta\epsilon\alpha,$$

$$\dots \dots \dots$$

and thence obtaining the linear factors of

$$C(\alpha\beta\gamma\delta, \alpha\beta\gamma\epsilon, \alpha\beta\delta\epsilon, \alpha\gamma\delta\epsilon, \beta\gamma\delta\epsilon)$$

in the form

$$5A, \quad 5B, \quad 5C, \quad 5D, \quad 5E.$$

The fourth theorem is that *if the elements of the first row of a circulant of the  $n^{\text{th}}$  order be multiplied by  $\omega^n, \omega^{n-1}, \dots, \omega$  respectively, the elements of the second row by  $\omega^{n-1}, \omega^{n-2}, \dots, \omega, \omega^n$  respectively, the elements of the third row by  $\omega^{n-2}, \omega^{n-3}, \dots, \omega, \omega^n, \omega^{n-1}$  respectively, and so on, where  $\omega$  is any  $n^{\text{th}}$  root of unity, the circulant is unaltered in value.* Here for proof we have only to multiply the rows in order by  $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4$ , and then divide the columns by  $\omega^5, \omega^4, \omega^3, \omega^2, \omega$ . The fifth theorem concerns skew circulants, being the analogue of the same author's theorem of 1881 regarding ordinary circulants.

In order to throw light on the law of the coefficients of the final expansion of  $C(a, b, c, d, e)$  the determinant

$$\begin{vmatrix} a\alpha & b\beta & c\gamma & d\delta & e\epsilon \\ e\beta & a\gamma & b\delta & c\epsilon & d\alpha \\ d\gamma & e\delta & a\epsilon & b\alpha & c\beta \\ c\delta & d\epsilon & e\alpha & a\beta & b\gamma \\ b\epsilon & c\alpha & d\beta & e\gamma & a\delta \end{vmatrix}$$

is framed by attaching to each element of  $C(a, b, c, d, e)$  the corresponding element of  $(-1)^{3+2+1}C(\alpha, \beta, \gamma, \delta, \epsilon)$ . This, which degenerates into the ordinary five-line circulant on putting  $a=b=c=d=e=1$  or  $\alpha=\beta=\gamma=\delta=\epsilon=1$ , is found to be equal to

$$\begin{aligned} & \sum a^5 \cdot \alpha\beta\gamma\delta\epsilon - \sum a^3 b e \cdot \sum a^3 \gamma\delta + \sum a^2 b^2 d \cdot \sum a^2 \beta\gamma^2 \\ & + \sum a^5 \cdot abcde - \sum a^3 \beta\epsilon \cdot \sum a^3 cd + \sum a^2 \beta^2 d \cdot \sum a^2 bc^2 - 10abcde \cdot \alpha\beta\gamma\delta\epsilon \end{aligned}$$

where all the coefficients of the ordinary circulant except the last are in a manner "sifted" into their unit constituents.

ROBINSON, L. W. (1889): THYAGARAGAIYAR, V. R. (1899).

[Question 10254. *Educ. Times*, xlii, p. 332: *Math. from Educ. Times*, liv, p. 54.]

[Question 14240. *Educ. Times*, lii, p. 270: *Math. from Educ. Times*, lxxiii, p. 56.]

Already known results.

ROGERS, L. J. (1891).

[Question 10992. *Educ. Times*, xliv, p. 199.]

This concerns the first primary minor of  $C(a, b, c, \dots)$  when  $a+b+c+\dots=0$ , the problem being to resolve the said minor into linear factors. No solution ever appeared. Probably the proposer had in his mind special circulants of the form

$$C(a-b, b-c, c-d, d-a),$$

the first primary minor of which is

$$\begin{vmatrix} a-b & b-c & c-d \\ d-a & a-b & b-c \\ c-d & d-a & a-b \end{vmatrix} \text{ and } \therefore = \begin{vmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ 1 & 1 & 1 & 1 \end{vmatrix},$$

so that its factors are factors of  $C(a, b, c, d)$ .

WOODALL, H. J. (1894).

[Question 12426. *Educ. Times*, xlvii, p. 305: *Math. from Educ. Times*, lxiii, pp. 35-36.]

The real point of interest here concerns the product of two circulants. If the circulants be zero-axial, the ordinary multiplication-theorem, though of course producing a circulant, does not produce one that is zero-axial; so that, strictly speaking, the product is in this case not of the same form as its factors. It is shown, however, that what is not true of the zero-axial circulant is true of the zero-axial circulant divided by the sum of its elements. Thus, by row-multiplication

$$\begin{vmatrix} . & a & b \\ b & . & a \\ a & b & . \end{vmatrix} \cdot \begin{vmatrix} . & x & y \\ y & . & x \\ x & y & . \end{vmatrix} = \begin{vmatrix} ax+by & bx & ay \\ ay & ax+by & bx \\ bx & ay & ax+by \end{vmatrix},$$

and consequently on removing the factors  $a+b, x+y, (a+b)(x+y)$  from the three determinants respectively we have

$$\begin{vmatrix} 1 & 1 & 1 \\ b & . & a \\ a & b & . \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ y & . & x \\ x & y & . \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ ay & ax+by & bx \\ bx & ay & ax+by \end{vmatrix},$$

where, by diminishing in the determinant on the right the second and third rows by  $ax+by$  times the first, there results

$$\begin{vmatrix} 1 & 1 & 1 \\ ay-ax-by & . & bx-ax-by \\ bx-ax-by & ay-ax-by & . \end{vmatrix}.$$

In other words,

$$(a^2-ab+b^2)(x^2-xy+y^2)=P^2-PQ+Q^2,$$

if  $P, Q$  be taken equal to  $bx-ax-by, ay-ax-by$  respectively.

MUIR, T. (1896).

[On the resolution of circulants into rational factors. *Proc. Roy. Soc. Edin.*, xxi, pp. 369-382.]

Viewing the circulant  $C(a_1, a_2, \dots, a_n)$  as the eliminant of the equations

$$\left. \begin{aligned} a_1x^{n-1}+a_2x^{n-2}+\dots+a_n &= 0 \\ x^n-1 &= 0 \end{aligned} \right\},$$

the author concludes that *corresponding to every rational factor of  $x^n-1$  there must be a rational factor of the circulant*; and his object is to determine such factors and to present them, when found, in the most convenient forms. The means employed for the purpose is elimination.

Thus, to begin with, Catalan's factor corresponding to

$$(x^n-1)/(x-1)$$

is obtained by deducing from the equations

$$\left. \begin{aligned} ax^4+bx^3+cx^2+dx+e &= 0 \\ x^4+x^3+x^2+x+1 &= 0 \end{aligned} \right\}$$

the cubic

$$(b-a)x^3+(c-a)x^2+(d-a)x+(e-a)=0,$$

thence by cyclical substitution three other equations, and finally

$$\begin{vmatrix} b-a & c-a & d-a & e-a \\ c-b & d-b & e-b & a-b \\ d-c & e-c & a-c & b-c \\ e-d & a-d & b-d & c-d \end{vmatrix},$$

which being inherently persymmetric is more conveniently written

$$P(b-c, c-d, d-e, e-a, a-b, b-c, c-d).$$

The process is then extended to find the factor corresponding to

$$(x^{2m}-1)/(x^2-1);$$

in other words, to find the cofactor of

$$(a_1 + a_2 + \dots + a_{2m})(a_1 - a_2 + a_3 - \dots - a_{2m}) \text{ in } C(a_1, a_2, \dots, a_{2m}).$$

Thus, in the case where  $2m=8$ , from the equation

$$ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h = 0$$

the seventh and sixth powers of  $x$  are removed with the help of the other equation

$$x^6 + x^4 + x^2 + 1 = 0,$$

the result being

$$(c-a)x^5 + (d-b)x^4 + (e-a)x^3 + (f-b)x^2 + (g-a)x + (h-b) = 0,$$

whence by cyclical substitution and elimination we obtain

$$\begin{vmatrix} c-a & d-b & e-a & f-b & g-a & h-b \\ d-b & e-c & f-b & g-c & h-b & a-c \\ e-c & f-d & g-c & h-d & a-c & b-d \\ f-d & g-e & h-d & a-e & b-d & c-e \\ g-e & h-f & a-e & b-f & c-e & d-f \\ h-f & a-g & b-f & c-g & d-f & e-g \end{vmatrix},$$

which, again, can be expressed persymmetrically, namely,

$$P(c-e, d-f, e-g, f-h, g-a, h-b, a-c, b-d, c-e, d-f, e-g).$$

In the next place, the factor of  $C(a_1, a_2, a_3, a_4, a_5, a_6)$  corresponding to the factor  $x^2+x+1$  of  $x^6-1$  is obtained. We simply, as before, remove in order from the equation

$$a_1x^5 + a_2x^4 + a_3x^3 + a_4x^2 + a_5x + a_6 = 0,$$

with the help of the equation

$$x^2 + x + 1 = 0,$$

the fifth, fourth, third, second powers of  $x$ , with the result

$$(a_5 - a_4 + a_2 - a_1)x + (a_6 - a_4 + a_3 - a_1) = 0,$$

whence the eliminant

$$\begin{vmatrix} a_5 - a_4 + a_2 - a_1 & a_6 - a_4 + a_3 - a_1 \\ a_6 - a_5 + a_3 - a_2 & a_1 - a_5 + a_4 - a_2 \end{vmatrix},$$

which, as before, can be expressed persymmetrically.

Proceeding in this way, the prime factors of the circulants up to and including that of the tenth order are calculated, and are tabulated for reference. By reason of the persymmetry and the notation for it, the table occupies only one page; and it is pointed out how it might have been made still shorter by using the fact that when once the first element of any of the persymmetric determinants is obtained, the others follow

at once in cyclical procession. For example, if we desire to have the factor of  $C(a_1, a_2, \dots, a_{10})$  corresponding to the factor  $x^4 + x^3 + x^2 + x + 1$  of  $x^{10} - 1$ , and know from the table that the first element of it is  $a_7 - a_8 + a_2 - a_3$ , we have the whole of it immediately, namely,

$$\begin{vmatrix} a_7 - a_8 + a_2 - a_3 & a_8 - a_9 + a_3 - a_4 & a_9 - a_{10} + a_4 - a_5 & a_{10} - a_1 + a_5 - a_6 \\ a_8 - a_9 + a_3 - a_4 & a_9 - a_{10} + a_4 - a_5 & a_{10} - a_1 + a_5 - a_6 & a_1 - a_2 + a_6 - a_7 \\ a_9 - a_{10} + a_4 - a_5 & a_{10} - a_1 + a_5 - a_6 & a_1 - a_2 + a_6 - a_7 & a_2 - a_3 + a_7 - a_8 \\ a_{10} - a_1 + a_5 - a_6 & a_1 - a_2 + a_6 - a_7 & a_2 - a_3 + a_7 - a_8 & a_3 - a_4 + a_8 - a_9 \end{vmatrix}.$$

The process is seen to be general, and to consist in deducing from the equation of higher degree, by means of the other equation, an equation of lower degree than the latter, and in then using cyclical substitution and elimination.

The next part of the paper, however, makes it clear that the same results can be obtained, when known, by operating on the circulants themselves in accordance with the laws of determinants. Thus, from the eight-line circulant it is shown how to remove the second linear factor after the first, and how then to resolve the remaining six-line determinant into the two previously found persymmetric determinants of the second and fourth orders respectively.

Finally, the results are applied to the special case of circulants whose elements, when the first is left aside, read forward the same as backward: and it is shown that, when from such circulants the rational linear factor or factors are removed, the remaining factor is a complete square. This is possible from the fact that the persymmetric cofactors above obtained are then zero-axial and skew. Thus it is found that

$$\begin{aligned} & C(a_1, a_2, a_3, a_4, a_4, a_3, a_2) \\ &= (a_1 + 2a_2 + 2a_3 + 2a_4) \begin{vmatrix} a_3 - a_4 & a_2 - a_3 & a_1 - a_2 & a_2 - a_1 & a_3 - a_2 \\ & a_3 - a_4 & a_2 - a_3 & a_1 - a_2 & a_2 - a_1 \\ & & a_3 - a_4 & a_2 - a_3 & a_1 - a_2 \\ & & & a_3 - a_4 & a_2 - a_3 \\ & & & & a_3 - a_4 \end{vmatrix}^2 \end{aligned}$$

and that

$$\begin{aligned} & C(a_1, a_2, a_3, a_4, a_5, a_4, a_3, a_2) \\ &= (a_1 + 2a_2 + 2a_3 + 2a_4 + a_5)(a_1 - 2a_2 + 2a_3 - 2a_4 + a_5) \\ & \cdot \begin{vmatrix} a_3 - a_1 & a_4 - a_2 & a_5 - a_3 & . & a_3 - a_5 \\ a_3 - a_1 & a_4 - a_2 & a_5 - a_3 & . & \\ a_3 - a_1 & a_4 - a_2 & a_5 - a_3 & & \\ a_3 - a_1 & a_4 - a_2 & a_5 - a_3 & & \\ a_3 - a_1 & a_4 - a_2 & a_5 - a_3 & & \end{vmatrix}^2. \end{aligned}$$

Further, it is shown that such circulants being centrosymmetric can be resolved into factors in quite a different way, the repeated factor now appearing not as a pfaffian but as a determinant; so that by the equatement of the two resolutions we obtain equality between the two determinants

$$\begin{vmatrix} a_1 - a_3 & a_2 - a_4 & a_3 - a_4 \\ a_2 - a_4 & a_1 - a_4 & a_2 - a_3 \\ a_3 - a_4 & a_2 - a_3 & a_1 - a_2 \end{vmatrix}, \quad \begin{vmatrix} a_1 - a_3 & a_2 - a_4 & a_3 - a_5 \\ a_2 - a_4 & a_1 - a_5 & a_2 - a_4 \\ a_3 - a_5 & a_2 - a_4 & a_1 - a_3 \end{vmatrix}$$

and the above-given pfaffians respectively.

BICKMORE, C. E. (1898).

[Question 13748. *Educ. Times*, li, p. 40: *Math. from Educ. Times*, lxi, p. 85.]

The result here recorded is that the skew circulant of the fourth order,  $C'(x, y, z, w)$ , whose final expansion is

$$x^4 + y^4 + z^4 + w^4 + 2(x^2z^2 + y^2w^2) + 4(x^2yw - y^2zx - z^2wy + w^2xz),$$

is expressible in each of the forms

$$a^2 + b^2, \quad c^2 + 2d^2, \quad e^2 - 2f^2$$

where  $a, b, c, d, e, f$  are definite quadratic functions of  $x, y, z, w$ , namely,

$$\begin{aligned} x^2 + 2yw - z^2, & \quad y^2 - 2zx - w^2, \\ x^2 - y^2 + z^2 - w^2, & \quad xy - yz + zw + wx, \\ x^2 + y^2 + z^2 + w^2, & \quad xy + yz + zw - wx. \end{aligned}$$

This is reached by taking the four linear factors

$$x + \zeta y + \zeta^2 z + \zeta^3 w, \quad x + \zeta^3 y - \zeta^2 z + \zeta w, \quad x - \zeta y + \zeta^2 z - \zeta^3 w, \quad x - \zeta^3 y - \zeta^2 z - \zeta w$$

of the circulant and multiplying them in the three different ways as a pair of pairs. For example, the product of the first and second factors being

$$x^2 - y^2 + z^2 - w^2 + (\zeta + \zeta^3)(xy - yz + zw + wx),$$

and the product of the third and fourth being

$$x^2 - y^2 + z^2 - w^2 - (\zeta + \zeta^3)(xy - yz + zw + wx),$$

the product of the four is got in the form

$$(x^2 - y^2 + z^2 - w^2)^2 + 2(xy - yz + zw + wx)^2.$$

We note for ourselves that by the multiplication-theorem we obtain

$$\begin{vmatrix} x & y & z & w \\ -w & x & y & z \\ -z & -w & x & y \\ -y & -z & -w & x \end{vmatrix}^2 = \begin{vmatrix} e & f & . & -f \\ f & e & f & . \\ . & f & e & f \\ -f & . & f & e \end{vmatrix} \\ = \begin{vmatrix} e & f & . & . \\ 2f & e & . & . \\ . & f & e & 2f \\ -f & . & f & e \end{vmatrix} = (e^2 - 2f^2)^2;$$

also

$$\begin{vmatrix} x & y & z & w \\ -w & x & y & z \\ -z & -w & x & y \\ -y & -z & -w & x \end{vmatrix} \cdot \begin{vmatrix} x & -y & z & -w \\ -w & -x & y & -z \\ -z & w & x & -y \\ -y & z & -w & -x \end{vmatrix} = \begin{vmatrix} c & -d & . & -d \\ -d & -c & d & . \\ . & d & c & -d \\ -d & . & -d & -c \end{vmatrix},$$

i.e.

$$C'(x, y, z, w) \cdot (-1)^2 C'(x, y, z, w) = (c^2 + 2d^2)^2;$$

and

$$\begin{vmatrix} x & y & z & w \\ -w & x & y & z \\ -z & -w & x & y \\ -y & -z & -w & x \end{vmatrix} \cdot \begin{vmatrix} x & w & -z & y \\ -w & z & -y & x \\ -z & y & -x & -w \\ -y & x & w & -z \end{vmatrix} = \begin{vmatrix} a & . & b & . \\ . & -b & . & a \\ b & . & -a & . \\ . & a & . & b \end{vmatrix},$$

i.e.

$$C'(x, y, z, w) \cdot (-1)^2 C'(x, y, z, w) = (a^2 + b^2)^2.$$

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## VII.—The Dynamics of Cyclones and Anticyclones. Part III.

By John Aitken, LL.D., F.R.S.

(MS. received December 22, 1915. Read January 24, 1916.)

IN 1900\* I communicated to this Society a paper on the above subject. Since that date a great deal of information has been obtained by means of free balloons carrying instruments which recorded the temperature, humidity, and pressure of the air up to great elevations. Much of this new knowledge seems to contradict our previous ideas, and does not seem to fit into the old convectional theory that cyclones are formed by the rising of the hot, moist air from the surface of the earth; their energy being due to their temperature and to the heat liberated by the condensation of the water vapour in them. We are told by those who have studied the bearing of the new knowledge on our atmospheric circulation that the old theory is "utterly untenable." Their reasons for this conclusion are, first, that the recent investigations show that the air is colder in cyclones than in anticyclones; second, that the isothermal layer is lower than the mean over cyclones, while it is higher than the mean over anticyclones. At first sight these discoveries seem to shatter the convectional theory, but before we come to any conclusion I should like to present certain facts which it appears to me will require to be considered before we scrap our old ideas.

Let us take the first objection to the convectional theory, namely, the lower temperature in cyclones. Now, while temperature plays a most important part, it is not the only factor in determining the circulation; the pressure has also much to do with the density of the air. The air in the cyclone is under less pressure than the air in the anticyclone; it has therefore less weight per unit of volume. Air expanded by reduction of pressure tends to rise just as air that has been expanded by heat.

Let us now see how far the expansion due to reduction of pressure affects the question of convection. In the *Journal of the Scottish Meteorological Society* for 1914 there is a paper on cyclones and anticyclones by W. H. Dines, F.R.S., in which there is an excellent abstract of these upper-air observations, given in tabular form. From that table I find

<sup>1</sup> *Trans. Roy. Soc. Edin.*, vol. xl, part i (No. 7).



that the cyclone up to 10 kilometres is on an average  $7^{\circ}4$  C. colder than the anticyclone; while from 10 to 14 kilometres it is  $7^{\circ}3$  warmer; and Mr Dines considers that, taken as a whole, the cyclone is colder than the anticyclone by  $5^{\circ}$ . Let us now turn to the effect of pressure. In the table referred to cyclones have an average pressure at the earth's surface of 29.2 inches, while the anticyclones have a pressure of 30.3. This gives a difference of pressure of 1.1 inch, which is  $1/28.5$  of the anticyclonic pressure. The air in its passage to the cyclone will be expanded by fall of pressure by  $1/28.5$  of its volume. Now, air is only expanded by heat by  $1/273$  of its volume at  $0^{\circ}$  per degree C. With these figures we see that the reduction of pressure reduces the density of the air as much as heating it  $9^{\circ}5$  would do. So that the reduction in density by loss of pressure more than compensates for the lower temperature in the cyclone at the earth's surface, and it seems probable that the same relative difference, or even a greater difference, will continue up to great heights owing to the velocity of the winds increasing with the elevation.

It may be objected that we should not take the extremes of pressure and allow for so great a reduction of pressure, because the cyclone does not rise in the anticyclone, but in the air at a lower pressure at a distance from the anticyclone. That is quite the case; but if we allow for a fall of only half the amount of pressure, we must also allow for the less difference in temperature, as the air surrounding a cyclone has a temperature much lower than that of the anticyclone. The above figures work out very much the same if we use half the difference in pressure and the mean temperatures as given in the table referred to.

We will turn now to the second objection to the convection theory, namely, the lower level of the stratosphere over cyclones, and the higher level over anticyclones. But before proceeding to consider these objections I should like to call attention to a phenomenon which all of us have witnessed. It is so common that we have either not seen the wonder of it, or it has ceased to interest us. The wonderful thing to which I refer is a jet of steam issuing from a boiler under high pressure. And yet that jet is a most perfect illustration of the conversion of potential energy into energy of motion. Let us look more closely at the facts. Suppose that there is a short open pipe connected directly to the boiler, and that steam is issuing from the open end. If you ask, "What is the pressure of the steam in the pipe?" many people would say, "It is the same as in the boiler." But those who have studied the matter know there is no pressure in the steam in the pipe; that is, its pressure is the same as that of the air into which it is escaping. A moment's consideration will convince us that this is correct,

because, if there were any pressure, the moment the steam escaped from the confinement of the tube it would at once expand to a much greater diameter; but the jet shows no signs of expanding—it flows straight out, only slowly increasing in diameter as its motion is retarded by its being mixed up by eddies with the air. In the jet, therefore, the whole of the potential energy of pressure of the steam is converted into energy of motion.

It is well known that this energy of motion in the jet can be reconverted into potential energy of pressure. If another pipe of the same diameter as the one from which the steam is issuing be placed directly opposite it, the jet will raise the pressure in that pipe to the pressure of the boiler from which the steam is issuing. Practically we may not quite accomplish this, owing to loss of energy due to friction. The points to be kept in mind from this illustration are that gases moving from a higher to a lower pressure lose potential energy and gain velocity, and that gases having velocity gain pressure while losing their velocity.

Let us now apply these principles to the circulation in our atmosphere, and see how far they help to explain the difference in level of the stratosphere over cyclones and anticyclones. In the anticyclone the pressure is high and the motion very slight, and the energy is all potential. On the other hand, in the cyclone the pressure is lower but the motion is very much greater. Some of the potential energy which was in the anticyclone is converted into energy of motion in the cyclone: what is lost in pressure is gained in velocity. This distribution of energy is retained by the cyclone, the air circling inwards and upwards to great heights, up as far as the strato-cumulus clouds. Above that level the circulation seems to change and begin to flow outwards towards the high-pressure area. The air in its movement towards the anticyclone loses its velocity, and in so doing regains pressure. If that represents what takes place, we can easily see why the isothermal layer is lower than the average over cyclones, because the air there has great velocity but has not regained its pressure. While its centrifugal force confers on it a quasi-horizontal pressure which enables it to act against the side pressure, it has no corresponding vertical pressure; hence the descent of the stratosphere in that area. On the other hand, the low-pressure air with high velocity flowing out of the upper part of the cyclone towards the anticyclone loses its velocity and regains sufficient pressure to enable it to enter the upper part of the anticyclone, where it gives rise to an increase of pressure which tends to push the isothermal layer higher than the mean. It would thus appear that the change of

energy from potential to actual and back again to potential gives the explanation of the lowering of the isothermal layer over cyclones, and its rising over anticyclones.

The increase in velocity of cyclonic air also shows itself in other ways when passing from higher to lower pressures. As the air in cyclones is rising it is at the same time passing from higher to lower pressure, and in so doing is gaining velocity. This seems to account for the increase in velocity with height observed in cyclones. Up to the height of the cirro-cumulus clouds its centrifugal force is not sufficient to hold back the higher pressure surrounding it; but by the time it has risen above these clouds its velocity is so great that its tangential force enables it to overcome the surrounding pressure and to flow outwards towards the high-pressure area. The increase of velocity with height in cyclones is not simply a question of the retardation of the lower layers by friction with the earth's surface, as the increase in velocity goes on up to heights far above that to which we would expect the earth's surface to have any influence. Of course, all this expansion is accompanied by a loss of temperature; but, as the observations show, the loss of volume from this cause is more than compensated for by the fall in pressure.

There is another point to which I should like to call attention. If the table of the temperatures at the different elevations in the upper air, already referred to, is examined, it will be seen that, while the temperatures in the lower part of the cyclone differ from those in the anticyclone, at the elevation of 10 kilometres they have the same temperature; and that, while the temperature over the cyclone ceases to fall, that over the anticyclone continues to fall to a considerable amount. One naturally asks, Why this difference? I have already pointed out why the stratosphere should be lower over the cyclone than elsewhere; but why should the air over the anticyclone at higher elevations than the cyclone be colder than that over the cyclone? If I may be allowed to hazard an explanation, I would suggest that the top of the anticyclone is formed of the air which has escaped from the upper part of the cyclone and flowed towards the high-pressure area, and in so doing has been forced upwards—its energy of motion being converted into potential energy of elevation—and at the same time has expanded and fallen in temperature; the top of the anticyclone being thus in a certain sense the top of the cyclone. According to the generally received opinion, the air which rises in the cyclone comes down in the anticyclone, and this suggestion as to the passage of the air between the two systems seems to explain the process.

The lower temperature at the top of the anticyclone and its greater

elevation seem to suggest that the air in the stratosphere will tend to flow from the areas over anticyclones to those over cyclones, and that some exchange of air may take place between the two systems in the isothermal layer.

If the above represents the true condition of matters in the cyclone, it is evident it is not in accordance with the generally received ideas. If it is correct, then the cyclone must receive energy from outside sources. It is of course known that it uses up the latent heat of the vapour it carries up with it from the earth. Further, it will absorb more heat from the sun than does the anticyclone, owing to the presence in it of more clouds and more water vapour. But it is not easy to say whether or not the heat from these sources is sufficient.

There are two points connected with the temperature observations of the upper air to which I wish briefly to refer. First, there is the accuracy of the balloon records. Observations taken in anticyclones during the day will likely give too high readings of the thermographs, owing to the greater amount of sunshine in anticyclones than in cyclones. If we cannot avoid getting readings two or more degrees too high in screens in sunshine at low level, the balloon instruments are likely to have a still greater error, which will be greater in the anticyclone than in the cyclone, where they will be less exposed to sunshine owing to the greater cloudiness, and where they will also be exposed to an occasional wetting, which will still further lower their readings. These considerations seem to point to the probability that the temperature in cyclones is not so much lower than that in anticyclones as the observations indicate.

The second point to which I wish to refer is that, in the table of temperatures referred to, no readings at the earth's surface are given: the table begins with temperatures at 1 kilometre above the surface. Now, according to the convection theory, this lower stratum under 1 kilometre plays a most important part, as it is there that the start of the cyclone is made and from it much of its energy is drawn. These last words naturally cause us to ask the question: If we must abolish the convection theory of cyclones, where are we to find a source of energy sufficient to drive the winds? We can hardly expect to find it in the highly attenuated atmosphere under the troposphere.

#### NOTES ON CYCLONIC MOTION.

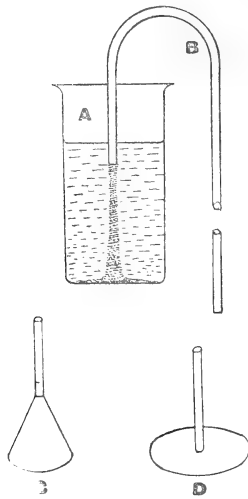
The following notes on cyclonic motion may be useful in helping us to understand what is taking place in cyclones. Let us take a vessel, say an ordinary washhand basin, with an outlet in or near the centre, and fill it

full of water. If before opening the outlet we allow the water to come to rest, then in passing out it flows in radial lines towards the outlet, converging from all directions towards the opening, but without any cyclonic motion being formed. If now, instead of allowing the water to come to rest before opening the outlet, we give it a slight circular motion, the flow of the water towards the opening is entirely changed. The water is not drawn from the bottom, but from a small area over the outlet, and soon the whole core is sucked out of the centre of the circling water, leaving the well-known hollow space.

One naturally asks, Why this difference? Why does the water in one case run out from all directions, while in the other it tends to be drawn from the upper layers? A little consideration of what is taking place in the basin will enable us to understand the reason for the difference in the two cases. When the water is free from motion the whole of the moving force acts radially from the outlet and the water flows direct to the opening. But if the water is rotating, this radial force has to contend against the centrifugal force of the water, which centrifugal force opposes the flow towards the centre; and this opposition is increased by the suction drawing the water towards the centre, so augmenting its angular velocity and its centrifugal action. The result of this is that the circular movement of the water, increased by the inward suction, surrounds the outlet with a resisting tube of circling water. But while the sides of this tube offer resistance to the entering water, the top of the tube is, so to speak, open. The horizontally circling water offers no resistance to the vertical pressure, so the water flows in at the top; but in the act of flowing in, its centrifugal force is increased, with the result that the resisting tube of water is lengthened, and this lengthening process goes on till the resisting water tube reaches the surface, after which the indraught increases the centrifugal force to such an extent that it is able to hold back the weight of the surrounding water—the depth of the hollow depending on the amount of suction and the initial rate of revolution given to the water.

To elucidate this point a simple experiment may be made which is illustrated in the accompanying sketch. Fig. 1, A, is a cylindrical glass vessel; a large-sized beaker, 20 by 10 cm., does very well for the purpose. The beaker is filled with water, into which is stirred some sawdust made from a heavy wood, to show the movements in the water. B is a siphon about 5 mm. in diameter, by means of which the water is emptied out—from the top, in this case; the tube being lowered as the level of the water falls. If the water be free from motion before the

siphon is started, the sawdust is seen to be drawn in radially from all directions, and there is no cyclonic motion. Now repeat the experiment, but before putting the siphon in action give the water a circular movement. On now starting the siphon a quick circular, or rather spiral, motion will be seen round the outlet, and this quickly circling tube will be seen to develop downwards till it reaches the bottom of the vessel, where it will suck up the heavier particles of sawdust, the upward spiral current being densely packed with these particles, making the cyclone clearly visible, as represented in fig. 1, A.\* After the cyclone has reached



*Fig. 1*

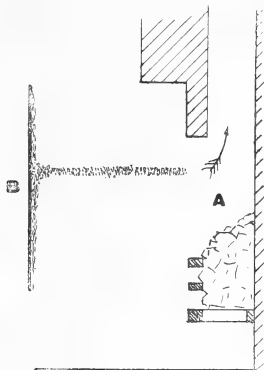
the bottom of the vessel it continues to draw its supplies mainly from that part, as the velocity of rotation of the water there is less than elsewhere, partly owing to its being reduced by friction on the bottom of the vessel.

It will be noticed that the cyclones in these experiments have a very small diameter, that the outflowing water surrounds itself with a quickly rotating tube of water, and that the outer water does not take much part in the action. This is greatly due to the outside water not being drawn nearer the centre owing to the intake being at the open end of the cyclone. Part of the smallness might be thought to be due to using a small pipe for the suction. This, however, does not seem to be the case, because we get the same small cyclones if we fix to the end of the siphon the wide-mouthed conical funnel C (fig. 1). This provides a large low-pressure

\* This experiment was shown at the meeting.

area, but it makes no difference; the small cyclones develop in the funnel and proceed downwards as before. Or we may use a very much larger siphon without making any great difference. Or again, if we fix to the end of the siphon the disc D (fig. 1), thus making the conditions similar to those in which the cyclone is made in water run out at the bottom, we effect no change in the diameter of the cyclone. This seems to depend on the strength of the suction and the initial angular velocity given to the water.

We can see the same intense small cyclonic movements in air by making the following simple experiment. All that is required is a good fire and a wet towel. The towel (B, fig. 2) is hung up in front of the



*Fig. 2*

fire (A), so as to be heated and a good amount of steam caused to rise from it. Before a cyclone can form on the surface of the towel some circular motion must be communicated to the air, or some tangential force applied to the one side of the air current moving towards the chimney, as has been shown in Parts I and II \* on this subject. If the draught in the room in front of the fire blows across the fireplace with equal velocity at all parts, no cyclone will form; but if we shield, say, the lower part, and allow the cross current to flow over the upper part, then a cyclone at once forms, and the steam rising from the wet towel is seen to be collected from all parts of its surface, flowing spirally towards the centre, where it is drawn into the cyclone and is seen forming a small pillar of rapidly spirally-moving steam extending from the towel to the upward opening of the chimney. If there are no cross currents in front of the fire, they must be made by opening a window or by other means. The appearance of these cyclones is roughly indicated in fig. 2. It will be noticed that all the steam is

\* *Trans. Roy. Soc. Edin.*, vol. xl, part i (No. 7).

collected in a small cyclone of about 2 or 3 cm. diameter. It will also be seen that in this case, as in the water cyclones, the diameter is very small, and that the low-pressure area into which it is drawn is large—very many times larger than the cyclone. The high angular velocity of the air surrounding the centre makes it act like a solid tube, through which the steam is drawn.

The general appearance of these suctionally-driven cyclones in water and air, as shown in figs. 1 and 2, presents a certain likeness to water-spouts, and possibly to tornadoes; and if we could only discover the existence of sufficiently low-pressure areas over them, we might then be a little nearer to understanding their formation.

It is well known that rapid motion confers on chains and bands certain properties which we associate with solid bodies. If a long loop of chain be hung over a pulley and driven at a great velocity, the chain behaves very much as if it were a solid at rest. If we strike it a blow anywhere below the pulley, we simply make a dent at the place. The chain acts very much as if it were a bar of lead; only, the dent made by the blow will slowly straighten out owing to the friction of the links, and other causes, the chain behaving very much as if it had viscid properties. If the loop of chain be put on a pulley large enough to give it nearly a circular form, and set in rapid motion, and while still moving dropped off the pulley, it will run along like a hoop and go many yards before its motion is spent and it collapses on the floor. Cyclonic motion seems to confer on water and gases similar properties, giving them a certain degree of rigidity which they otherwise do not possess. It is well known that vortex rings vibrate after encountering an obstacle which tends to deform them. Or if we make the rings oval, they are seen to vibrate; their long diameter, if originally horizontal, changes to a vertical position, and then back to the horizontal, and continues to oscillate between the two positions till they end their existence. They act very much like an elastic solid. The cyclonic movements in water and air (figs. 1 and 2) have also acquired properties associated with solids; they tend to keep straight and develop in a direction at right angles to the plane of the rotational component of the spiral movement in them. This is well seen in the case illustrated in fig. 2. Here the centre of the cyclone has developed in a direction at right angles to the wet surface which determined the plane of its rotational movement. It will be noticed that the cyclone is uninfluenced as regards straightness and direction by the movements of the surrounding air, till it meets with the strong cross current going up the chimney, where it gets broken up, as cyclones seem to have a distinct objection to bending.



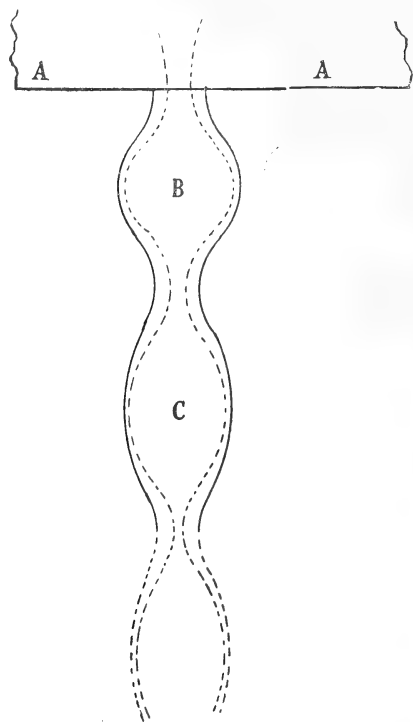
It is not that they cannot be bent—as a vortex ring is in many respects simply a cyclone with its two ends joined,—but that bending puts a considerable strain on the structure, and gives it its elasticity, which causes it to vibrate when distorted. The low pressure in the centre of the circling air which forms the ring keeps the ring from expanding or straightening out; but if a small piece be cut out of the ring, the low pressure inside is weakened by the entrance of air, and the ring begins to straighten out and is rapidly dissipated.

These small cyclones do not help us much towards understanding the movements taking place in cyclones in our atmosphere. The conditions are all very different. The height of these small cyclones is many times their diameter, while in the atmosphere the height may not be  $1/200$  of the diameter. It is evident, therefore, that the air passing through a cyclone in the atmosphere will only undergo a proportionately small amount of rotational movement.

*Note added February 23, 1916.*

There is an interesting difference in the appearance of a water jet escaping from still water and one escaping with circular motion, which appears to be worth recording here. Let us use a circular metal vessel 30 cm. in diameter and 8 cm. deep, with a hole in the centre 1 or 2 cm. in diameter which can be closed from below. This vessel is filled, and a circular motion given to the water. Some time should be allowed for the water to get into a uniform circulation. When the circulation is sufficiently slow the outlet in the bottom is opened. When this is done the well-known jet at first appears like a solid tapering rod. But soon the outflowing water develops a cyclone strong enough to give rise to an air space in the centre. This air space develops downwards and passes through the outlet, the *vena contracta* disappears, and a series of hollow globes of water is formed, the uppermost one being more than twice the diameter of the outlet. See fig. 3, where AA is the bottom of the vessel containing the water, B is the first bulb which develops immediately below the outlet, while below it another globe, C, appears, somewhat longer and narrower than the top one. Below the second bulb there sometimes appears another, or others, gradually decreasing in diameter and increasing in length. The dotted lines in the figure show the air space extending down from the whirling water in AA through the bulbs. As the shape of the bulbs depends greatly on the rate of rotation, the upper bulb is often not so spherical as shown, but may be more like the second bulb, C.

The explanation of the formation of these hollow globes would appear to be as follows. When the water flows out without circular motion, the lines of flow are all radial, and as these converge on the centre of the opening they give rise to a reduction in the area of the jet a short distance from the opening. But when the escaping water has a circular motion, whenever it escapes from the constraint of the surrounding water its centrifugal force tends to cause it to move outwards, and it would fly off



*Fig. 3*

in spray but for the surface tension of the water. This tension, however, yields to the centrifugal force and the jet expands; but in expanding the water loses its angular velocity and therefore some of its centrifugal force. In this process the water carries the expansion further than it can maintain it, and the surface tension draws it back, but in so doing increases the angular velocity of the water, thus enabling it to form a second bulb; and this see-saw between the circling water and its surface tension goes on as long as the circular motion continues. The loss of circular motion accounts for the diminishing diameter of the globes, and the increasing rate of fall of the water for their greater length lower down.

In Part I on this subject it was pointed out that, when water escapes

from a vessel such as is shown in fig. 3, in which a circular motion is given to the water, and precautions are taken to prevent air entering either from above or from below, though the outlet be full of water the outflow is greatly retarded. It is found that the amount of retardation depends on the diameter of the outlet. In a vessel of the dimensions given above, if the outlet be 2 cm. the retardation amounts to 50 per cent., more or less according to the rate of rotation; but if the opening be only 1 cm. the retardation is much less, and it is very small in the cyclone of the proportions shown in fig. 1, where the outlet is only 0·5 cm.

*(Issued separately September 30, 1916.)*

## VIII.—Clinical Methods of Estimation of Sugar in the Blood.

By Dr Harry Rainy and Miss Christina M. Hawick, B.Sc.

(MS. received March 18, 1916. Read May 1, 1916.)

THE variation in the sugar content of the blood in certain pathological conditions has created the need for modes of estimation which require only a small amount of blood and at the same time give sufficiently reliable results. The more recent methods of estimation fall under three types: (1) those in which the method of Rona and Michaelis for the precipitation of the protein is combined with Bertrand's method for sugar estimation, (2) MacLean's more recent method, and (3) the "micro-method" of Ivar Bang.

Clinically, the third method possesses the great advantage that, as very little blood is required for an estimation, the observation can be repeated at short intervals during the day without incommoding the patient. In its original form it has several sources of error which prove a great drawback to its use, but by certain modifications these can be overcome.

(1) *a.* Gardner and MacLean in 1914 published a method whereby blood sugar could be estimated, and in which 2 c.c. of blood were required. The blood is measured into a small mortar, treated with 7 c.c. of water, 14 c.c. of dialysed iron, and 1 c.c. of saturated sodium sulphate. The whole is stirred and the thick pasty mass squeezed through a cloth. The resulting somewhat turbid fluid is filtered, and exactly 16 c.c. of the filtrate are used for a sugar estimation by Bertrand's method. The cuprous oxide is filtered off through asbestos, washed with water, dissolved in Bertrand's ferric sulphate solution, and the amount of sugar estimated in the ordinary way from the subsequent permanganate titration.

*b.* In June 1915 Stein and Wiseley published a similar method in which 0.5 c.c. of blood is used. Into a centrifuge tube, accurately calibrated at the 0.5, 1, 5, and 10 c.c. points, are put a sodium fluoride solution to the 0.5 c.c. mark, blood to the 1 c.c. mark, and water to the 5 c.c. mark. A pinch of Rochelle salt is then added, the whole shaken and allowed to stand until the Rochelle salt has dissolved. A 5 per cent. solution of dialysed iron is added up to the 10 c.c. mark, the tube corked, and the whole vigorously shaken and then centrifugalised for from 3 to 5 minutes.

5 c.c. of the supernatant fluid is used in the sugar estimation. The sugar is estimated by a modified Bertrand process in which a known quantity of glucose is added to the copper solution. The cuprous oxide is filtered off through asbestos, washed, treated with acid ferric sulphate solution, and the amount of sugar calculated from a permanganate titration.

(2) Recently MacLean has published still another method for the estimation of blood sugar in which 1 c.c. of blood is used. The method for removing the protein is that employed by Gardner and MacLean, the only difference being that a small quantity of phosphoric acid is added to hasten filtration. The total bulk of fluid before filtration is 40 c.c., and 20 c.c. of the filtrate are used in the subsequent estimation. The glucose solution is boiled for 10 minutes with an alkaline copper solution containing potassium iodide and a known amount of potassium iodate, a reflux condenser being used to prevent reduction in bulk. The flask and its contents are cooled and a measured amount of pure concentrated hydrochloric acid added. The iodine thus liberated oxidises the cuprous salts to cupric, and the excess of iodine is estimated by means of a standard thiosulphate solution.

(3) According to the method of Ivar Bang, about 100 mg. of blood are soaked up into a small piece of specially prepared filter paper, which is weighed before and after on a torsion balance. The paper is transferred to a clean, dry, rather wide test tube, and 6.5 c.c. of a boiling solution of potassium chloride containing a trace of hydrochloric acid are poured upon it. By this means the albumen is coagulated and the sugar diffuses into the surrounding solution. After half an hour the liquid is poured into a small Jena flask, the flange of whose neck has been cut off, the paper is washed with 6.5 c.c. of the potassium chloride solution, and the washings added to the contents of the flask. 1 c.c. of a solution containing 4.4 gm.  $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ ; 160 gm.  $\text{KHCO}_3$ ; 100 gm.  $\text{K}_2\text{CO}_3$ ; and 66 gm.  $\text{KCl}$  made up to a litre with boiled-out distilled water is added. A piece of thick-walled rubber tubing is fitted over the neck of the flask and the contents heated over a flame so adjusted that boiling begins in from 1 minute 25 seconds to 1 minute 35 seconds. Boiling is continued for exactly 2 minutes, the rubber tubing is closed with an efficient clip and the flask cooled in a current of cold water for 1 minute. The rubber tubing is now removed and the cuprous chloride estimated by titration with  $\frac{\text{N}}{200}$  iodine from a micro-burette, starch solution being used as an indicator. To prevent oxidation by the air, a current of  $\text{CO}_2$  is passed into the flask while this titration is being

performed. If  $n$  be the number of c.c. of  $\frac{N}{200}$  iodine used in the titration, the amount of glucose is calculated from the formula

$$\frac{n \text{ c.c.} - .16 \text{ c.c.}}{4} = \text{glucose in milligrams.}$$

By introducing certain modifications into Bang's method we have been able to secure very satisfactory results. Our process is as follows:—

Instead of using a torsion balance the filter paper is weighed in a small weighing bottle with a well-fitting stopper, the blood is then soaked up into it, care being taken to leave the margins fairly clean, and the paper replaced in the weighing bottle and weighed again. A small balance which can weigh accurately to milligrams is used, and the time occupied in weighing the blood is very short. Loss due to evaporation is almost completely avoided, and the time required for the frequent determination of the constants of a torsion balance is saved. The inaccuracy of a torsion balance, unless frequently calibrated, is well known to all physicists. The solution used for coagulating the albumen is boiled for some minutes to render it air-free, a small quantity of distilled water being added to allow for evaporation. A few minutes—about five—should be allowed for the blood to dry before it is coagulated, and it is well not to shake the tube after the boiling solution has been poured in. Usually the albumen is completely retained in the paper, but should coagulation be incomplete the specimen may be rejected at once, as estimations of glucose in solutions containing traces of albumen are most inaccurate. An hour is allowed for the glucose to diffuse into the surrounding solution, which is then transferred to the small flask, the paper washed with a further quantity of potassium chloride solution, which has been boiled to render it air-free and cooled, and the washings added to the contents of the flask. Neither in Bang's original papers, nor in any account of his method which we have seen, is mention made of having the coagulating solution air-free. If no precaution be taken to attain this, the method amounts to adding 1 c.c. of an air-free solution to 13 c.c. of an air-containing one. The copper solution is also boiled shortly before it is to be used, to render it air-free. A small quantity of distilled water is added to a few c.c. of the solution and the whole boiled for some minutes and cooled. 1 c.c. is then added to the glucose solution in the small flask. Experiments with pure glucose led to the belief that the most accurate results are obtained with the flame so adjusted that the contents of the flask begin to boil in from 1 minute 15 seconds to 1 minute 25 seconds. The efficient sealing of the flask is

of the utmost importance. Various kinds of clips were tried, but a pair of surgical artery forceps was found to be much the best instrument for this purpose. A very rapid current of cold water must be used for cooling the flask and its contents, and care must be taken to insert the leading tube from the  $\text{CO}_2$  generator into the flask as soon as the rubber tubing is removed from the neck. The starch solution may afterwards be added by means of a small pipette pushed well down into the flask. The  $\frac{\text{N}}{200}$  iodine is made up with cold boiled-out distilled water. After each estimation the flask is cleaned with a small quantity of concentrated hydrochloric acid and washed several times with water.

Contrasted with the three more recent methods of Gardner and MacLean, Stein and Wiseley, and MacLean, the method of Bang has several advantages. The quantity of blood required is exceedingly small—only about .1 c.c.—so that several estimations may be made upon the same person in one day without inconvenience. Also any method for blood analysis which permits of the blood being weighed is to be preferred to one in which it is measured. The quantity of blood delivered by an ordinary pipette is very difficult to estimate, and is almost certain to differ in different people. As regards the method of Stein and Wiseley for measuring blood, the error involved in measuring .5 c.c. of blood by dropping into an ordinary graduated centrifuge tube may be very considerable. In Bang's method there is no difficulty about filtration. The paper used in soaking up the blood retains all the albumen and the reduced copper salt is held in solution. Successful filtration through an asbestos mat requires considerable experience and a very reliable make of asbestos wool. The adding of a known amount of glucose to the copper solution in order to obtain a filterable amount of cuprous oxide in the Bertrand estimation makes the process easier but no more exact, for any error that occurs affects the blood sugar alone. Glucose is, moreover, a difficult substance to standardise. The new method of MacLean has the obvious advantage over his earlier method that no filtration of cuprous oxide is required, but compared with the method of Ivar Bang the quantity of blood used is large.

The outstanding advantage which the Michaelis-Bertrand methods have over that of Bang is that the final titration is performed with permanganate, whereby an outside indicator is avoided and a very clean end point obtained. Still, with a little practice, the exact point at which the green-blue of the copper solution gives place to the clear blue of the starch-iodine compound can be readily enough determined.

The time required to make one blood-sugar estimation by Bang's method is rather long, but if several have to be made the average time required for each estimation is 10 minutes. In addition to this, about 8 minutes are occupied in weighing the blood and coagulating it, so that the average whole time required for one of a series of estimations is somewhat under 20 minutes. The only piece of special apparatus required is a micro-burette.

The method of Bang has this further advantage that it is as applicable to the estimation of glucose in urine as in blood. Bertrand's method of glucose estimation is not satisfactory for urine, as some urinary constituents keep cuprous oxide in solution. In studying the variation of urine sugar with blood sugar, the gain in being able to use the same method for the estimation of both is obvious.

The following experiment will serve to illustrate the points brought out in the foregoing paper. It was one of a series performed to find how the blood sugar varied as compared with the urine sugar when a known considerable excess of glucose had been administered. The subject was a young adult whose sugar metabolism was normal. He received 280 gm. of glucose by the mouth, specimens of blood and urine having been obtained 10 minutes before the commencement of the experiment in order to determine their normal sugar content. Specimens of blood and urine were thereafter taken at intervals during the next few hours. The blood sugar was estimated in the manner already described. For the estimation of sugar in the urine the latter was diluted in proportions varying from 1 in 10 to 1 in 80, according to the amount of sugar present, and 2 c.c. of the diluted urine were used for each estimation.

The results of the experiment are shown in the annexed table:—

J. C., 1ST JULY 1915.  
280 gm. glucose at 12 noon.

Time.	Blood Sugar per cent.	Time.	Urine Sugar per cent.
11.50	·120	11.50	·085
12.40	·284	12.35	·151
1.10	·270	1.5	·909
1.45	·245	1.45	1·305
2.40	·189	2.45	·852
3.30	·186	3.30	·740
4.30	·186	4.35	·629

It will thus be seen that the blood sugar rises very rapidly, having reached or probably passed its maximum by the time that the first specimen



was obtained at 12.40 p.m. The urine sugar does not reach its maximum until one hour later, and then falls off much more rapidly than does the blood sugar.

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(Issued separately September 30, 1916.)

IX.—Note on Captain Weir's Azimuth Diagram and its anticipation of a Spherical Triangle Nomogram. By Herbert Bell, M.A., B.Sc. *Communicated by THE GENERAL SECRETARY.*

(MS. received May 15, 1916. Read June 5, 1916.)

IN a paper \* read before this Society in 1889, Captain Weir gave a diagram for solving graphically, by means of a chart and parallel ruler, the polar spherical triangle whose vertices are the pole, zenith, and a star, so as to obtain the Azimuth when the declination, hour-angle, and latitude are known. The purpose of this note is to show that this diagram is in reality equivalent to the later straight-line nomogram, and to deduce a similar diagram to Weir's for another problem in spherical trigonometry.

When three of the six parts of a spherical triangle (three sides and three angles) are given, and we require a fourth part, the most frequently recurring cases in practice are:—

- (i) The case (3.1), in which three of the parts concerned are consecutive, and the fourth—a side—stands alone.
- (ii) The cases (4.0), in which all the four parts concerned are consecutive.

If  $a, b, c, A, B, C$  are the sides and angles, then the case (3.1) corresponds to four such parts as  $b, A, c, -$ ,  $a$ , and case (4.0) to four such parts as  $c, B, a, C$ .

The corresponding well-known formulæ connecting the four parts are:—

Case (3.1):

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad . \quad . \quad . \quad (1)$$

Case (4.0):

$$\cos B \cos a = \cot c \sin a - \cot C \sin B \quad . \quad . \quad . \quad (2)$$

Captain Weir's construction for the case (4.0) is as follows:—

Draw the circle  $x = r \cos C, y = -r \sin C$  about the origin and graduate it in terms of  $C$ , taking  $r$  of any convenient length. Graduate the  $y$ -axis

\* "Theoretical Description of a new 'Azimuth Diagram' by Captain Patrick Weir," *Proc. Roy. Soc. Edin.*, vol. xvi, No. 129, p. 354.

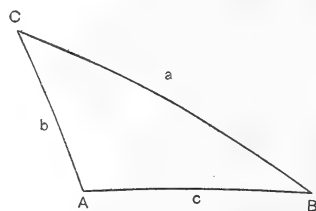


FIG. 1.

## INSTRUCTIONS TO AUTHORS.

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SESSION 1915-16.

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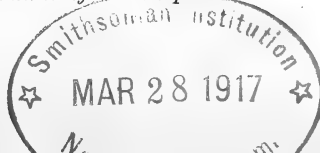
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*[Continued on page iii of Cover.]*

according to the formula  $y = \cot c$ . Construct the network consisting of the family of ellipses

$$\frac{x^2}{\operatorname{cosec}^2 a} + \frac{y^2}{\cot^2 a} = 1 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

intersected by the family of hyperbolæ

$$\frac{x^2}{\sin^2 B} - \frac{y^2}{\cos^2 B} = 1 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The resultant diagram is reproduced in fig. 2. Since in practice we are concerned with positive values of  $x$  only, the other half of the plane is not drawn.

An  $a$ -ellipse in the network and a  $B$ -hyperbola intersect in the point  $(\operatorname{cosec} a \sin B, \cot a \cos B)$ . The line joining this point to the point on the  $y$ -axis  $y = \cot c$  has for equation

$$y = x \frac{\cot a \cos B - \cot c}{\operatorname{cosec} a \sin B} + \cot c \quad . \quad . \quad . \quad . \quad (5)$$

This line is parallel to the line  $y = -x \cot C$  through the origin cutting the  $C$ -circle in the  $C$ -reading, since by (2)

$$\cot C = \frac{\cot c \sin a - \cos B \cos a}{\sin B}$$

Hence the method of using: Join the  $C$ -reading to the origin by one arm of a parallel ruler; make the other arm pass through the point in the network determined by  $a$  and  $B$ . This second arm then passes through the required value of  $c$  on the  $y$ -axis.

As used in navigation to find the azimuth  $A$  of a star given the declination  $\delta$ , hour-angle  $H$ , and latitude  $\phi$ , the four consecutive parts  $c, B, a, C$  (fig. 1) are replaced successively by  $90^\circ - \delta$ ,  $H$ ,  $90^\circ - \phi$ ,  $100^\circ - A$ , corresponding to the spherical triangle whose vertices are the star, the pole, and the zenith.

The chart as used in H.M. Navy measures 30 inches  $\times$  15 inches, and enables one to read to an accuracy of about a quarter of a degree.

Since two parallel lines intersect at an infinite distance, the arm of the parallel ruler, which always passes through the origin, and the  $C$ -reading on the  $C$ -circle may be regarded as pointing towards a point  $C$  at infinity, towards which the other arm also points. Let us then project stereographically the  $x, y$  plane into an  $x', y'$  plane, bringing  $C$  into a finite part of it. We can do this by projecting some line parallel to the  $y$ -axis to infinity, *e.g.* the line  $x = -1$ ; we shall let the  $x$ -axis become the  $x'$ -axis, and





The scale  $x=0$ ,  $y=\cot c$  becomes  $x'=-1$ ,  $y' (=y) = \cot c$ . The line (5) becomes

$$2y' = (1+x')(\cos \alpha \cos B - \sin \alpha \cot c) / \sin B + (1-x') \cot c \quad . \quad (10)$$

while the parallel line  $y = -x \cot C$  becomes

$$2y' = -(1+x') \cot C \quad . \quad (11)$$

These lines (10) and (11) intersect in the point C given by the scale

$$x' = 1 \quad y' = -\cot C,$$

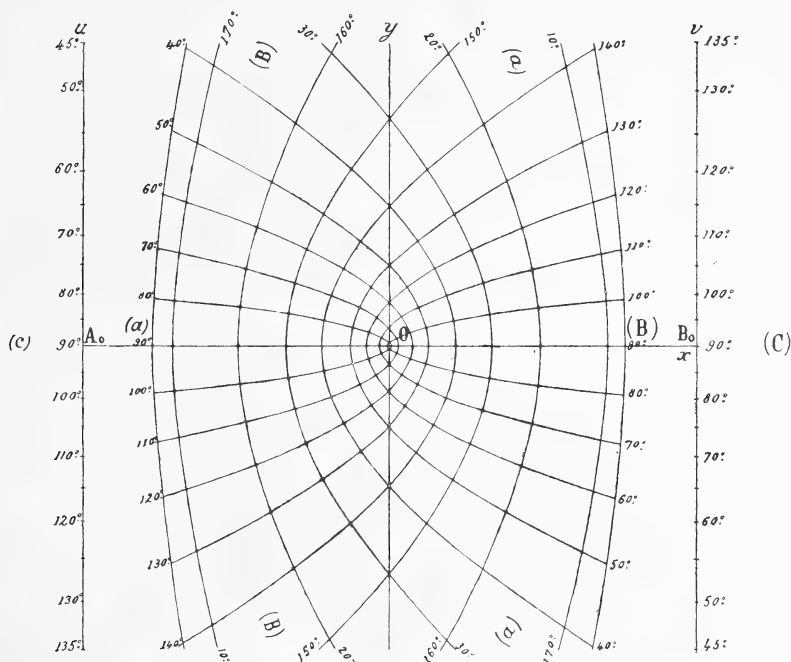


FIG. 3.

so that in the  $x', y'$  plane the new network point  $(a, B)$  is collinear with the point  $c$  on the scale  $x'=-1$ , and the point  $C$  on the scale  $x'=+1$ . The resulting straight-line nomogram is reproduced in fig. 3. It was first drawn by Perret \* in 1904, following a method due to Professor d'Ocagne of Paris. To use it we have merely to draw a straight line across it joining up the three points  $c$ ,  $(a, B)$ , and  $C$ . When any three of the variables are given, the fourth is then read from the diagram. We thus see that Captain Weir was the first to construct and use this type of nomogram, for fig. 3 is merely a projection of fig. 2. D'Ocagne's form has, of course, the advantage of dispensing with the parallel line.

\* *Annales Hydrographiques*, Paris, 1904.

Although more convenient in some respects than Captain Weir's diagram, it has the disadvantage of throwing the scale-readings both of  $c$  and  $C$ , which are near  $0^\circ$  or  $180^\circ$ , too far away from the centre,\* whereas Weir's does this only for the variable  $c$ . If we wish to have all values of  $c$  on the diagram, we could transform d'Ocagne's nomogram so that the  $c$ -scale goes to infinity and is replaced by a  $c$ -circle about the new origin. The required transformation is analogous to the one just given.

D'Ocagne has also published† a straight-line nomogram for the case (3.1) formula (1). It consists of two scales  $x = -1$ ,  $y = -\cos a$ ;  $x = +1$ ,

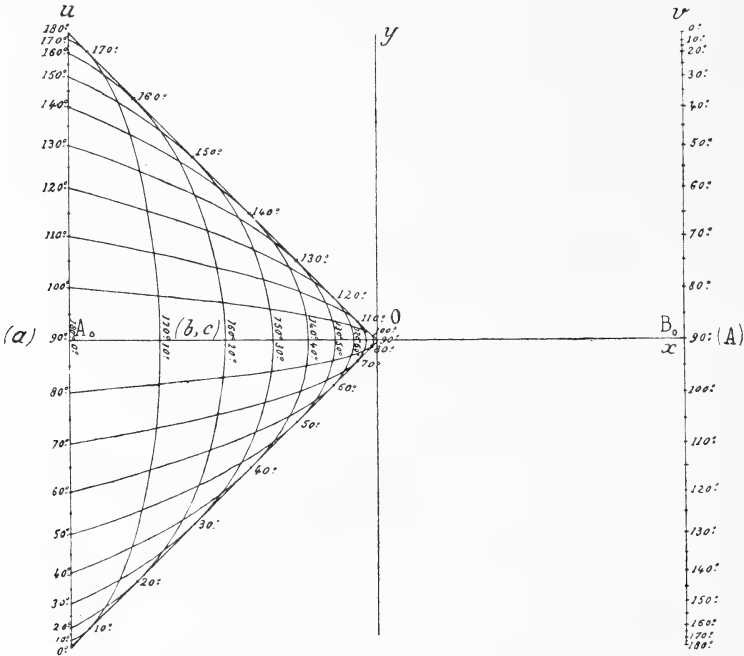


FIG. 4.

$y = -\cos A$ , together with the network made up of the two identical families of ellipses,

$$\begin{aligned} (x-1)^2 &= (x+1)^2 \operatorname{cosec}^2 b + 4y^2 \sec^2 b \} \\ (x-1)^2 &= (x+1)^2 \operatorname{cosec}^2 c + 4y^2 \sec^2 c \} \end{aligned} \quad \dots \quad (12)$$

The intersection of a  $b$ -ellipse and a  $c$  one determine a point  $(b, c)$  on the diagram which is collinear with the point  $a$  on the  $a$ -scale and the point  $A$  on the  $A$ -scale, provided the four variables  $b, A, c, a$  satisfy the equation (1). It is reproduced in fig. 4.

\* This was subsequently partially remedied by Perret by an additional construction. See *Assoc. française pour l'avancement des sciences*, 1905, p. 93.

† *Bulletin Astr.*, t. xi (1894), p. 5; *Soc. Mach. France Bull.* (1904), p. 196.

Applying to it the transformation

$$x' = \frac{1+x}{1-x} \quad y' = \frac{2y}{1-x},$$

equations (12) readily become

$$\frac{x'^2}{\sin^2 b} + \frac{y'^2}{\cos^2 b} = 1,$$

$$\frac{x'^2}{\sin^2 c} + \frac{y'^2}{\cos^2 c} = 1.$$

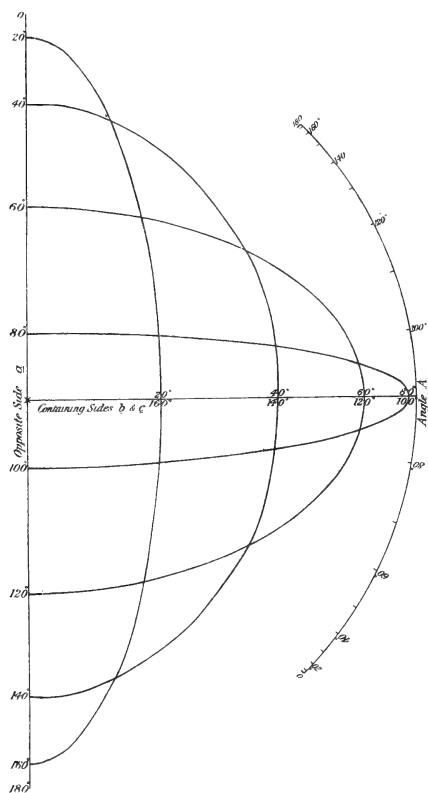


FIG. 5.

The scale  $x = -1, y = \cos a$  becomes  $x' = 0, y' (= y) = \cos a$ , while the A-scale ( $x = +1, y = -\cos A$ ) goes to infinity. The variable line  $\frac{y}{x+1} = \frac{-\cos A}{2}$ , which joins any point A on it to the point  $x = -1, y = 0$ , becomes  $y' = -x' \cos A$ , a line through the origin cutting the line  $x' = +1$  in the scale  $y' = -\cos A$ .

We can now construct diagram 5 to be used with parallel rulers which bears the same relation to d'Ocagne's (3.1) nomogram as Captain

Weir's azimuth diagram does to the (4.0) one. Instead of the A-scale ( $x=1, y=-\cos A$ ) there has been substituted the scale on the circle  $x^2+y^2=1$  given by the radial line through the origin to this A-scale. When one arm of the ruler joins the point  $a$  on the  $a$ -scale to the point  $(b, c)$  in the network, the other arm, when made to pass through the origin, cuts the circular A-scale in the required value of  $A$ .

This modification has the advantage over the original of being only half its size for a given accuracy, and of avoiding some of the excessive compression which occurs near where  $b$  and  $c$  approach the value  $90^\circ$ .

We could project the  $a$ -scale of d'Ocagne's nomogram to infinity instead of the A one by a similar transformation, but it does not seem to offer any special advantages.

*(Issued separately November 29, 1916.)*

## X.—On a Possible Explanation of the Satellites of Spectral Lines.

By R. A. Houstoun, M.A., Ph.D., D.Sc., Lecturer on Physical Optics in the University of Glasgow.

(MS. received April 24, 1916. Read May 15, 1916.)

It is well known that, when examined with a modern high-power spectro-scope, the bright lines in the spectra of the different elements exhibit an individual character. Some are diffuse, some are sharp, some are diffuse on the one side and sharp on the other, and some are accompanied by fainter lines in their neighbourhood. To the latter the name of satellites has been given. The most celebrated satellites are those of the green line of mercury, which have been investigated very often owing to the ease with which the mercury spectrum can be produced. They are much fainter than the main line, and at least six can be seen. But there are cases occurring in which a satellite is so strong that it is hardly possible to say which is the main line. Thus the red line of hydrogen is a close doublet with a separation of  $\cdot 14$  A.U., the two components of the doublet being of approximately equal brightness.

The purpose of this short paper is to suggest that satellites may in some cases be caused by sudden changes of amplitude or phase in an oscillator of one definite period. The explanation is interesting principally on account of the use it makes of certain well-known diffraction formulæ.

Let us suppose that an oscillator is emitting a wave given by  $\cos \beta t$ , and that the wave starts suddenly when  $t=0$ , and stops suddenly when  $t=l$ . Then, expanding the wave in a Fourier integral, we have

$$\begin{aligned} & \frac{1}{\pi} \int_0^\infty da \int_{-\infty}^{+\infty} \cos \beta \xi \cos a(\xi - t) d\xi \\ &= \frac{1}{2\pi} \int_0^\infty da \int_0^l [\cos \{(a - \beta)\xi - at\} + \cos \{(a + \beta)\xi - at\}] d\xi \\ &= \frac{1}{\pi} \int_0^\infty \left[ \frac{\sin \frac{1}{2}(a - \beta)l \cos \{\frac{1}{2}(a - \beta)l - at\}}{a - \beta} + \frac{\sin \frac{1}{2}(a + \beta)l \cos \{\frac{1}{2}(a + \beta)l - at\}}{a + \beta} \right] \end{aligned}$$

Thus the original limited cosine wave of period  $2\pi/\beta$  is equal to the superposition of an infinite number of unlimited cosine waves, the period of the typical unlimited wave being given by  $2\pi/a$ . It is clear, however, from the denominator of the first term, that in the integration the only waves of

importance are those for which  $\alpha$  is nearly equal to  $\beta$ . We can therefore neglect the second term entirely.

We then find by the aid of a theorem given by Lord Rayleigh\* that the energy of the waves lying between  $\alpha$  and  $\alpha + d\alpha$  is given by

$$\frac{1}{\pi} \frac{\sin^2 \frac{1}{2}(\alpha - \beta)l}{(\alpha - \beta)^2} d\alpha.$$

This expression obviously has a maximum value for  $\alpha = \beta$ , and falls away rapidly on both sides of this maximum to a minimum value, zero, given by  $(\alpha - \beta)l = \pm 2\pi$ . It has a succession of minima given by  $(\alpha - \beta)l = \pm 2n\pi$ , where  $n$  is any integer. Only the central maximum is of importance; it is twenty-one times as high as those on each side of it. The expression, in fact, is the same as that representing the diffraction pattern seen when an unlimited plane harmonic wave falls normally on a slit, and, as such, it is discussed in most books on optics.

Let us suppose, now, that the original wave is received by a spectroscope of infinite resolving power and with an infinitely narrow slit. It does not converge to a mathematical line at the point in the spectrum specified by  $\beta$ , as it would do if it were an infinitely long train, but, owing to its being limited, forms a maximum of finite breadth at this point, accompanied by fainter lines arranged symmetrically on each side.

Let the initial wave be given by  $e^{-ht} \cos \beta t$  from  $t=0$  to  $t=\infty$ ; i.e. it starts with a finite value, and slowly damps down to zero. Then the Fourier integral becomes

$$\frac{1}{\pi} \int_0^\infty d\alpha \int_0^\infty e^{-h\xi} \cos \beta \xi \cos \alpha(\xi - t) d\xi = \frac{1}{2\pi} \int_0^\infty d\alpha \int_0^\infty e^{-h\xi} \cos \{(\alpha - \beta)\xi - \alpha t\} d\xi,$$

since the other term can be neglected as in the previous case. On integrating and substituting the limits this becomes

$$\frac{1}{2\pi} \int_0^\infty \frac{\cos \{ \alpha t - \tan^{-1}(\alpha - \beta)/h \}}{\{(\alpha - \beta)^2 + h^2\}^{\frac{1}{2}}} d\alpha,$$

and the energy contained between  $\alpha$  and  $\alpha + d\alpha$  is consequently

$$\frac{1}{4\pi} \frac{d\alpha}{\{(\alpha - \beta)^2 + h^2\}^{\frac{1}{2}}}.$$

Now  $h$  is supposed to be small in comparison with  $\beta$ . Consequently the above expression has a sharp maximum falling away to zero on both sides. If the train represented by  $e^{-ht} \cos \beta t$  from  $t=0$  to  $t=\infty$  is received by the same ideal spectroscope, a line of finite breadth is formed at the point in

\* *Phil. Mag.*, xxvii, pp. 460-469, 1889; or *Collected Works*, vol. iii, p. 268.

the spectrum specified by  $\beta$ ; this line is, however, more diffuse than in the previous case, and is not accompanied by fainter components.

Let the oscillator emit a wave given by  $\cos \beta t$  from  $t=0$  to  $t=l$ ; let it then rest from  $t=l$  to  $t=l+k$ , and again emit the wave given by  $\cos \beta t$  from  $t=l+k$  to  $t=2l+k$ ; there are thus two periods of activity of equal duration separated by a period of rest. The sole difference between this case and the first one lies in the limits of integration. The second term may be neglected as before. After the limits are substituted the Fourier integral takes the form

$$\begin{aligned} \frac{1}{\pi} \int_0^\infty \frac{1}{(a-\beta)} & \left[ \sin \frac{1}{2}(a-\beta)l \cos \left\{ \frac{1}{2}(a-\beta)l - at \right\} + \sin \frac{1}{2}(a-\beta)l \cos \left\{ \frac{1}{2}(a-\beta)(3l+2k) - at \right\} \right] da \\ &= \frac{2}{\pi} \int_0^\infty \frac{\sin \frac{1}{2}(a-\beta)l}{a-\beta} \cos \frac{1}{2}(a-\beta)(l+k) \cos \left\{ \frac{1}{2}(a-\beta)(2l+k) - at \right\} da. \end{aligned}$$

The energy contained between the periods specified by  $a$  and  $a+da$  is consequently

$$\frac{4 \sin^2 \frac{1}{2}(a-\beta)l}{\pi(a-\beta)^2} \cos^2 \frac{1}{2}(a-\beta)(l+k).$$

The first factor is the same as occurs in the first case; it has a maximum at  $a=\beta$ , whence it descends to minima at  $(a-\beta)l = \pm 2\pi$ . The second factor is zero at  $(a-\beta)(l+k) = \pm(2n+1)\pi$ . Let  $l=k$ ; then three of the zeroes occur in the original central part of the pattern. Considering the two factors together, we see that the main part of the original line is now resolved into five components.

We might proceed further in the same way, making suppositions as to the manner in which the oscillator starts and stops, or introducing sudden changes of phase, and calculating the way in which this affects the appearance of the line. Generally speaking, however, the calculation becomes heavy, and it will probably bring out no new features. It seems to me more promising to follow up the investigation experimentally.

The analysis here is practically the same as that used in the calculation of Fraunhofer diffraction patterns. Our first case is the case of the single slit; our third case is the case of the two parallel slits separated by an opaque interval; and similarly we might have passed to the case of the diffraction grating. The integration with respect to  $\xi$  is simply the summation of the S.H.M.s which according to Huygen's principle take place at each point in the aperture. To every Fraunhofer diffraction pattern there corresponds a method of stopping and starting the oscillator. The question, therefore, as to whether the structure of a certain line can be explained on the method suggested in this paper resolves itself into the

question as to whether a Fraunhofer diffraction pattern can be produced having the structure of the line. This will be best solved by trial, by fixing an arrangement of slits on the table of a spectrometer so that the rays from the collimator fall on them normally, and examining the appearance seen in the telescope. As bright a sodium flame as possible should be used as source, and there should be some means of setting the slits accurately parallel to the slit of the collimator.

The question arises as to how the broadening of the line due to the train being limited is related to the other causes of broadening of a spectral line. The whole question of the widening of spectral lines has been recently considered by Lord Rayleigh.\* He comes to the conclusion that at low pressures, when the line is narrow, its width is due solely to the Doppler effect, but at high pressures the influence of impacts becomes apparent. The impacts produce sudden changes in the phase and intensity of the vibrations, and the method of taking account of such changes is to represent the vibration by a Fourier integral, as has been done here. To explain the occurrence of satellites, however, we cannot rely on the random impacts of molecules or atoms. The change of phase postulated must be due to something in the atom itself, something inseparable from the process of radiation from the atom. For example, in the third case considered above, after the oscillator has acted for time  $l$ , it must rest for  $k$  and then act again for  $l$ . It next rests for an interval which must be determined by chance, and then goes through its cycle again. Superimposed on the cycle of changes there are of course irregular changes due possibly to impact, and these cause the widening of the line at high pressures. Lord Rayleigh considers that, although it is certain that damping of the vibrations must operate as a cause of widening, we cannot at the present time point to any experimental verification of its influence.

The objection that may be urged against the suggestion put forward in this paper is, that it is incompatible with the high path difference at which interference can be observed in the case of monochromatic radiations. Michelson obtained interference with the green line of mercury when there was a path difference of 540,000 wave-lengths. This seems to require no change of phase for 540,000 periods, a number that would bring the attendant lines much closer in than is required for the satellites, *i.e.* if we use the result for the first of our three cases. I do not consider that the objection is a fatal one.

In any case, the possibility of successive stages in the emission of an

\* "On the Widening of Spectrum Lines," *Phil. Mag.*, xxix, p. 274, 1915.



oscillation of one definite frequency is worthy of consideration as a means of explaining the complexity of spectra. The old idea, according to which we had a number of particles moving under a system of attractions according to the laws of dynamics, and which in view of the complexity of the subject usually relapsed into generalised co-ordinates, has not proved successful; and the modern view, which puts electrons into the molecule, as if they were stones into a bag, assigning a definite electron to each line, does not seem very promising either. So there is room for fresh hypotheses.

*(Issued separately November 29, 1916.)*

XI.—The Optical Rotation and Cryoscopic Behaviour of Sugars dissolved in (a) Formamide, (b) Water. Part II. By John Edwin Mackenzie and Sudhamoy Ghosh, D.Sc. (Edin.). *Communicated* by Professor J. WALKER, F.R.S.

(MS. received December 8, 1915. Read January 10, 1916.)

INTRODUCTION.

IN a recent communication (*Proc. Roy. Soc. Edin.*, 1914–15, vol. xxxv, p. 22) measurements of the optical rotation and cryoscopic behaviour of the following sugars, viz.  $\beta$ -*l*-arabinose, *l*-xylose,  $\alpha$ -*d*-glucose,  $\alpha$ -*d*-galactose, *d*-mannose, *d*-fructose,  $\alpha$ - and  $\beta$ -lactose, dissolved in (a) formamide, (b) water, were described. The investigation has been carried a step further by the addition of  $\beta$ -*d*-glucose,  $\beta$ -*d*-galactose and maltose to the above list. Thus the change in rotation or mutarotation in formamide solution has now been measured, starting from both the  $\alpha$ - and  $\beta$ -form of *d*-glucose, *d*-galactose and lactose. As in the case of the water solutions of these sugars, the constant rotation shown when there is equilibrium between the two modifications is found to be the same whether the starting-point be the  $\alpha$ - or the  $\beta$ -modification.

The phenomenon of mutarotation having been demonstrated to be of similar character in non-aqueous and aqueous solutions, it is obvious that any explanation of the mechanism of mutarotation reactions must account for such reactions taking place in the absence of water. The theories hitherto set forth by various authors are mainly concerned with the change of rotation in aqueous solutions. It has been shown that in aqueous solutions the presence of even traces of alkali increases to an enormous extent the rate of mutarotation of sugars (*cf.* Lowry, *Chem. Soc. J.*, 1899, lxxv, 212). But the presence of alkali causes an enormous increase in the electrical conductivity of water and it has been assumed that the increase in the number of hydroxide ions is responsible for the increase both of electrical conductivity and rate of mutarotation.

In the course of voluminous researches, Nef and others have theorised upon the changes undergone by sugars in presence of alkali and have come to the conclusion that, in aqueous alkaline solutions, sugars exist, at any rate to some extent, in enolic modifications, *e.g.* glucose as  $\text{CH}(\text{OH}) : \text{C}(\text{OH}) . (\text{CHOH})_3 . \text{CH}_2\text{OH}$ ; but it is very doubtful if

mutarotation is in any way dependent on internal changes undergone by such enolic forms. On the other hand, the recent recognition of the existence of glucose in the two forms termed collectively ' $\gamma$ -glucose' (Irvine, Fyfe, and Hogg, *Trans. Chem. Soc.*, 1915, cvii, 524) opens up the possibility that the phenomenon of mutarotation may involve these new varieties of sugars, so that detailed discussion of the results which follow may be deferred.

Irvine and Steele have, however, discussed the mechanism of mutarotation in aqueous solution in a paper (*Chem. Soc. J.*, 1915, cvii, 1230) published after the completion of the experimental work described below. On the evidence of measurements of the conductivities of certain methylated sugars in aqueous and in dilute boric acid solutions, they favour E. F. Armstrong's oxonium theory of the mechanism of mutarotation in which the  $\gamma$ -oxidic oxygen atom displays quadrivalency, a hydrogen atom and a hydroxyl group satisfying the two latent valencies. What effect boric acid has upon the  $\beta$ - form of the sugars has not yet apparently been determined experimentally, but it is tacitly assumed that the  $\beta$ - form would, under similar circumstances, arrive at the same equilibrium mixture as that attained by the  $\alpha$ - form.

The mutarotation of sugars has been most fully studied in the case of aqueous solutions, but measurements in pyridine solution have also been made, and now those in formamide solution are available. The last-mentioned solvent probably possesses the  $-\text{NH}_2$  group, though it may

be argued that formamide has an imino  $\text{H} \cdot \text{C} \begin{array}{l} \text{NH} \\ \text{OH} \end{array}$ , and not the amide

constitution  $\text{H} \cdot \text{C} \begin{array}{l} \text{NH}_2 \\ \text{O} \end{array}$ . The very low conductivity of the formamide

used in the following experiments makes it, on the whole, improbable that the imino modification was present. Now a definite compound of  $\beta$ -glucose and pyridine is known, and, this being so, it is conceivable that the mechanism of mutarotation is due to the formation of such compound and subsequent splitting off of the pyridine molecule with formation of  $\alpha$ - and  $\beta$ - forms. Under certain conditions only the  $\beta$ - form is actually formed.

The authors, of whom one is on military duty and the other has returned to India, are unable at the present time to present a full theoretical discussion of the subject, and must content themselves with

the statement that mutarotation is a reversible process represented by the equation



## EXPERIMENTAL.

### $\beta$ -*d*-GLUCOSE.

Ten grams of pure  $\alpha$ -*d*-glucose were dissolved by boiling in 38 c.c. of freshly distilled pyridine (B.P. 114–115° C.) previously treated with potassium hydroxide and the solution filtered. The solution was cooled in ice water, and, on scratching the sides of the vessel, crystals came out after about an hour. The solution was filtered off and the crystals washed with some pyridine and then with ether. On drying at 105° C. to constant weight pure  $\beta$ -glucose in a very fine state of division was obtained. The yield was about 3·4 grams. The melting-point of the substance was 146–148° C. and the specific gravity at 13° C. (determined by the method of flotation) 1·547.

The cryoscopic measurements in water were not repeated. The figures for formamide solution were as follows:—

Concentration per 100 grams of solvent.	Mol. weight found.	Mol. weight calculated.
0·35	181·4	180
0·68	181·4	„
0·98	183·5	„

The polarimetric figures (*cf.* figs. 1 and 2, in which the data for both  $\alpha$ - and  $\beta$ -*d*-glucose are graphed) were as follows:—

Solvent . .	Water.		Formamide.	
Graph . .	$\otimes$		$\oplus$	
Concentration grams in 100 c.c. solution.	2.3356.		1.7148.	
	Time.	$[\alpha]_D^{20}$ .	Time.	$[\alpha]_D^{20}$ .
	8 min.	+ 24.83°	10 min.	+ 17.20°
	12 "	26.97	27 "	18.66
	18 "	28.47	45 "	20.41
	27 "	31.90	1 hr. 0 "	21.87
	32 "	33.39	1 " 19 "	23.33
	37 "	34.89	2 " 14 "	26.24
	42 "	36.61	2 " 42 "	27.99
	47 "	37.46	3 " 51 "	31.78
	53 "	38.96	4 " 30 "	33.82
	1 hr. 0 "	40.46	5 " 0 "	34.99
	1 " 49 "	46.67	5 " 36 "	36.45
	2 " 7 "	47.74	6 " 9 "	37.91
	2 " 55 "	50.09	24 " 34 "	48.11
	20 " 33 "	52.02	25 " 39 "	48.69
	48 " 0 "	+ 52.02	27 " 15 "	49.57
			29 " 0 "	51.03
			30 " 10 "	52.19
			47 " 45 "	54.23
			49 " 47 "	54.53
			53 " 6 "	55.69
			72 " 0 "	+ 56.28
			14 days.	56.28
			K=0.000996.	
			Calc. initial $[\alpha]_D^{20} = +15.74^\circ$ .	
			Extrapolated initial $[\alpha]_D^{20} = +15.90^\circ$ .	

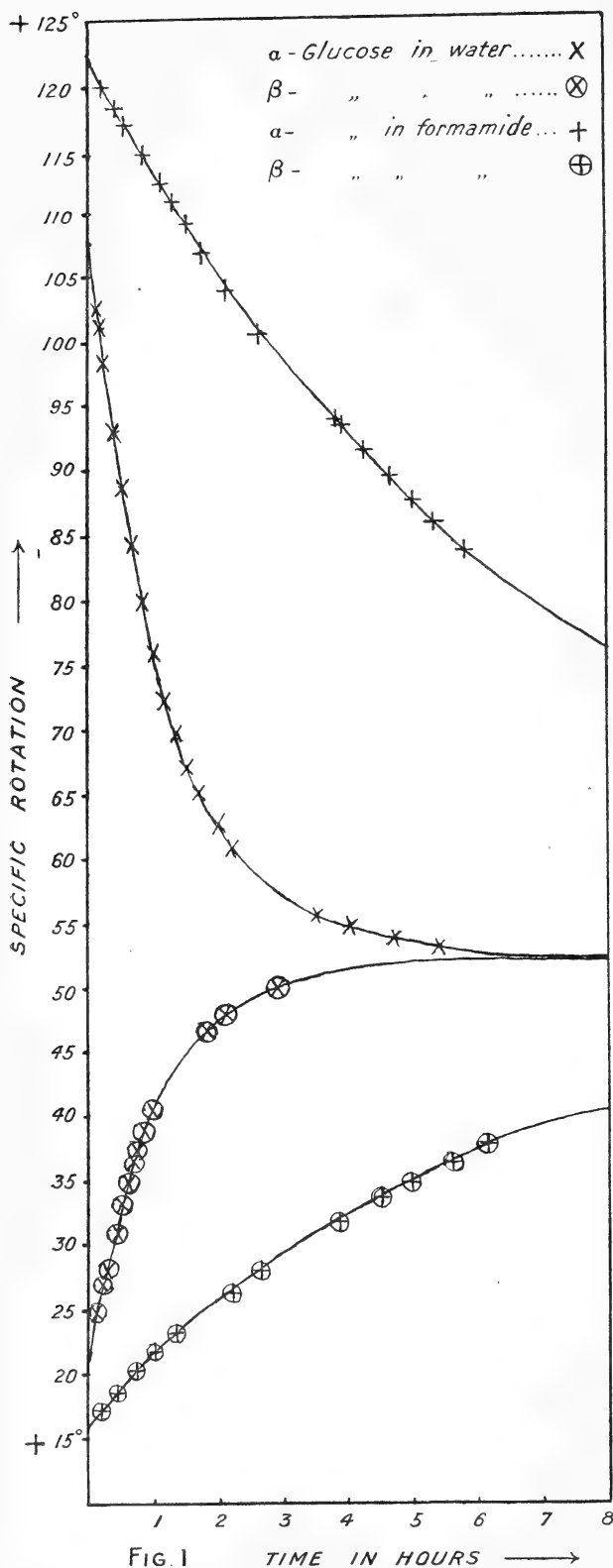


FIG. 1

TIME IN HOURS →

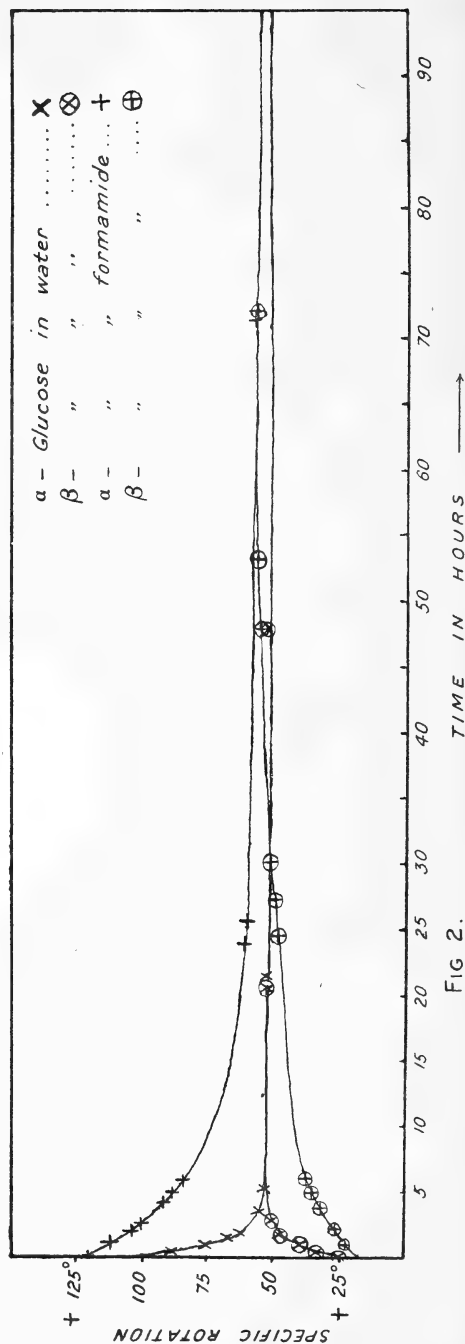


FIG. 2.

$\alpha$ -D-GALACTOSE.

In our previous paper the mutarotations of  $\alpha$ -D-galactose in water at 12° and in formamide at 18° were described. Professor J. C. Irvine having kindly supplied us with some specially pure galactose, recrystallised from glacial acetic acid, it was dried to constant weight in the apparatus previously described, and measurements were made at 20°, with the following results (*cf.* figs. 3 and 4):—

Solvent . .	Water.		Pyridine G. and B.		Formamide.	
Graph . .	×		⊙		+	
Concentration grams in 100 c.c. solution.	2.2460.				1.7488.	
	Time.	$[\alpha]_D^{20}$	Time.	$[\alpha]_{red}^{20}$	Time.	$[\alpha]_D^{20}$
	6 min.	+131.57°	23 min.	+120.98°	9 min.	+152.96°
	10 "	127.78	30 "	112.34	12 "	151.25
	12 "	125.34	45 "	106.17	17 "	150.10
	15 "	122.22	60 "	102.47	22 "	148.96
	19 "	118.21	90 "	100.00	27 "	147.82
	25 "	113.09	120 "	97.50	34 "	145.53
	30 "	110.42	180 "	93.88	41 "	144.10
	35 "	106.86	240 "	91.31	52 "	140.96
	42 "	102.85	1 day	74.04	1 hr. 5 "	137.81
	48 "	99.78	2 days	55.53	1 " 15 "	136.09
	54 "	97.28	3 "	46.94	1 " 25 "	134.38
	1 hr. 0 "	94.61	4 "	+ 46.94	1 " 35 "	132.66
	1 " 7 "	92.61		Constant	1 " 45 "	130.66
	1 " 15 "	90.38			1 " 55 "	128.95
	1 " 25 "	88.38			2 " 10 "	126.94
	1 " 35 "	86.82			2 " 25 "	124.94
	1 " 45 "	85.26			2 " 43 "	122.94
	1 " 58 "	83.93			3 " 0 "	120.65
	2 " 15 "	82.37			4 " 37 "	112.65
	2 " 30 "	81.03			4 " 51 "	110.93
	2 " 45 "	80.59			5 " 12 "	108.65
	3 " 2 "	79.70			5 " 35 "	107.22
	4 " 11 "	79.25			6 " 2 "	105.50
	5 " 30 "	79.25			6 " 22 "	103.50
	24 "	+ 79.25			23 " 42 "	89.20
					24 " 20 "	88.63
					26 " 32 "	87.77
					29 " 30 "	87.77
					48 "	+ 87.77
	K=0.009601.				K=0.001988.	
	Calc. initial $[\alpha]_D^{20} =$				Calc. initial $[\alpha]_D^{20} =$	
	+139.36°.				+155.25°.	
	Extrapolated initial				Extrapolated initial	
	$[\alpha]_D^{20} = +139.50°.$				$[\alpha]_D^{20} = +155.0°.$	

*$\beta$ -d-GALACTOSE.*

*$\beta$ -d-galactose* was prepared by a modification of Tanret's method, for a description of which we are indebted to Professor J. C. Irvine.

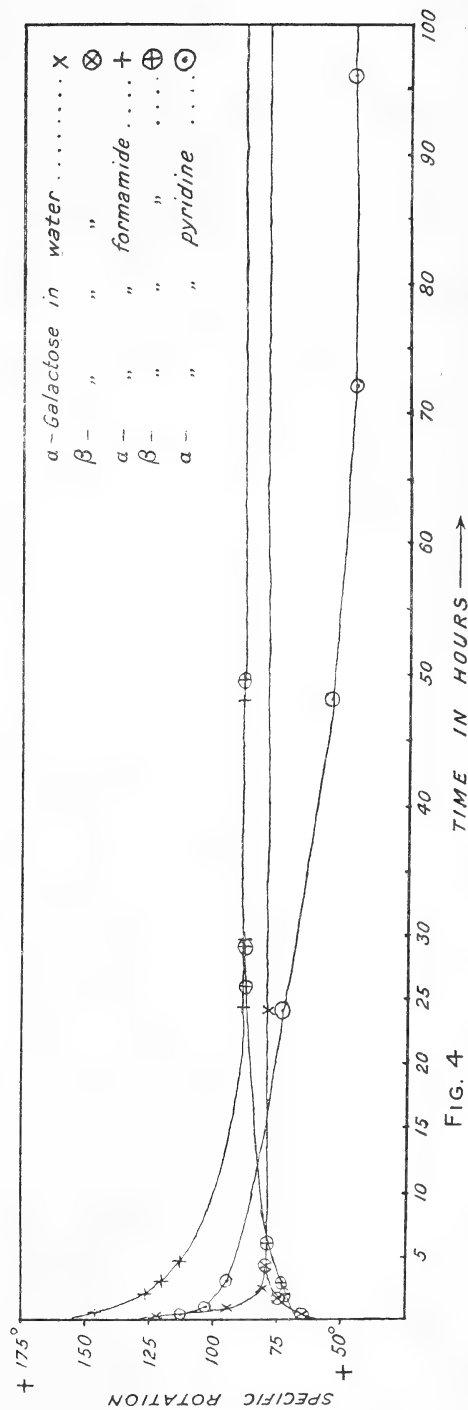
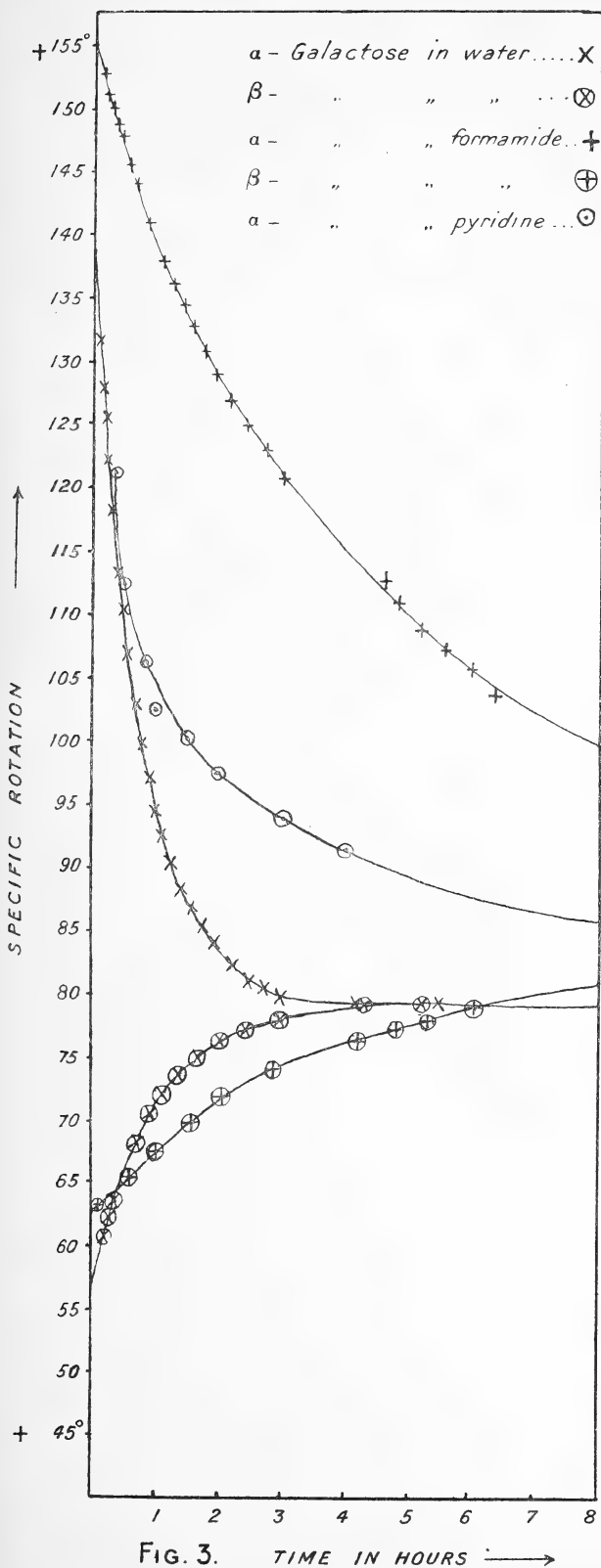
The substance thus obtained was found to be partly mixed with  *$\alpha$ -galactose*, as shown by its initial specific rotation. The discrepancy in the values for *K* for  *$\beta$ -* with those for  *$\alpha$ -galactose* is possibly due to the impurity.

The polarimetric results (*cf.* figs. 3 and 4) were as follows:—

 *$\beta$ -d-GALACTOSE.*

Solvent . . .	Water.		Formamide.	
Graph . . .	⊗		⊕	
Concentration grams in 100 c.c. solution.	1·8668.		1·8064.	
	Time.	$[\alpha]_D^{20}$	Time.	$[\alpha]_D^{20}$
	12 min.	+ 60·53°	10 min.	+ 63·39°
	18 "	62·14	22 "	64·22
	24 "	63·48	37 "	65·32
	34 "	65·35	50 "	66·43
	45 "	68·03	1 hr. 3 "	67·54
	55 "	70·44	1 " 18 "	68·64
1 hr. 7 "	72·05		1 " 33 "	69·75
1 " 22 "	73·65		1 " 50 "	71·13
1 " 42 "	75·00		2 " 5 "	71·97
2 " 2 "	76·33		2 " 25 "	73·07
2 " 27 "	77·14		2 " 53 "	74·18
3 " 0 "	77·94		4 " 15 "	76·39
4 " 20 "	79·01		4 " 50 "	77·06
5 " 15 "	+ 79·01		5 " 20 "	77·78
	Constant		6 " 5 "	78·88
			23 " 35 "	86·63
			29 " 0 "	87·19
			48 " 0 "	+ 87·19
	K=0·007222.		K=0·001573.	
	Calc. initial $[\alpha]_D^{20}$ =		Calc. initial $[\alpha]_D^{20}$ =	
	+ 56·51°.		+ 62·30°.	
	Extrapolated initial		Extrapolated initial	
	$[\alpha]_D^{20}$ = + 56·5°.		$[\alpha]_D^{20}$ = + 62·50°.	





## MALTOSE.

Considerable difficulty was found in the purification of samples of maltose commercially obtainable. We are indebted to Mr John Ford, F.I.C., for a sample which, after recrystallisation from alcohol, was perfectly colourless, and on heating melted between  $125^{\circ}$  and  $130^{\circ}$ .

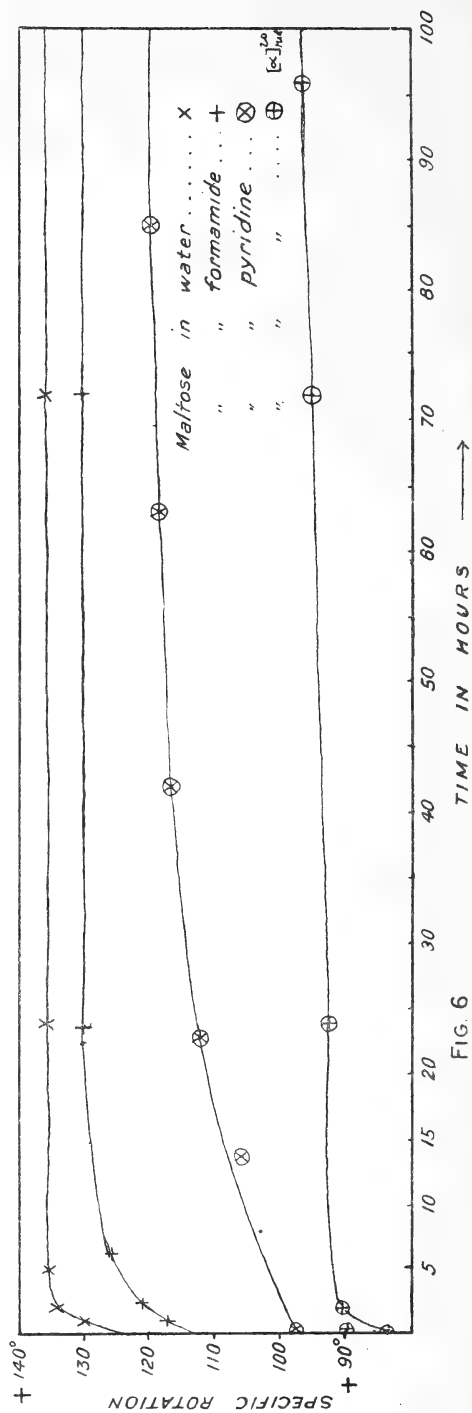
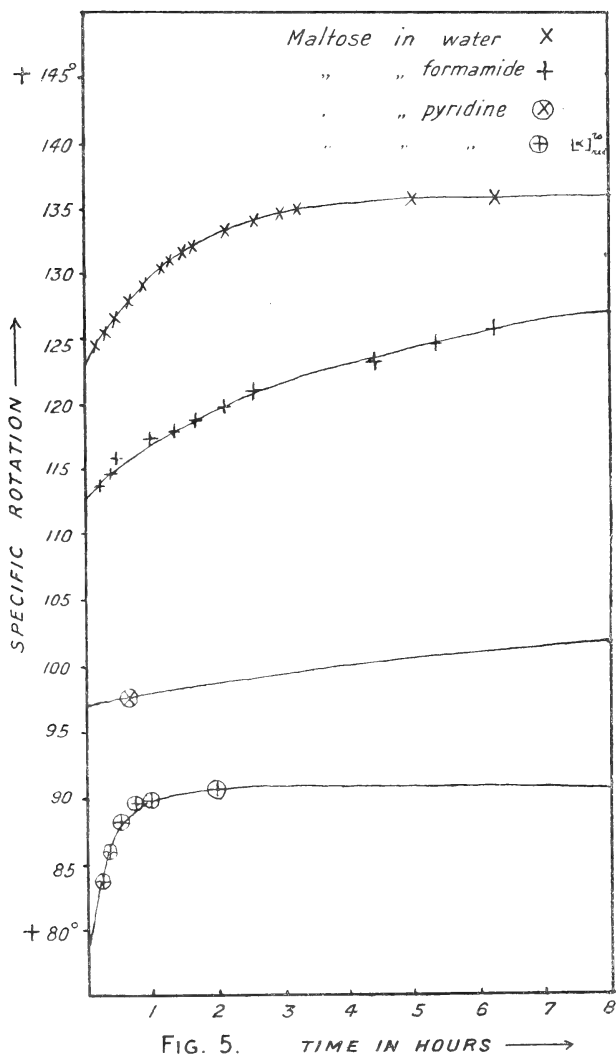
The cryoscopic measurements in water and formamide solutions gave the following values:—

Solvent.	Concentration per 100 grams solvent.	Mol. weight found.	Mol. weight calculated.
Water . . .	0.45	341.8	342
” . . .	0.56	333.0	”
Formamide . .	0.35	301.6	”
” . . .	0.69	297.6	”

The slow rate of solution of maltose in formamide probably accounts for the low values obtained.

The polarimetric results (*cf.* figs. 5 and 6) were as tabulated. The values for pyridine solution are those of Grossmann and Block (for red light) and Schliephacke (*Ann.*, 1910, **377**, 164):—

Solvent	Water.	Formamide.	Pyridine G. and B.	Pyridine S.
Graph	×	+	⊕	⊗
Concentration grams in 100 c.c. solution.	2.5188.	2.1224.		5.00.
	Time. $[a]_D^{20}$	Time. $[a]_D^{20}$	Time. $[a]_{red}^{20}$	Time. $[a]_D^{20}$
	9 min. + 124.46°	12 min. + 113.79°	15 min. + 83.59°	40 min. + 97.7°
	12 " 124.86	23 " 114.73	20 " 85.93	13 hr. 40 " 105.4
	16 " 125.26	29 " 115.67	30 " 88.28	23 " 113.0
	25 " 126.65	1 hr. 0 " 117.32	45 " 89.84	42 " 117.0
	31 " 127.05	1 " 21 " 117.79	60 " 89.84	63 " 119.0
	39 " 127.84	1 " 42 " 118.73	120 " 90.66	85 " 120.1
	45 " 128.23	2 " 8 " 119.68	1 day 92.67	166 " 121.0
	53 " 129.03	2 " 35 " 121.09	3 days 95.31	214 " 121.8
	1 hr. 1 " 129.83	4 " 25 " 123.21	4 " 96.87	14 days 122.2
	1 " 9 " 130.42	5 " 20 " 124.62	5 " 97.66	Const. + 123.5
	1 " 17 " 131.02	6 " 15 " 125.80	6 " 98.43	
	1 " 28 " 131.61	23 " 45 " 130.28	8 " 98.43	K = 0.000210.
	1 " 39 " 132.21	72 " + 130.28	9 " 99.22	Calc. initial $[a]_D^{20} =$
	1 " 54 " 132.80		11 " 100.00	+ 97.2°.
	2 " 9 " 133.40	K = 0.00163.	12 " 100.00	Extrapolated initial
	2 " 26 " 134.00	Calc. initial $[a]_D^{20} =$	15 " 100.00	$[a]_D^{20} = + 97.0°.$
	2 " 43 " 134.20	+ 113.09°.		
	3 " 0 " 134.60	Extrapolated initial		
	3 " 17 " 135.18	$[a]_D^{20} = + 113.0°.$		
	5 " 0 " 135.58			
	6 " 17 " 135.78			
	24 " 5 " 136.18			
	72 " 25 " + 136.18			
	K = 0.00503.			
	Calc. initial $[a]_D^{20} =$			
	+ 123.20°.			
	Extrapolated initial			
	$[a]_D^{20} = + 123.0°.$			



In the following table a summary of the values of  $K$  for the various sugars is given.

$$K = \frac{1}{t_2 - t_1} \log \frac{\beta_1 - \phi}{\beta_2 - \phi},$$

where  $t_1$  and  $t_2$  are times from moment of solution till polarimetric readings were made,  $\beta_1$  and  $\beta_2$  are the actual polarimetric readings at times  $t_1$  and  $t_2$ , and  $\phi$  is the actual reading when constant.

It will be observed that in the case of the three sugars—*d*-glucose, *d*-galactose, and lactose—in which measurements from both modifications were made, the values for the constant in each solvent are in close agreement. Attention may also be called to the much smaller value of  $K$  in formamide solution than in water solution.

Sugar.	K in water.				K in formamide.			
	$\alpha \rightarrow \beta$ .		$\beta \rightarrow \alpha$ .		$\alpha \rightarrow \beta$ .		$\beta \rightarrow \alpha$ .	
		Temp.		Temp.		Temp.		Temp.
<i>l</i> -arabinose			0.0134	12°	0.00306	20°	0.00154	13°
<i>l</i> -xylose .	0.0188	20°			0.00109	20		
<i>d</i> -glucose .	0.00627	20	0.00690	20	0.001988	20	0.000996	20
<i>d</i> -galactose	0.00960	20	0.00722	20	0.00326	20	0.00157	20
* <i>d</i> -mannose .	0.0273	20			0.00839	20		
<i>d</i> -fructose .					0.00163	20		
maltose .	0.00503	20			0.000387	15°		
lactose .	0.00378	14°	0.00297	17°			0.0004	17°

\* ERRATUM.—*Proc. Roy. Soc. Edin.*, 1914-15, xxxv, 37, the value of  $K$  in formamide is given as 0.000326 instead of 0.00326.

The authors desire to express their gratitude for a grant from the Moray Bequest Fund towards the expenses incurred in the above experiments.

CHEMISTRY DEPARTMENT,  
UNIVERSITY OF EDINBURGH.

(Issued separately November 29, 1916.)

XII.—Note on the Sublimation of Sugars.\* By Sudhamoy Ghosh,  
D.Sc. (Edin.). Communicated by Professor J. WALKER, F.R.S.

(MS. received December 8, 1915. Read January 10, 1916.)

THE literature of the sugars would appear to show that no sugars have hitherto been observed to sublime with the exception of glycolose,  $\text{CH}_2\text{OH} \cdot \text{CHO}$ , which is described as being "perceptibly volatile with water and alcohol vapour under diminished pressure, especially from a pure, concentrated solution" (Lippmann, *Chemie der Zuckerarten*, p. 4). This substance, being the first of the sugar series, might be expected to have properties somewhat different from the other members of the series, which are usually looked upon as non-volatile. Experiments with rhamnose and fructose appear to show that under diminished pressure they do sublime.

RHAMNOSE.

In an experiment in which a sample of rhamnose (Kahlbaum) was heated under diminished pressure in the steam-jacketed drying apparatus described in a paper by Mackenzie and Ghosh (*Proc. Roy. Soc. Edin.*, 1914-15, vol. xxxv, p. 22) a sublimate was obtained, which had a sweet taste and charred on heating.

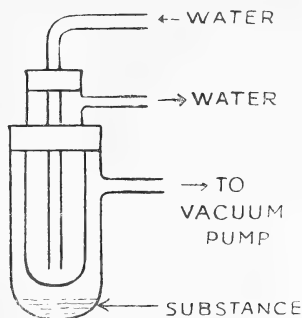
In further experiments a sample of rhamnose hydrate, for which I am indebted to Professor Walker, was used. This sample gave the following data:—Melting-point  $82^\circ$  to  $92^\circ$ ;  $[\alpha]_{\text{D}}^{20} = +8.33^\circ$  (Fischer and Piloty give for the monohydrate  $[\alpha]_{\text{D}}^{20} = +8^\circ$  to  $9^\circ$  [*Ber.*, 1890, 23, 3102]); on analysis:—

Found.	Calc. for $\text{C}_6\text{H}_{12}\text{O}_5, \text{H}_2\text{O}$ .	Calc. for $\text{C}_6\text{H}_{12}\text{O}_5$ .
C=39.38 per cent. H= 7.71    „	39.55 7.69	43.90 7.31

The sublimation was carried out in a simple apparatus consisting of a larger and a smaller test-tube, each having a side tube, the inner test-tube

\* [The author's departure for India prevented him continuing the work of which this is a preliminary note. The Council considered that in the circumstances the observation was worthy of being recorded.—C. G. K., Sec.]

being used as a condenser, cold water passing through it, the arrangement being as in the diagram. The sugar was placed in the bottom of the outer test-tube, which was then immersed in an oil-bath. With a pressure of 1



to 2 mm. and a temperature of  $105^{\circ}$ , sublimation took place, a transparent layer forming on the lower end of the condenser tube. Analysis of some sublimate kept over phosphoric anhydride for two days (I) and for seven days (II) gave the following results:—

Found.		Calculated	
I.	II.	for $C_6H_{12}O_5, \frac{1}{4}H_2O$	for $C_6H_{12}O_5$ .
C = 42.90 H = 7.39	42.93 7.38	42.73 7.42	43.90 7.31

The substance being hygroscopic may account for the discrepancy from the theoretical values for the anhydrous substance, but it is curious that the figures coincide so closely with those required for  $C_6H_{12}O_5, \frac{1}{4}H_2O$ . The quantity of sublimed substance being small, the specific rotation was measured in a one-decimetre tube of about one cubic centimetre capacity. To test the accuracy of such measurement, a comparison of the values obtained with this tube and those obtained with a two-decimetre jacketed tube using a less pure specimen of rhamnose hydrate was made, and the values found to be in close agreement. The values for the sublimed substance were:—

$$\text{For conc.} = 4.1266 \quad . \quad . \quad . \quad [\alpha]_{\frac{12.5}{D}} = +9.21^{\circ}$$

$$\text{For conc.} = 4.000 \quad . \quad . \quad . \quad [\alpha]_{\frac{12}{D}} = +9.00^{\circ}$$

which closely agrees with the value obtained by Fischer:—

$$\text{For conc.} = 3.4208 \quad . \quad . \quad . \quad [\alpha]_{\frac{20}{D}} = 9.24^{\circ}$$

(Ber., 1895, 28, 1162).

Further confirmation of the identity of this sublimate with anhydrous rhamnose was given by the formation of the phenylosazone, the needle-shaped crystals melting at 179–180°, whereas the melting-point given by Fischer is 180°.

There can therefore be no doubt that the sublimate thus formed is anhydrous rhamnose.

It was thought advisable to examine the melt which remained in the outer tube, and from the results of the examination it is clear that it is also anhydrous rhamnose, though it is difficult to explain the variation in specific rotation. The analysis:

Found.	Calc. for $C_6H_{12}O_5$ .
C=43.75 H= 7.26	43.90 7.31

showed close concordance between the observed and the calculated values. The rotation values were:—

$$\text{For conc.} = 2.932 \quad . \quad . \quad . \quad [\alpha]_{\text{D}}^{11} = 5.80^\circ$$

$$\text{For conc.} = 4.904 \quad . \quad . \quad . \quad [\alpha]_{\text{D}}^{12} = 3.65^\circ$$

Neither this substance nor the sublimate showed mutarotation. The phenylosazone obtained from the melt was identical with that from the sublimate.

#### FRUCTOSE.

The low melting-point of this sugar suggested that it might behave similarly to rhamnose, and preliminary experiments show that at from one to two millimetres pressure and about 100° fructose does sublime, though much less rapidly than is the case with rhamnose.

CHEMISTRY DEPARTMENT,  
UNIVERSITY OF EDINBURGH.

(Issued separately November 30, 1916.)



XIII.—On the Size of the Particles in Deep-sea Deposits. By  
Dr Sven Odén, Uppsala. *Communicated by THE GENERAL  
SECRETARY.*

(MS. received December 13, 1915 ; lost by fire May 15, 1916. Second copy received  
July 1916. Read January 24, 1916.)

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I. INTRODUCTION.

THE sediments which carpet the floor of the ocean have hitherto chiefly been subject only to biological and chemical investigations, undertaken either in order to determine the character of their organic residues or to ascertain their chemical composition. Very few authors have investigated the *mechanical* constitution of deep-sea deposits, *i.e.* the numbers and dimensions of their particles, and the few papers dealing with that subject which have hitherto been published are based on measurements of only very few groups of particles.

By means of Schöne's "elutriation method" O. B. Bøggild\* has separated four groups of particles of the following diameters: >0·5 mm.; 0·5 to 0·05 mm.; 0·05 to 0·02 mm.; <0·02 mm., from the deposits collected by the Danish Ingolf-Expedition and has measured their amount. J. Thoulet† has made some of the Atlantic deposits collected by S.A.S. the Prince of Monaco subject to a mechanical analysis by means of a set of sieves. Evidently the results of a similar separation process with subsequent weighing of the different fractions can only give a very incomplete idea of the real composition of a deposit, unless the number of fractions isolated is very large, which again would require a great deal of labour. In his well-known

\* *Den danske Ingolf-Expedition*, Bd. i, 3, Havbundens aflagringer af O. B. Bøggild, S. 18-24, Kjøbenhavn, 1899.

† *Résultats des campagnes scientifiques, etc., par Albert I*, fasc. xix, "Etude des fonds marins, etc.," par J. Thoulet, Monaco, 1901. See also J. Thoulet, "Analyse mécanique des sols sous-marines," *Annales des Mines*, avril 1900.

text-book on oceanography Krümmel also remarks: "The present state of mechanical analysis of deep-sea deposits is therefore anything but satisfactory. However, an improvement appears to be in progress, and we especially hope for a thorough revision of the whole method." In the sequence he recommends an application of the method of E. A. Mitscherlich,\* viz. to determine the total surface area of one gramme of the deposit by measuring its hygroscopic power instead of by weighing the different fractions as in earlier investigations.

However, apart from the serious objections which can be raised against the theoretical side of Mitscherlich's method, it is obvious that a determination of the surface is by no means sufficient if we want to define a deposit. For example, a sample consisting of coarse sand intermingled with some high-colloidal clay may have the same total surface area as a loamy deposit consisting of more uniform particles of intermediate size, yet it will in most other respects differ profoundly from the latter, so that there are no reasons for classing them together.

An ideal characterisation can evidently not be attained by dividing the sample into a smaller or greater number of groups of various sizes or by determining their hygroscopic power or total surface, but only by obtaining a continuous *distribution-curve* of the well-known Maxwellian type, viz. by plotting a variable representing in some way the weight or number of the individual particles against another variable representing their linear dimensions.

The following investigation has been undertaken by me in order to solve this problem, especially with the intention of finding a successful mechanical analysis of the medium-sized and the finest particles contained in some deep-sea deposits.

These samples, which belong to the collection of the late Sir John Murray, were put at my disposal by the courtesy of Mr J. Chumley of the *Challenger* Office, to whom I here wish to express my indebtedness.

Although the new method has already been applied by me to the study of certain continental soils, it is advisable to give here a brief description of its main features.†

## II. THEORETICAL DISCUSSION.

It has already been pointed out that an immense amount of labour would be required to split each sample up into a series of fractions (each consisting of particles of approximately the same size), sufficiently numerous

\* *Bodenkunde*, Berlin, 1905, S. 56.

† For further particulars see *Internat. Reports on Pedology*, vol. v, 257-312 (1915).

for the construction of a "distribution-curve."\* I have therefore found it necessary to evolve a new method by which a certain characteristic physical property of the sample is studied without any previous mechanical separation into different fractions. For this property, which must obviously be a function of the size of the particles in the sample, *i.e.* a function of their "distribution," I have taken the rate of sedimentation from an aqueous suspension. The weight of sediment which settles down on a certain bottom-area during a unit of time gives a quantitative measure of the aforesaid property. Plotting this (increasing) weight against time, one obtains a "precipitation-" or "accumulation-curve" from which the "distribution-curve" may be calculated by a mathematical analysis.

From a colloid-chemical point of view we may define the aqueous suspensions of bottom-sediments as "coarse disperse systems." It may be mentioned in this connection that The Svedberg has described a method for measuring the distribution of size in disperse systems, either by measuring the different rates at which a number of particles sink in the suspension, or by measuring the amplitudes of their Brownian motion. The first method was employed by The Svedberg and Knud Estrup† with the aid of a microscope, and curves representing the number of particles as a function of their size ("distribution-curves") were obtained for certain suspensions such as milk, cream, and vegetable extracts of different kinds, etc. So far as I know, these are the only distribution-curves for disperse systems which have hitherto been published. This microscopic method fails with the bottom samples, because a considerable part of their particles sink much too rapidly for direct measurements, so that a special method, simpler and more effective, had to be devised for these samples.

We will first consider the rate at which a small spherical particle sinks through a liquid. A theoretical investigation by Sir G. G. Stokes‡ (1845) has given the well-known equation:

$$v = \frac{2}{9}gr^2 \frac{\sigma - \sigma_1}{\eta} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{or} \quad v = Cr^2 \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

where  $v$  = rate of fall,  $g$  = gravitation-acceleration,  $r$  = radius of the particle,  $\sigma_1$  = density of the liquid,  $\sigma$  = density of the particle,  $\eta$  = viscosity of the

\* This term is used in the Maxwellian sense, *i.e.* to represent how the number of particles of a certain size varies with that quantity, or rather how the particles are *distributed* amongst the different sizes.

† "Ueber die Bestimmung der Häufigkeitsverteilung der Teilchengrößen in einem dispersen System," *Koll. Zeitschr.*, ix, 259-261 (1911).

‡ *Cambr. Phil. Trans.*, viii, 287 (1845); ix, 8 (1851); *Mathematical and Physical Papers*, vol. i, 75.

liquid, so that the multiplier of  $r^2$  is a constant dependent on the nature of the liquid and of the sphere. Consequently, by measuring  $v$  in some way or other we can calculate the value of  $r$ , *i.e.* the size of the particle.

This purely theoretical law has been experimentally investigated by H. S. Allen,\* H. D. Arnold,† I. Nordlund,‡ and others. For particles of dimensions exceeding a certain limit it has been found that the actual velocity is *higher* than its theoretical value. Thus Allen has found that quartz spherules in water fall strictly according to the law, provided their radius does not exceed  $85\mu$  ( $=0.085$  mm.). Consequently, if the experiments should give a value for the radius *higher* than  $85\mu$ , we can only say that the true value does not *exceed* that found from the experiments.

We will now consider the case of a particle which is not spherical. It is then rather difficult to define its dimensions, especially if its shape is very irregular. In order to evade this difficulty, it is convenient to define a new quantity, the "*effective radius*," *i.e.* the radius of a perfect sphere of the same material which sinks at the same average rate as the particle in question, the latter being supposed to retain during its fall a certain orientation with respect to its line of motion. The case of a particle shaped like an *ellipsoid of rotation* may be given here, the effective radii for a fall parallel to either *axis*,  $a$  and  $b$ , being : §

$$R_a = \frac{a}{b} \left( \frac{a^2}{b^2} - 1 \right) + \left( 3 - 2 \frac{a^2}{b^2} \right) \sqrt{\frac{a^2}{b^2} - 1} \log \left( \frac{a}{b} - \sqrt{\frac{a^2}{b^2} - 1} \right)$$

$$R_b = 2 \frac{a}{b} \left( \frac{a^2}{b^2} - 1 \right) + 2 \left( 2 \frac{a^2}{b^2} - 1 \right) \sqrt{\frac{a^2}{b^2} - 1} \log \left( \frac{a}{b} - \sqrt{\frac{a^2}{b^2} - 1} \right).$$

That the effective radius must vary according to the orientation of the particle is especially obvious in the case of particles shaped like discs or rods. One would perhaps for this reason be inclined to consider any attempt to analyse sediments as hopeless. This conclusion would be true for extremely small samples consisting of a very limited number of particles, or for a suspension of small depth where the total distance through which each particle falls is very short. The present case is, however, quite different, as the number of particles of a certain size present in the sample is extremely large, so that we are not measuring the effective radius of any single particle, but are determining the *mean* effective radius of the

\* "The Motion of a Sphere in a Viscous Fluid," *Phil. Magazine* (5), 1, 323-338 (1900).

† "Limitations imposed by Slip and Inertia Terms upon Stokes's Law for the Motion of Spheres through Liquids," *Phil. Magazine* (6), xxii, 755-775 (1911).

‡ "Ueber die Gültigkeit des Stokes'schen Gesetzes, etc.," *Arkiv f. Matematik, etc.*, edited by K. Svenska Vet. Akad. i Stockholm, ix, Nr. 13 (1913).

§ Viz. The Svedberg, "Ueber die Gestalt der Moleküle," II, *Arkiv f. Kemi, etc.*, utg. av K. Svenska Vet. Akad. i Stockholm, v, Nr. 11 (1914).

enormous number of particles which enter into the composition of even a comparatively small fraction of the sample.

As the probability of a certain orientation is about the same for all the particles, we may assume that the average rate of fall for a large number of particles (which is the only quantity measured by the new method) will give a fairly accurate value of their mean effective radius. Also the orientation of the smaller particles will vary considerably during their fall, owing to an irregular Brownian rotation due to molecular impacts, so that the resistance encountered by each of these particles will undergo incessant variations.

In cases where the shape of the particles deviates considerably from that of an ellipsoid of rotation, the relationship between the effective radius and the real dimensions of the particle remains, of course, unknown. However, by stating the effective radius of a particle, or rather the mean effective radius of not excessively heterogeneous fractions, we obviously define a quantity which will on the whole give a fairly accurate idea of the real size of the particles.

As a further argument in favour of this reasoning and of the applicability of Stokes's law to the process of sedimentation, I have given in Table I the effective radius of certain sediments (silt and clay) investigated by A. Atterberg,\* viz. the values measured directly by him with the microscope, compared with those found by calculation according to Stokes's law from the time required for their sedimentation.†

As the Swedish soils considered are chiefly made up from practically unaltered fragments of feldspar, mica, and quartz, I have taken the average density of the particles to be 2·7.

TABLE I.

*Time of Sedimentation for Fall through 10 cm., according to Atterberg.*

$t$  = Time of fall through 10 cm.

$v$  = Velocity in cm./sec.

$r$  = Mean effective radius calculated according to Stokes's law.

$r'$  = Mean radius measured with the microscope by Atterberg.

$t$	$v$ in cm./sec.	$r$	$r'$
5 seconds	2	$78\mu$	$100\mu$
50 seconds	0·2	$24\cdot8\mu$	$30\mu$
7 minutes 30 seconds	$222\cdot2 \cdot 10^{-4}$	$8\cdot3\mu$	$10\mu$
1 hour	$27\cdot78 \cdot 10^{-4}$	$2\cdot9\mu$	$3\mu$
8 hours	$3\cdot472 \cdot 10$	$1\cdot03\mu$	$1\mu$

\* "Die mechanische Bodenanalyse und die Klassifikation der Mineralböden Schwedens," *International Reports on Pedology*, ii, 319 (1912).

† See also A. D. Hall, *Journ. of Chem. Soc. Trans.*, lxxxv, 959 (1904).

Considering the difficulties of making exact measurements with the microscope, the agreement must be regarded as satisfactory. In this table the value of the viscosity at 15° C. is taken to be  $\eta=0.0114$ . Considering that the viscosity (which enters into Stokes's equation) decreases rapidly with temperature, viz. 46 per cent. between 10° C. and 25° C., it is of vital importance to measure the temperature and to introduce a corresponding correction into the mechanical analysis of soils—a fact which seems to have been completely overlooked by previous authors.

If we now take the case of a suspension of particles of different sizes contained in a cylindrical vessel, we can, by an experimental arrangement described below, measure how the weight of the deposit on the bottom,  $P=f(t)$ , increases with time, the concentration of the suspension being supposed to be uniform at the start,  $t=0$ . Evidently  $f(t)$  or the curve of accumulation must depend on the height of the liquid,  $h$ , the total number of the particles, as well as their dimensions, and on their distribution.

It follows from a simple reasoning, which I have also verified by experiments, that the rate of accumulation, or  $\frac{dP}{dt}$ , is a linear function both of the total number of particles and of the depth of the liquid. Therefore, if all weights measured during the sedimentation of a sample are expressed in percentages of its *total* weight taken as an arbitrary unit,  $P_{\infty}=100$ , and if the results are further corrected to a standard depth of liquid, 10 cm. (the times observed being multiplied by  $h/10$ ), then the result will be independent both of the absolute weight of the sample,  $P_{\infty}$ , and of the actual value of the depth of the liquid column.\*

Before entering on the mathematical transformations required for finding the "distribution-curve" from the "accumulation-curve" given by the experiments, I shall give a brief description of the experimental arrangements used.

### III. EXPERIMENTAL ARRANGEMENTS.

In order to measure the rate of deposition from an aqueous suspension of the sediment accumulating on the bottom of the vessel, I have constructed an apparatus which, after having undergone various improvements, gives fairly exact results. As the instrument is still open to improvements in several respects, especially with regard to making it self-recording so as to

\* For further particulars of these investigations, and of the limits within which these simplifications can be assumed to give correct results, see my paper in *International Reports on Pedology*, v, 257–312 (1915).

avoid subjective errors of observation, I shall reserve the detailed description of the instrument and its technique for further publication, and give here only the main principles on which it is based.

Suppose an aqueous suspension of the sample to be contained within a cylindrical vessel, in which, close to the bottom, there is a thin circular disc of slightly smaller diameter suspended from one arm of a sensitive balance. On that disc will then be accumulated all the deposit which settles from a liquid column having the upper surface area of the disc for its base, and a height equal to the vertical distance from that surface to the free surface of the liquid. After a sufficiently long time all the particles originally in the liquid column will have accumulated on the disc, and their joint weight,  $P_{\infty}$ , will be slightly less than the total weight,  $P$ , of the sample used for the suspension (owing to the fact that the particles outside the circumference of the disc, as well as those contained within the space below it, will not fall on the disc but on the bottom of the vessel). By means of the balance we can measure from time to time how the weight of the deposit accumulated on the disc increases with time, and plot an "accumulation-curve" of the type mentioned above.

Taking a mean value of the density of the sample,  $\sigma$ , and always expressing the accumulated weights in percentages of the total weight (the latter measured after all particles have settled, if necessary after coagulation), we need not consider the weight of the deposit in air.

Fig. 1 gives a schematic idea of the experimental arrangements. The disc A was of thin copper foil, heavily gilt, with a surface area of 100 cm.<sup>2</sup> It was suspended from the arm of a sensitive balance by a very fine wire of gilt silver, so as to be quite close to the bottom of the cylindrical vessel L. As it is indispensable for an undisturbed accumulation of the sediment that the distance  $h$  between the disc and the surface of the liquid remain practically unaltered throughout the experiment, and also because movements of the disc will give rise to errors in the weighings due to convection currents, it was necessary to use a compensating arrangement (K, to the right on the figure) limiting the displacements of the disc to a minimum. When the particles accumulated on the disc outweigh the counterweight G, so that the disc begins to sink from its zero position, an electric current becomes automatically closed by the mercury contacts at E. A small Osram lamp, O, in the circuit then lights up for a moment, at which the time is read off a chronograph, and simultaneously a small counterweight is added to G. Both these operations may be executed either by the observer or automatically by means of an electromagnetic arrangement. Before the beginning of each experiment the balance is brought to its zero

position by means of tara and sliding weight. The balance is then arrested, the water is removed by means of a siphon, H, and a new sample suspended in water is poured into the vessel as quickly as possible. The balance is now released, and the chronograph started, after which the observer has simply to record the values of the counterweights added, as well as the moments when the Osram lamp lights up.

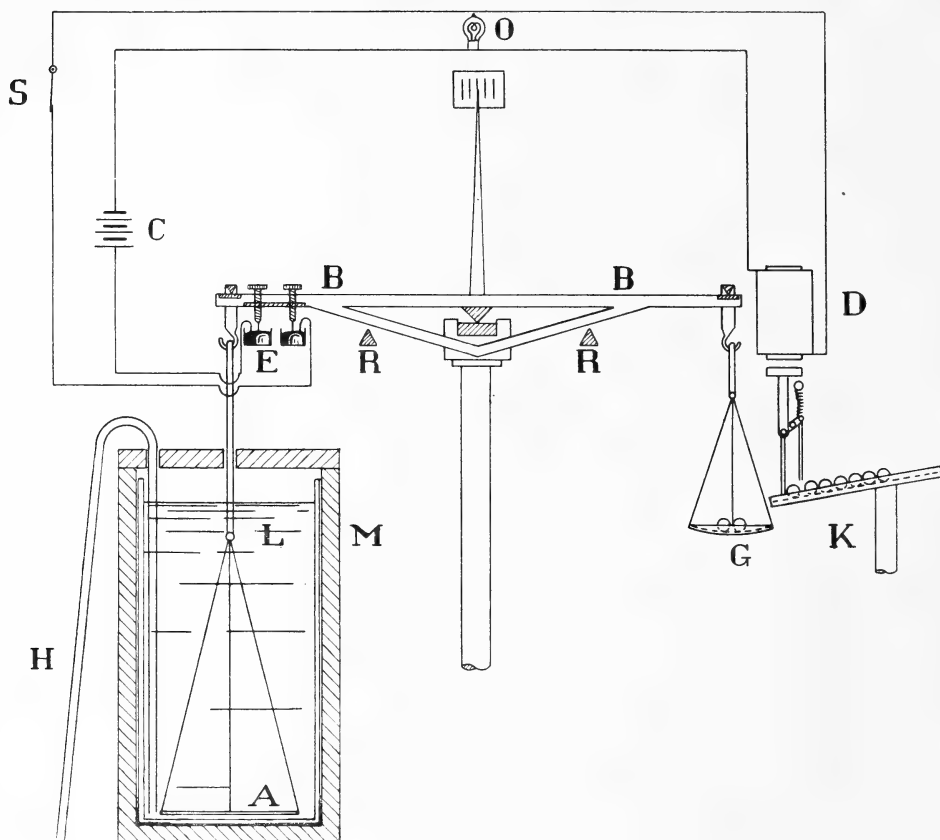


FIG. 1.

In order to avoid convection currents due to variations of the temperature, all the experiments were carried out in the room for constant temperature in the Chemical Institute at Uppsala, the aqueous suspensions being always allowed to assume the prevailing temperature of the room. The cylindrical vessel was kept within a non-conducting envelope, M, and covered with a lid which had a small aperture for the suspension-wire from the balance to the disc. Fig. 2 gives a photograph of the arrangement.

From observations made with the apparatus just described we can plot



a certain "accumulation-curve" characteristic of each sample; and a mathematical analysis, which I shall now briefly describe, allows us to

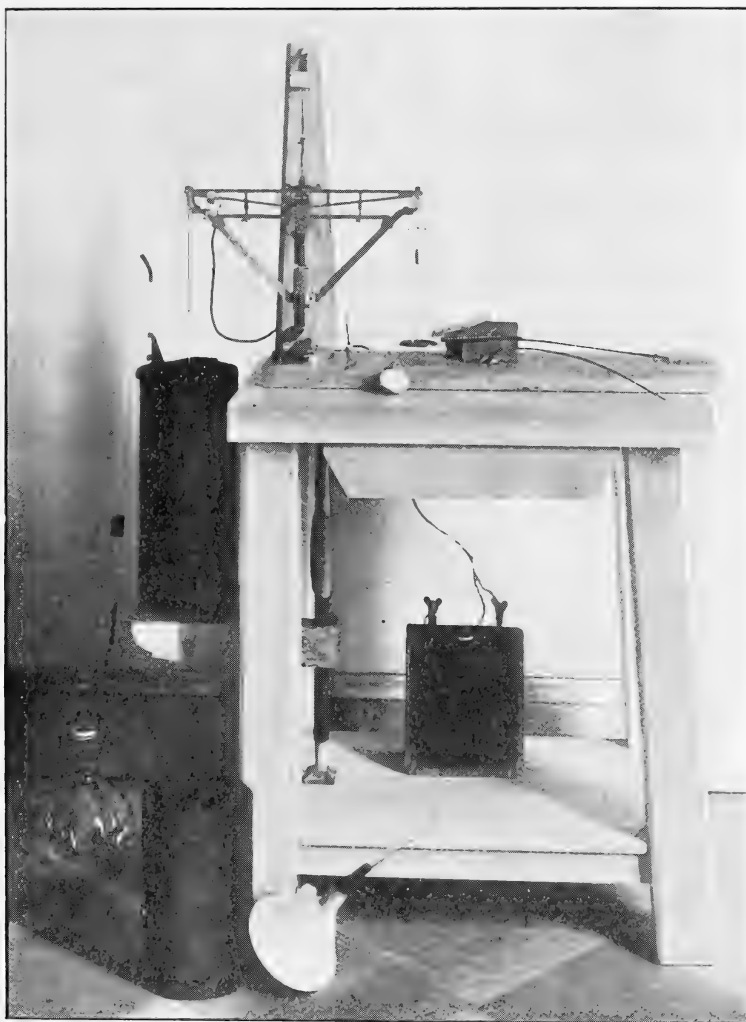


FIG. 2.

transform this accumulation-curve into a "distribution-curve," *i.e.* a graphical representation of the statistical composition of the sample.

#### IV. MATHEMATICAL ANALYSIS OF THE ACCUMULATION-CURVES.

Thanks to the method described in the preceding paragraphs, we are now able to construct a new type of curve, the "accumulation-curve,"

the properties of which are characteristic of each individual sample. In another paper I have also proved that these curves are independent of the depth of the liquid,  $h$ , and also of the total quantity,  $P_\infty$ , of the suspended matter, provided that all weights are expressed in percentages of the latter value, and that the observed intervals of time are reduced to the standard depth of 10 cm. by multiplication by  $10/h$ .

I shall now proceed to a closer analysis of the accumulation-curve.

By  $q(r)$  I denote the quantity of particles the effective radii of which exceed a certain value  $r$ . Expressing  $q(r)$  in per cent. of the total quantity,  $P_\infty$ , we find that  $\Delta q$  per cent. of the particles have an effective radius of between  $r$  and  $r + \Delta r$ ,  $\Delta q$  being the decrease in the value of  $q$  which corresponds to a small increase,  $\Delta r$ , in the argument. The function  $q(r)$  obviously decreases from its maximum value of 100 (corresponding to the smallest particles in the sample) to 0 (for the largest value of  $r$ ).

It is now convenient to define another function,  $F(r)$ , more susceptible to mathematical transformations, by which the quantity of particles itself is graphically represented by an *area*. Taking  $\Delta r$  quite small, we have

$$\lim_{\Delta r \rightarrow 0} \frac{\Delta q}{\Delta r} = -\frac{dq}{dr}$$

(the negative sign is retained because  $q$  decreases with growing values of  $r$ ).

Calling  $-\frac{dq}{dr} = F(r)$ , we can, by integrating the function  $F(r)$  between  $r_1$  and  $r_2$ , find the percentage of such particles as have values of the effective radius comprised within these limits.

$$q_1 - q_2 = \int_{r_2}^{r_1} \frac{dq}{dr} dr = \int_{r_1}^{r_2} F(r) dr \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It remains to be shown how this curve  $F(r)$ , which may conveniently be called the "distribution-curve," can be found from the accumulation-curve given by the experiments.

Consider a very small interval of  $F(r)$  in the vicinity of  $r$ ; then  $F(r)dr$  will represent the fraction of the particles which has an effective radius between  $r$  and  $r + dr$ . After the lapse of a certain time,  $t$ , a part  $\phi(r)$  of this fraction will have fallen to the bottom.

Then

$$\phi(r) = \frac{v}{h} F(r) dr t \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$v$  being the velocity with which the particles of radius  $r$  sink through the liquid, and  $h$  the height from bottom to surface.



Substituting  $\rho$  for  $\sqrt{\frac{h}{Ct}}$  and solving  $\rho$  from this equation, we have

$$F(\rho) = -2\sqrt{\frac{C}{h}} \cdot t^{5/2} \frac{d^2P(t)}{dt^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

$$\rho = \sqrt{\frac{h}{Ct}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8b)$$

We thus have arrived at an expression for  $F(\rho)$  (*i.e.* for the total quantity of particles with an effective radius  $\rho$ ), where  $F$  is expressed in terms of  $t$  and of the second derivative of the experimental "accumulation-curve"  $P(t)$  taken at the same moment  $t$ . The radius itself is also expressed in terms of  $t$  together with constant quantities.

The simplest way is now to plot the logarithm of the first derivative,  $z = \log \frac{dP(t)}{dt}$ , against  $x = \log t$ . We can then find graphically from this auxiliary curve the value of

$$\frac{dz}{dx} = \frac{\frac{d^2P(t)}{dt^2}}{\frac{dP(t)}{dt}} \cdot t \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Again, the equation (8a) can be written in terms of this auxiliary curve

$$F(\rho) = -\frac{2t}{\rho} \cdot \frac{dP(t)}{dt} \cdot \frac{dz}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8c)$$

For any given value of the radius  $\rho$ , we obtain from equation 8b the corresponding value of  $t$ , whereas the two last factors are easily found from the auxiliary curve by a graphic construction.

For further details of the technique of the necessary mathematical operations, reference may be made to my original paper\* quoted above.

## V. DISCUSSION OF THE RESULTS.

As too much space would be required were my original observations to be reproduced *in extenso*, I have given at the end, in Tables II and III, as a typical example, only the observations and the results calculated for a sample of red clay from the Pacific Ocean. From 60 to 110 observations were taken with the other deposits, according to the character of the sample. In all cases the depth of the liquid was  $h = 20$  cm. The measurements were made (May 5th to 25th, 1915) by my assistant, Mr Gustav Larsson, under my supervision and control.

Each sample was treated in the following manner. According to the

\* *International Reports on Pedology*, v, 257-312 (1915).

method of Beam\* and Atterberg,† the sample was carefully rubbed under water with a stiff brush, after which a trace of ammonia was added to the suspension. After thoroughly investigating different methods of treatment, I have found this method to be the most efficient for obtaining the "ultimate particles." The old method of boiling the sample with water for hours is quite unsuitable, as the finest particles will then get partly aggregated, whereas the largest particles may be split up into smaller fragments.

In Plate I is given the accumulation-curve for the sample of red clay (according to the third and fourth columns of Table II), and in the first figure of Plate II the distribution-curve constructed from the former (according to Table III). In order to reduce the latter curve to more convenient dimensions, I have found it necessary to plot  $r \cdot F(r)$  against  $\log r$  (instead of  $F(r)$  against  $r$ ). However, as  $d(\log r) \cdot r \cdot F(r) = \frac{1}{r} dr \cdot r \cdot F(r) = F(r) dr$ , the area between any two given ordinates will also in this case represent the number of particles having an effective radius between the same limits.

It is seen from the curve that 96 per cent. of the total amount have values of the effective radius of from  $2\mu$  to  $16\mu$  ( $1\mu = 0.001$  mm.), whereas 4.1 per cent. are still smaller,  $r < 2\mu$ . In the curves, this last quantity, the very finest particles, is everywhere represented by a square at the left end of the curve. It is obviously not always possible to prolong the experiments until all particles have been deposited. It is therefore in some cases necessary to coagulate the very finest particles by adding an electrolyte to the suspension, and then to measure their joint weight.

This is conveniently done after, say, twenty-five hours' observation by removing the liquid with the aid of a siphon, after which the vessel is cleaned and the liquid again introduced, together with some electrolyte, *e.g.*  $\text{BaCl}_2$ . Even the very finest particles are then precipitated in the course of a few hours, so that their weight can be measured.

On the other hand (as has already been mentioned), the amount of those particles which fall too rapidly to be measured by my method is represented by a square to the right in the figures.

It is worthy of note that, strictly speaking, only the full-drawn parts of the curves are real "distribution-curves" in the proper sense, the squares serving only to demonstrate the amount of very fine and very large particles

\* W. Beam, *Fourth Report of the Wellcome Tropical Research Laboratories*, Khartoum, Sudan, 1911, p. 37.

† A. Atterberg, *International Reports on Pedology*, ii, 314 (1912).

which evade actual measurement. If correctly and completely drawn, these parts should obviously approach the axis almost asymptotically, and finally coincide with it for certain values of  $r$ , corresponding respectively to the largest and the smallest particle in the sample. The total surface enclosed by them would be equal to the shadowed areas in the figures.

The following deposits were investigated:—

1. Plate II, first figure. Red clay; Pacific Ocean: lat.  $13^{\circ} 28' S.$ , long.  $144^{\circ} 30' W.$  *Challenger* Station 276; depth 2350 fathoms.

No data for the mean specific density of these samples being obtainable, I measured it myself by means of a pycnometer:  $\sigma = 2.17$ .

The distribution-curve is seen to have a well-developed maximum corresponding to a radius of  $12\mu$ —in fact, the most pronounced maximum found in any of the curves I have studied. By far the largest part of the particles, viz., 89 per cent., are comprised within the comparatively narrow limits  $16\mu$ – $5\mu$ , whereas the percentage of very fine particles is quite small, 4.1 per cent., and that of very coarse particles almost vanishing.

2. Plate II, second figure. Red clay; Atlantic Ocean: lat.  $24^{\circ} 20' N.$ , long.  $24^{\circ} 28' W.$  *Challenger* Station 5; depth 2740 fathoms. Mean specific density 2.03.

This curve showed marked differences from No. 1. The number of very fine particles is much higher, no less than 44.3 per cent. falling below  $1.1\mu$ , so that I have only been able to measure their joint weight after coagulation. On the other hand, a comparatively large percentage of the particles are more or less evenly distributed over the larger sizes, the number having  $r > 16\mu$  being 8 per cent., whereas the Pacific red clay had no particles larger than  $16\mu$ .

3. Plate II, third figure. Atlantic Globigerina ooze; Atlantic Ocean: lat.  $21^{\circ} 15' S.$ , long.  $14^{\circ} 2' W.$  *Challenger* Station 338; depth 1990 fathoms. Mean specific density 1.72.
4. Plate II, third figure. Pacific Globigerina ooze; Pacific Ocean: lat.  $38^{\circ} 6' S.$ , long.  $82^{\circ} 2' W.$  *Challenger* Station 296; depth 1825 fathoms. Mean specific density 2.28.

A distinctly new type of curve is found for both samples of Globigerina ooze, in spite of the widely different localities from which the samples were collected. There are two pronounced maxima separated by an empty space, *i.e.* no particles whatever of intermediate size were to be found in the sample. On the other hand, it is seen that the Pacific ooze has its first maximum considerably smaller and situated at a smaller value of  $r$ , whereas

its second maximum is larger and falls at higher values of  $r$  than the corresponding maxima of the curve P for the Atlantic sample.

5. Plate III, first figure, A. Radiolarian ooze; Pacific Ocean: lat.  $7^{\circ} 25' S.$ , long.  $152^{\circ} 15' W.$  *Challenger* Station 274; depth 2750 fathoms. Mean specific density 2.02.

The curve for the sample of Radiolarian deposits is notably free from any maxima whatsoever in the interval ( $16.5\mu$ – $1.1\mu$ ) within which direct observations have been made. The comparatively large fraction of still smaller particles (56.3 per cent.) may of course have a maximum somewhere between 0 and  $-\infty$ ,\* although there is no indication of it in the curve.

6. Plate III, first figure, P. Radiolarian ooze; Pacific Ocean: lat.  $3^{\circ} 48' S.$ , long.  $152^{\circ} 56' W.$  *Challenger* Station 272; depth 2600 fathoms. Mean specific density 1.70.

This Radiolarian ooze is represented by a somewhat less regular curve than the former. There is a trace of a maximum somewhere about  $2.7\mu$ , and there is a distinct rise at the left-hand side of the curve.

7. Plate III, second figure. Blue mud; Atlantic Ocean: lat.  $38^{\circ} 34' N.$ , long.  $72^{\circ} 10' W.$  *Challenger* Station 45; depth 1240 fathoms. Mean specific density 2.38.

In the rectilinear shape of the right-hand part this curve resembles somewhat that for the first Radiolarian ooze (5). Only a faint tendency to a maximum at about  $12\mu$  is to be seen. The amount of very fine particles,  $r < 0.8\mu$ , is seen to be surprisingly high. Whether a maximum is really hidden in this uninvestigated part of the curve it is of course at present impossible to say.

To draw any definite conclusions of a general character from these curves appears to be premature. It is, however, already obvious that a promising field for future research is opened by the new method, which enables us to define sharply such an important character of the bottom deposits as the size of their particles. This result will no doubt have important bearings on various interesting problems, such as the origin of the deep-sea deposits and their rate of deposition.

Attention may also be called to one interesting fact. One would *a priori* expect the deposits from the largest depth to be relatively richest in very fine particles. This is, however, not the case, as is particularly

\* Observe that  $\log r = 0$  and  $\log r = -\infty$  correspond respectively to  $r = 1\mu$  and  $r = 0$ .

evident from the curve for Red clay Pacific on Plate II, which may be compared to the curve on Plate III for the Blue mud (a terrigenous sediment deposited in much shallower water).

As a further still more striking contrast I also give on Plate III the distribution-curve for a sample of Yoldia Clay (from a layer of only 1 mm. thickness) taken at Uppsala, and probably deposited some 10,000 years ago at a depth not exceeding 150 m. in the old Yoldia Sea, which at that time occupied the space to the south from the Fennoscandian continent.

The curve is seen to run quite close to the axis at  $r=0.7$ , but rises abruptly from that point, no less than 91 per cent. of the amount having  $r < 1\mu$ .

Possibly this striking fact can be explained as due to an agglomeration to larger units of the finest particles under the enormous pressure at these depths. Another explanation might be found in a subsequent solution of the very finest particles, also furthered by the exceedingly high pressure. An investigation of this last assumption, viz. the influence of the size of a particle on its solubility, is at present in progress.

## VI. SUMMARY.

A new method for the mechanical analysis of soils and deposits has been developed. It consists in weighing, from time to time, on a specially constructed balance, the amount of sediment accumulated on a circular disc suspended from the balance near the bottom of a vessel containing an aqueous suspension of the sample. From the "accumulation-curve" thus obtained one finds by a series of mathematical operations a "distribution-curve," i.e. a curve showing how the amount of particles of a certain size varies with the latter quantity.

This method has been applied to the study of some deep-sea deposits from the *Challenger* Office. The distribution-curves thus obtained show marked differences for the different samples, and reveal a surprising lack of very fine particles in the deposits from the largest depths, besides suggesting interesting problems for new research.

UNIVERSITY OF UPPSALA, CHEMICAL INSTITUTE,  
September 1915.\*

\* The printing having been delayed by the fire at Messrs Neill & Co.'s printing works.



TABLE II.

*Accumulation of Red Clay, Pacific Ocean. See Plate I.*

W = Weight of centrepiece balancing deposit.  
t = Observed time of accumulation of deposit.  
P = Weight of deposit expressed in per cent. of P<sub>∞</sub>.  
T = Length of time in seconds corrected to standard depth  
of liquid ;  $T = \frac{10}{h}t$ .

} Accumulation-curve.

W	t	P	T	W	t	P	T
	m s				h m s		
0·512	34	7·42	17	5·88	22 40	85·25	680
0·768	57	11·13	28·5	5·93	24 30	86·02	735
0·896	1 14	13·00	37	5·98	26 12	86·75	786
1·024	1 34	14·85	47	6·03	28 13	87·47	846·5
1·152	1 55	16·70	57·5	6·08	31 20	88·20	940
1·28	2 20	18·55	70	6·10	32 15	88·49	967·5
1·38	2 38	20·00	79	6·14	33 53	89·07	1016·5
1·48	2 55	21·44	87·5	6·16	36 15	89·36	1087·5
1·58	3 15	22·90	97·5	6·18	36 40	89·65	1100
1·68	3 30	24·35	105	6·20	38 15	89·94	1147·5
1·78	3 46	25·80	113	6·22	39 58	90·23	1199
1·88	4 00	27·24	120	6·24	41 55	90·50	1257·5
1·98	4 14	28·70	127	6·26	44 08	90·78	1324
2·08	4 28	30·15	134	6·28	46 55	91·07	1407·5
2·28	4 54	33·04	147	6·30	48 50	91·35	1465
2·48	5 20	35·95	160	6·32	50 45	91·64	1522·5
2·68	5 37	38·85	167·5	6·34	53 45	91·93	1612·5
2·88	6 00	41·75	180	6·36	56 30	92·23	1695
3·28	6 46	47·55	203	6·38	1 01 35	92·50	1847·5
3·48	7 11	50·45	215·5	6·40	03 16	92·80	1898
3·68	7 38	53·35	229	6·42	06 23	93·08	1991·5
3·98	8 23	57·70	251·5	6·44	10 55	93·37	2127·5
4·28	9 18	62·05	279	6·48	19 58	93·95	2399
4·48	10 00	64·95	300	6·50	26 03	94·24	2581·5
4·68	10 48	67·85	324	6·54	38 40	94·82	2960
4·88	11 45	70·75	252·5	6·56	48 15	95·11	3247·5
5·08	12 53	73·65	386·5	6·58	2 01 22	95·40	3641
5·28	14 22	76·55	431	6·62	15 10	95·98	4055
5·38	15 15	78·00	457·5	6·64	32 32	96·25	4576
5·48	16 22	79·45	491	6·66	53 08	96·54	5194
5·58	17 25	80·90	522·5	6·72	4 21 45	97·40	7852·5
5·68	18 32	82·35	556	6·76	6 51 00	98·00	12330
5·78	20 45	83·80	622·5	6·80	16 00 00	98·55	28800
5·83	21 58	84·52	659	6·90	∞		

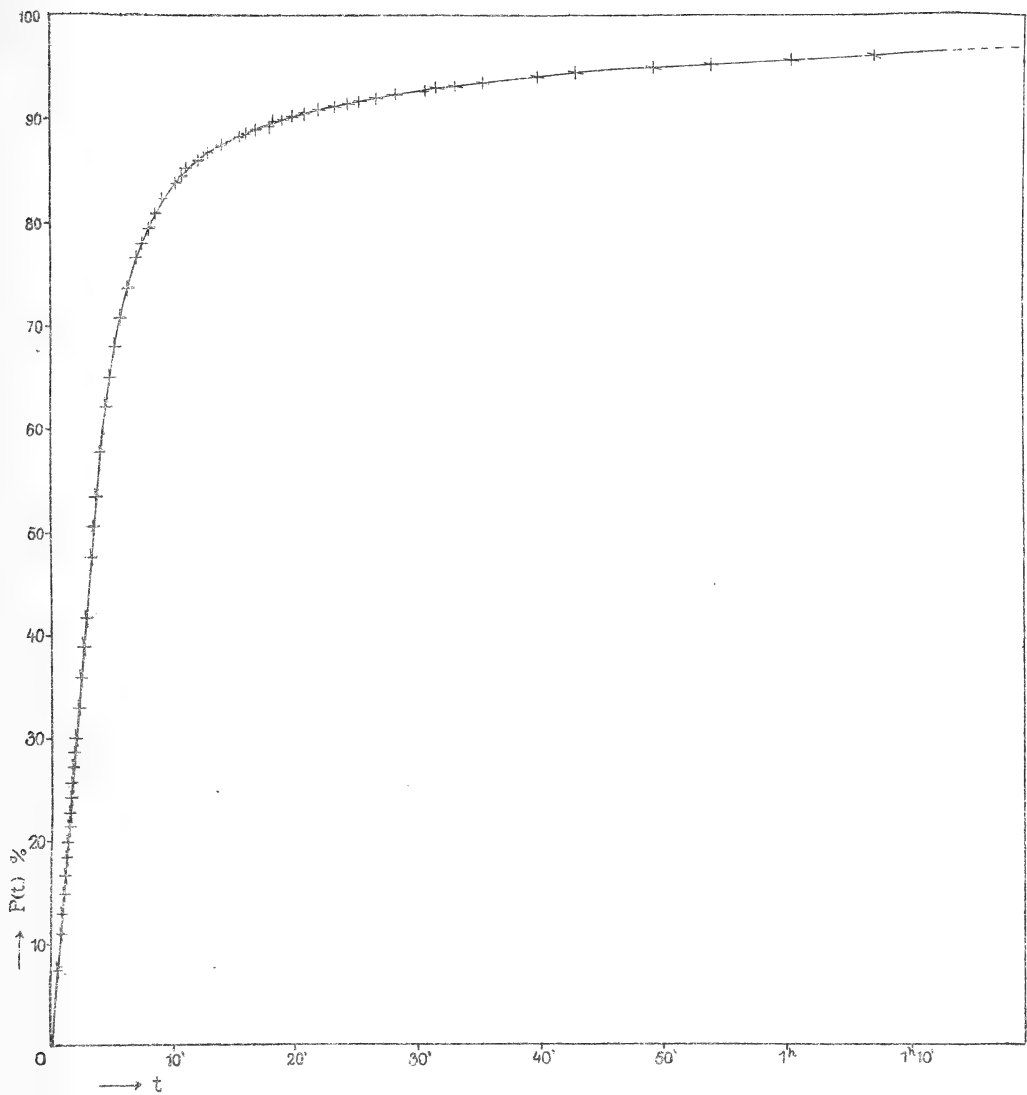
TABLE III.

*Results of Mathematical Transformations for constructing  
the Distribution-Curve.*

$r$  = Effective radius in  $\mu$  ( $= 0\cdot001$  mm.).  
 $\log t$  = Logarithm of the value of time in seconds calculated from equation (8b).  
 $\frac{dz}{dx}$ , as calculated from equation (9).  
 $F(\rho)$ , as calculated according to equation (8c).  
 $\left. \begin{matrix} \rho F(\rho) \\ \log \rho \end{matrix} \right\}$  co-ordinates of the distribution-curve ( $\log$  = Napierian logarithm).

$r$	$\text{Log } t$	$\frac{dz}{dx}$	$F(\rho)$	$\rho F(\rho)$	$\text{Log } \rho$
16	2·236	0·09	0·42	6·7	2·773
15	2·292	0·31	1·75	26·3	2·708
14	2·352	0·60	3·94	55·1	2·639
13	2·417	1·04	7·57	98·4	2·565
12	2·486	1·84	13·42	161·0	2·485
10	2·645	2·23	13·60	136·0	2·303
9	2·736	2·44	12·44	112·0	2·197
8	2·838	2·38	9·73	77·8	2·079
7	2·954	2·40	7·82	54·7	1·946
6	3·088	2·30	5·67	34·0	1·792
5·5	3·164	1·98	4·33	23·8	1·704
5	3·247	1·51	3·14	15·7	1·609
4	3·441	1·56	3·25	13·0	1·386
3	3·690	1·54	3·03	9·10	1·099
2·5	3·852	1·48	2·88	7·21	0·916
2	4·043	1·45	2·94	5·88	0·693

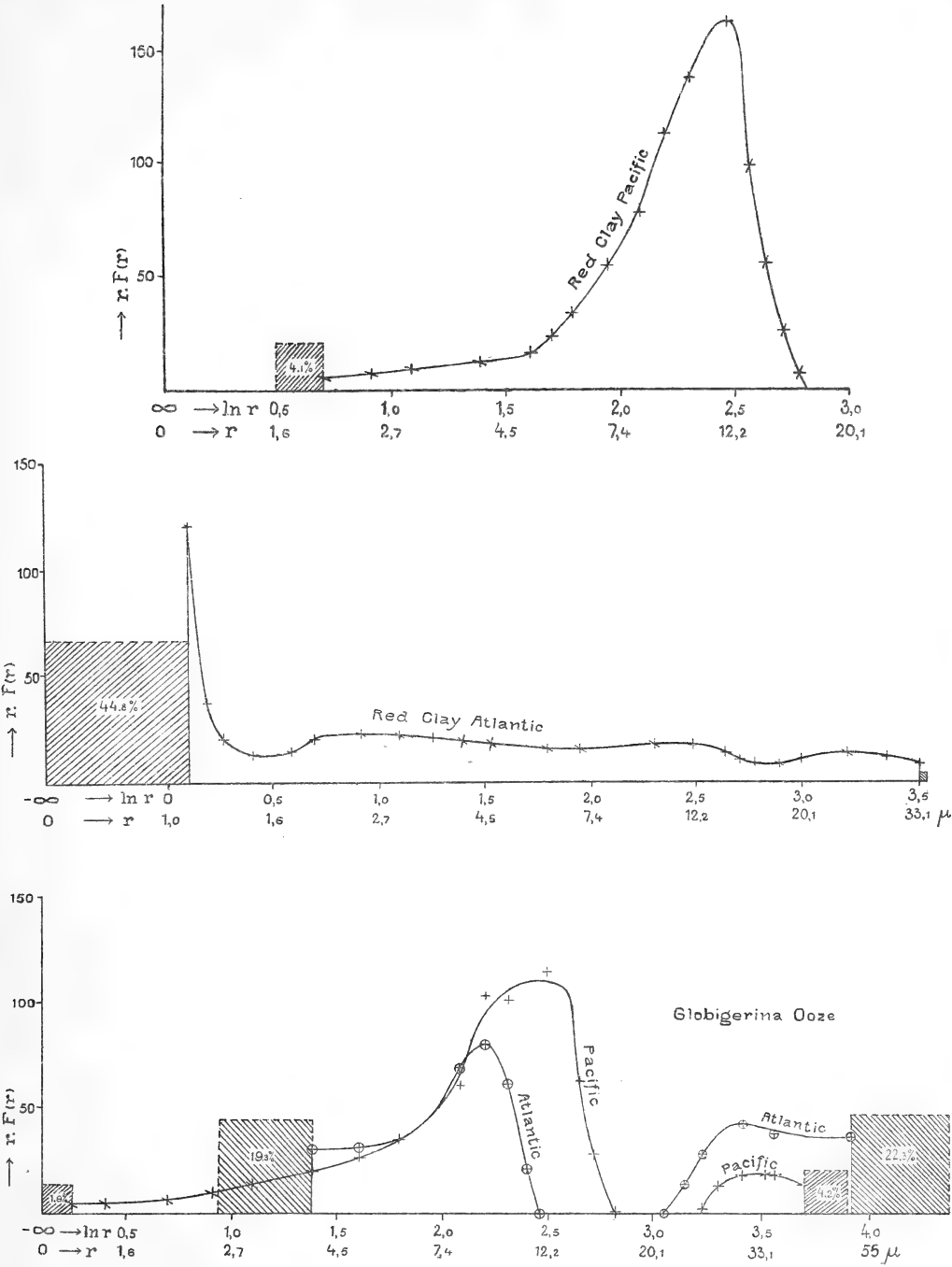
(Issued separately January 16, 1917.)



A. ACCUMULATION CURVE FOR RED CLAY.

ACCUMULATION CURVE FOR RED CLAY.

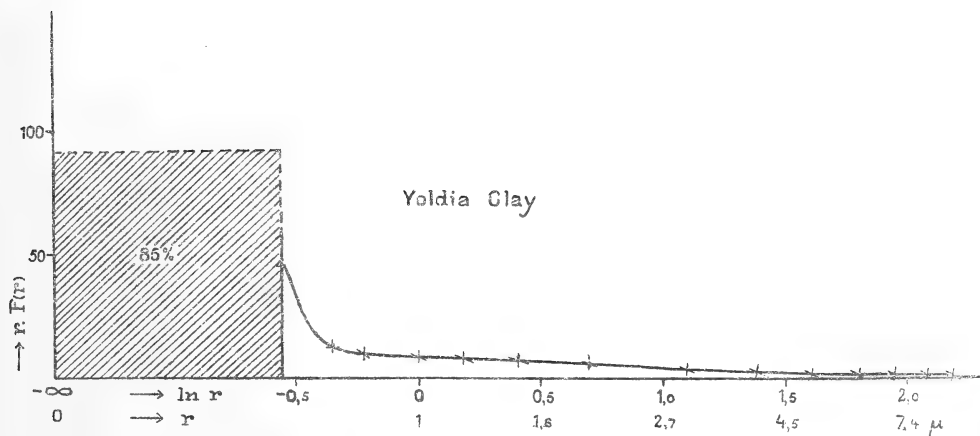
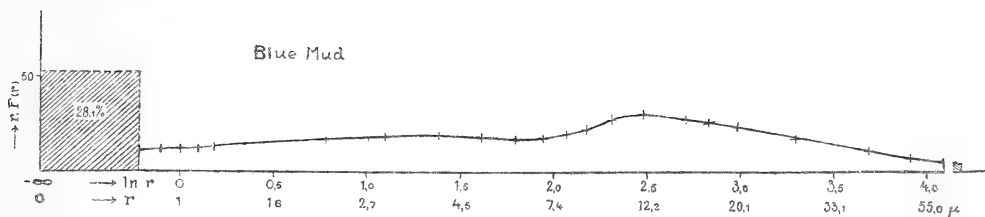
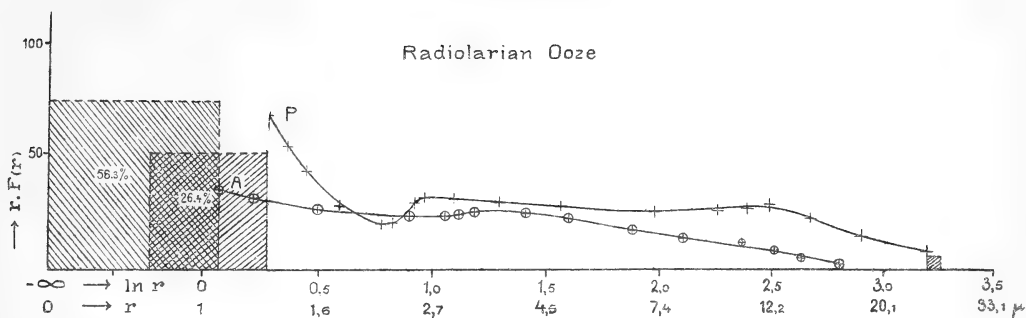




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XIV.—Mathematical Note on the Fall of Small Particles  
through Liquid Columns. By Professor C. G. Knott.

(MS. received March 20, 1916. Read March 20, 1916.)

THE following note presents in a simpler form the essence of Dr Sven Odén's mathematical discussion of the fall of small particles through a column of liquid, as given in his paper (immediately preceding) "On the Size of Particles in Deep-sea Deposits."

It has the further advantage of solving the problem without taking account explicitly or implicitly of Stokes's law (or modification thereof) as to the relation connecting the time of fall with the size, form, and density of the particles considered. It is important, I think, to refrain as long as possible from making the assumptions involved in such a law of fall, and to recognise how far we may carry the investigation before introducing these assumptions.

Imagine, then, a great number of small particles falling through a column of water. The terminal velocity and therefore rate of fall depend in some unknown way (unless the particles are spheres) upon their size and shape. Let them be partitioned off in bundles so that their times of fall form an arithmetical progression with small constant difference  $\delta t$ .\*

The "accumulation-curve" for any one of these groups will be a straight line whose final ordinate  $y_r$  corresponds to the time  $r\delta t$  taken by the corresponding set of particles to fall through the whole height. Particles of shorter time of fall will have already accumulated on the bottom. Particles of longer time of fall will have partially accumulated, the fraction of  $y_s$  ( $s$  greater than  $r$ ) which has accumulated up to time  $r\delta t$  being  $\frac{r}{s}y_s$ .

Hence the total accumulation up to time  $r\delta t$  is

$$P_r = \sum_1^r y_r \delta t + \sum_{p=1}^{p=n-r} \frac{r}{r+p} y_{r+p} \delta t$$

where  $n\delta t$  is the whole time for the completed accumulation.

\* We can imagine this being done by Maxwell's demons or other acute intelligences.

Similarly, for the next following intervals

$$P_{r+1} = \sum y_{r+1} \delta t + \sum_{p=2}^{p=n-r} \frac{r+1}{r+p} y_{r+p} \delta t$$

$$P_{r+2} = \sum y_{r+2} \delta t + \sum_{p=3}^{p=n-r} \frac{r+2}{r+p} y_{r+p} \delta t.$$

Hence

$$\delta P_r = P_{r+1} - P_r = \delta t \left[ \frac{1}{r+1} y_{r+1} + \sum_{p=2}^{p=n-r} \frac{1}{r+p} y_{r+p} \right]$$

$$\delta P_{r+1} = P_{r+2} - P_{r+1} = \delta t \left[ \frac{1}{r+2} y_{r+2} + \sum_{p=3}^{p=n-r} \frac{1}{r+p} y_{r+p} \right]$$

or

$$\left( \frac{dP}{dt} \right)_r = \frac{1}{r+1} y_{r+1} + \sum_{p=2} \frac{1}{r+p} y_{r+p}$$

$$\left( \frac{dP}{dt} \right)_{r+1} = \frac{1}{r+2} y_{r+2} + \sum_{p=3} \frac{1}{r+p} y_{r+p}$$

Then by subtraction

$$\left( \frac{dP}{dt} \right)_{r+1} - \left( \frac{dP}{dt} \right)_r = -\frac{1}{r+1} y_{r+1}$$

which may be written

$$\left( \frac{d^2 P}{dt^2} \right)_{r+1} = -\frac{y_{r+1}}{(r+1) \delta t} = -\frac{y_{r+1}}{t_{r+1}}.$$

Now the whole deposit due to  $y_{r+1}$  is

$$\frac{1}{2} y_{r+1} t_{r+1} = \left( -\frac{1}{2} t^2 \frac{d^2 P}{dt^2} \right)_{r+1}$$

where  $P$  is the measured deposit at the time  $t_{r+1}$  due to all the sets of particles falling together.

If we obtain, by Sven Odén's method of continuous weighing, the gradually accumulating deposit and plot this against the time, we get the accumulation-curve  $P = f(t)$ . From this we construct  $\frac{1}{2} t^2 \frac{d^2 P}{dt^2}$  for successive given instants of time, and plotting these values against the times we get a distribution-curve whose ordinates represent the relative quantities of particles whose times of fall are the corresponding abscissæ. Or we may use for abscissa the terminal velocity corresponding to the time of fall through the given height,  $h$ , of the column of liquid, namely,

$$v = h/t.$$

This is all that can be done without making more or less doubtful assumptions as to the density, form, and size of the particles. The application of Stokes's formula gives what Dr Sven Odén calls the "effective radius," and this may be regarded as affording a good approximation to

the size of particles which are of spheroidal or cuboidal form. Should the particles, however, be of a flat flaky form, it is doubtful if the "effective radius" calculated according to Stokes's law can be regarded as even a first approximation. Flaky particles will probably descend in zigzag courses in varying positions with fluctuating speed. It is possible that they might be set in rotation like thin slips of paper in their whirling oblique fall through air, a problem which was discussed qualitatively by Maxwell.\*

The distribution-curve obtained in the way described would certainly discriminate among sets of particles with different terminal velocities. Some interesting results might also be obtained by finding the distribution-curves for the same mixture of particles falling through columns of liquid of different density and viscosity.

\* *Cambridge and Dublin Mathematical Journal*, vol. ix, 1853; *Scientific Papers*, vol. i, p. 115.

XV.—Preliminary Communication on the Effects of Thyroid-Feeding upon the Pancreas. By Dr M. Kojima, Staff-Surgeon Imperial Japanese Navy. *Communicated by* SIR EDWARD A. SCHÄFER, F.R.S. (With Two Plates.)

(MS. received July 3, 1916. Read July 3, 1916.)

IN the course of a systematic inquiry, conducted in the Physiology Department of Edinburgh University, into the morphological and physiological changes produced in animals by thyroidectomy and thyroid-feeding—especially in rats—certain striking morphogenetic changes in the pancreas have come under observation. The full results of the investigations of which these observations form a part will be published subsequently, but it has been thought well to bring the effects herein described before the notice of this Society without waiting for the completion of every part of the investigation.

The morphogenetic changes produced in the pancreas by thyroid-feeding are of two kinds. The first is the causation of division and multiplication of the gland-cells, so that, after a few days' feeding with an adequate amount of thyroid, evidences of karyokinesis are observable in the shape of numerous mitoses at various stages, occurring so frequently that there may be as many as ten or twelve within the field of the ordinary high-power microscope. This is well seen in fig. 1, which is a drawing of a portion of such a field from the pancreas of a rat which had been fed for seven days with ordinary food, plus 1 grm. dried ox-thyroid per day. In this small portion of pancreas as many as ten nuclei in various stages of karyokinesis can easily be made out. This section was stained by Muir's method (alcoholic eosin and methylene blue): hæmatoxylin-stained preparations exhibit the mitoses equally well. Fig. 2 is from a control animal which received no thyroid with its food. It is of course a section of normal pancreas, in which, as is well known, mitoses are entirely absent. The change described begins to appear in rats after about three days' feeding, and continues for about ten days, after which time mitoses are fewer in number, and eventually disappear in spite of continuous feeding. The cells become after a time more numerous and smaller, but if the feeding be continued they again increase in size, the whole gland becoming enlarged and eventually resuming its normal microscopic appearance. After feeding for three days and subsequent

intermission of the feeding for as long as four days, a certain number of mitoses are still visible. After ten days' feeding, although, as we have seen, mitoses are becoming less frequent, the nuclei are often found in pairs as if they had recently divided.

Another morphogenetic effect of thyroid-feeding is upon the zymogen granules within the pancreas cells. Normally in the rat the zymogen is in large amount, filling the inner zone of the alveoli. Incidentally it may be remarked that it is in greater quantity in those alveoli which immediately surround the islets of Langerhans. After a few days' feeding the relative amount of zymogen in the gland-cells is considerably diminished, although the difference between the alveoli adjacent to the islets and the rest is still apparent. The diminution of zymogen continues, if the feeding be continued, for about three weeks. After that time zymogen begins again to accumulate within the gland-cells, so that after a month's feeding the alveoli again show abundance in the inner zone of the cells. These changes in the zymogen contents are well seen in preparations stained by Mallory's method, which colours the zymogen granules an intense red. Fig. 3 is a photograph of a section of the same (thyroid-fed) pancreas as that shown in fig. 1, but stained by Mallory instead of by Muir's method and magnified only 100 diameters; and fig. 4 of the same (normal) pancreas as that shown in fig. 2, also stained by Mallory and magnified 100 diameters. In these photographs the red zymogen granules appear black, and contrast strongly with the rest of the cells.

We have not been able to substantiate any distinct change in the cells constituting the islets of Langerhans as the result of thyroid-feeding.

This work has been aided by grants from the Moray Fund for scientific research in the University of Edinburgh and from the Carnegie Trust of the Scottish Universities.

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## EXPLANATION OF PLATES.

### PLATE I.

Fig. 1. Section of pancreas of white rat fed during seven days with addition of 1 gramme dry ox-thyroid per diem to the ordinary diet. (Drawn by Mr R. Muir under a magnifying power of 400 diameters.) Muir's staining method (alcoholic eosin and methylene blue). About ten mitoses are included in the field.

Fig. 2. Section of normal pancreas of white rat fed without addition of thyroid to the ordinary diet. (Drawn by Mr R. Muir under a magnifying power of 400

diameters.) Muir's staining method (alcoholic eosin and methylene blue). No mitoses are visible in this pancreas. The zymogen granules, which are coloured red, are far more abundant than in fig. 1.

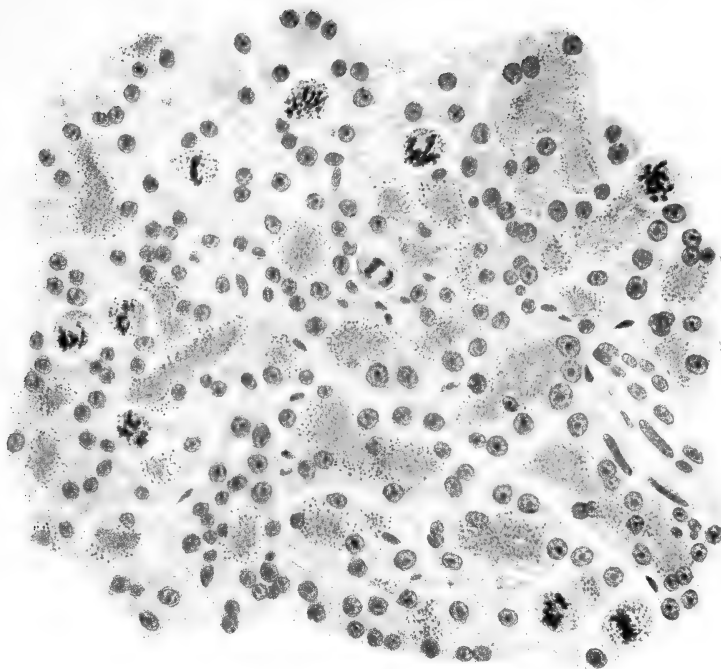
PLATE II.

Fig. 3. Section from the same pancreas as that shown in fig. 1, but stained by Mallory's method (acid fuchsin, orange G, and aniline blue) instead of by Muir's method. Photograph magnified 100 diameters. The zymogen masses are stained deep red by the acid fuchsin and come out black in the photograph. An islet of Langerhans is included in the field.

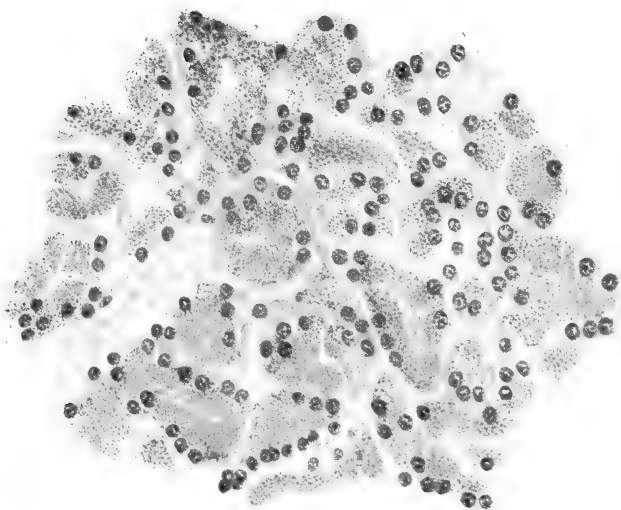
Fig. 4. Section from the same (normal) pancreas as that shown in fig. 2, but stained with Mallory instead of by Muir's method. Magnified 100 diameters. The deep red masses of zymogen come out black in the photograph. An islet of Langerhans is included in the field.

*(Issued separately January 19, 1917.)*

1.



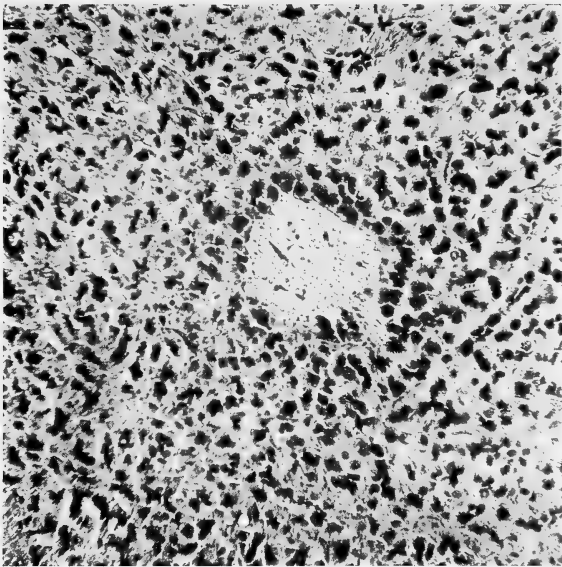
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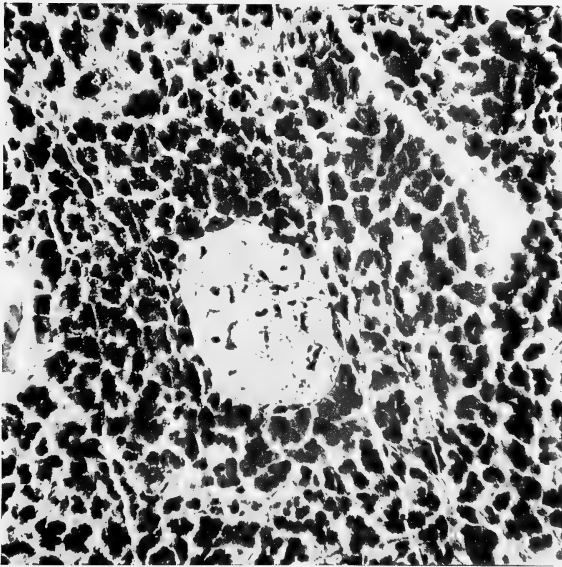




3.



4.





## XVI.—On the Theory of Continued-Fractions.

By Professor E. T. Whittaker.

(MS. received May 18, 1916. Read June 5, 1916.)

## I. INTRODUCTION.

THE value of the continued-fraction, as a means of representing an analytic function  $f(x)$ , is now fully recognised. Other representations, such as power-series or Fourier series or Dirichlet series or series of inverse factorials, converge in general only over limited regions of the  $x$ -plane, and fail to converge over the rest of the plane, whereas the representation of the function by a continued-fraction converges (in a large class of cases at any rate) over the whole  $x$ -plane, with the exception of certain singular curves. For example, the power-series

$$\frac{1}{2x} + \frac{1}{2! 2^2 x^3} + \frac{1 \cdot 3}{3! 2^3 x^5} + \frac{1 \cdot 3 \cdot 5}{4! 2^4 x^7} + \dots$$

converges only in the region which lies outside a circle of radius unity whose centre is the origin, although the function which it represents, namely,  $\{x - (x^2 - 1)^{\frac{1}{2}}\}$ , exists inside that circle; but the continued-fraction representation of the same function, namely,

$$\frac{1}{2x} - \frac{1}{2x} - \frac{1}{2x} - \frac{1}{2x} - ,$$

converges over the whole  $x$ -plane, with the exception of the part of the real axis of  $x$  between  $x = -1$  and  $x = +1$ .

The reason for the superiority of continued-fractions over power-series is that in a continued-fraction the numerators and denominators of the successive convergents are each helping on the approximation, whereas a power-series consists (so to speak) of a numerator alone, and being thus restricted, is unable to provide an approximation which is either so rapid or so widely applicable as that provided by the continued-fraction.

The great impediment to the use of continued-fractions in Theory of Functions and Differential Equations is the want of algorithms for adding, multiplying, and differentiating them. The object of the present paper is to supply in some measure this deficiency.

I think it would be a mistake to propose the problems in the form: *Given two continued-fractions, to find a third continued-fraction which is equal to their sum or product; or, Given a continued-fraction whose elements are functions of a variable, to find another continued-fraction which represents its differential coefficient:* for I doubt if the problems so formulated possess any simple solutions. But Sylvester showed long ago that any continued-fraction may be regarded as the quotient of two determinants: and if we regard continued-fractions in this light, advancing boldly from their theory to the theory of determinants, and aiming to express the products or sums of derivates of continued-fractions *in the form of determinants*, the situation becomes much more promising: we are, in fact, infusing into function-theory the marvellous flexibility and comprehensiveness of the determinant calculus.

In the present paper we are chiefly concerned with the expansion and differentiation of continued-fractions from this point of view.

The ultimate purpose of the work lies in its application to function-theory and the solution of differential equations: the continued-fractions are then non-terminating, and the determinants associated with them are of infinite order. But as the present paper is occupied only with the formal algorithms, the order may for convenience be supposed to be finite, and questions relating to the convergence of infinite processes need not here be considered.

## II. AN EXPANSION-THEOREM.

Our first object will be to show that the elementary algebraic identity

$$\frac{1}{b+x} = \frac{1}{x} - \frac{b}{x^2} + \frac{b^2}{x^3} - \frac{b^3}{x^4} + \dots \quad (1)$$

admits of a generalisation, which furnishes the transformation of a continued-fraction into a power-series, and which may be thus stated:

*Any continued-fraction*

$$\left. \frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x} \right\} \quad (2)$$

may be expanded as a power-series in  $1/x$ , in the form

$$\frac{1}{x} - \frac{b^{(1)}}{x^2} + \frac{b^{(2)}}{x^3} - \frac{b^{(3)}}{x^4} + \dots \quad (3)$$



But the first of these equations may be written

$$\frac{x_0}{X_0} = \frac{1}{b+x+\frac{c_1x_1}{x_0}},$$

and substituting for  $x_1/x_0$  from the second equation, and so on, we obtain ultimately

$$\frac{x_0}{X_0} = \frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x},$$

so the coefficient  $\beta_{00}$  is equal to the continued-fraction (2).

Now if  $E$  denotes the unit matrix

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & . & . \\ 0 & 0 & 1 & \dots & 1 \end{pmatrix}$$

the substitution (5) is evidently represented by the matrix  $M+xE$ , where  $M$  denotes the matrix (4): and as the substitution (6) is the reciprocal of this, it is represented in the symbolic matrix notation by  $\frac{1}{M+xE}$ . This may be expanded as if  $M$  and  $E$  were algebraical quantities, giving

$$\frac{E}{x} - \frac{M}{x^2} + \frac{M^2}{x^3} - \frac{M^3}{x^4} + \dots,$$

the numerators of all the fractions being matrices: and therefore  $\beta_{00}$ , which is the leading coefficient in this substitution, is equal to

$$\frac{1}{x} - \frac{b^{(1)}}{x^2} + \frac{b^{(2)}}{x^3} - \frac{b^{(3)}}{x^4} + \dots,$$

where  $b^{(n)}$  denotes the leading coefficient in the matrix  $M^n$ . Since we have already proved that  $\beta_{00}$  is equal to the continued-fraction, the theorem is now established.

#### *Example 1.*

As an example of the theorem, consider the continued-fraction

$$\frac{1}{3+x} + \frac{3}{(-\frac{1}{3})+x} + \frac{(\frac{1}{9})}{(\frac{1}{3})+x}.$$

Here the matrix is

$$M = \begin{pmatrix} 3 & 1 & 0 \\ -3 & -\frac{1}{3} & 1 \\ 0 & -\frac{1}{9} & \frac{1}{3} \end{pmatrix}$$

and raising it to the  $n^{\text{th}}$  power we obtain

$$M^n = \begin{pmatrix} \frac{1}{2}(n+1)(n+2) & \frac{1}{3}n(n+2) & \frac{1}{2}n(n-1) \\ -n(n+2) & 1 - \frac{2}{3}n(n+1) & -n(n-2) \\ \frac{1}{6}n(n-1) & \frac{1}{6}n(n-2) & \frac{1}{6}(n-2)(n-3) \end{pmatrix}$$

so the expansion of the continued-fraction as a power-series is

$$\frac{1}{x} - \frac{3}{x^2} + \frac{6}{x^3} - \dots + \frac{(-1)^n(n+1)(n+2)}{2x^{n+1}} + \dots$$

### Example 2.

Consider next the continued-fraction

$$\frac{1}{x + (\alpha\beta - \alpha\delta + \delta)} - \frac{\alpha(1 - \alpha)(\beta - \delta)^2}{x + (\alpha\delta - \alpha\beta + \beta)}.$$

Here we may write the matrix  $M$  in the form

$$\begin{pmatrix} \alpha\beta - \alpha\delta + \delta & \alpha(\beta - \delta) \\ (1 - \alpha)(\beta - \delta) & \alpha\delta - \alpha\beta + \beta \end{pmatrix}$$

and its  $n^{\text{th}}$  power is

$$M^n = \begin{pmatrix} \alpha\beta^n + (1 - \alpha)\delta^n & \alpha(\beta^n - \delta^n) \\ (1 - \alpha)(\beta^n - \delta^n) & (1 - \alpha)\beta^n + \alpha\delta^n \end{pmatrix}$$

so the continued-fraction is equivalent to the power-series

$$\sum_n \frac{(-1)^n(\alpha\beta^n + \delta^n - \alpha\delta^n)}{x^{n+1}}.$$

I did not originally arrive at the theorem by the way indicated in the above proof: it was suggested by combining Sylvester's theorem\* that any continued-fraction can be expressed as the quotient of two continuants with Cayley's theorem† that a matrix always satisfies a matrix-equation of its own order, and Kronecker's corollary‡ to Cayley's theorem, in which the elements of the reciprocal of a determinant are expanded in descending powers of a parameter, the coefficients being elements in the powers of a matrix. The combination of these theorems gives the result under discussion readily.

## III. STIELTJES' METHOD OF CONVERTING CONTINUED-FRACTIONS INTO POWER-SERIES.

So far as I am aware, the only previous discussion of the problem of converting a continued-fraction into a power-series is due to Stieltjes.§ He took the continued-fraction in the form

$$\left. \begin{aligned} & \frac{1}{x} + \frac{c_1}{1} + \frac{c_2}{x} + \frac{c_3}{1} + \dots + \frac{c_{2n-1}}{1} + \frac{c_{2n}}{x} + \dots \end{aligned} \right\} \dots \dots \dots (7)$$

and arrived finally at the following solution:—

\* *Phil. Mag.* (4), **5**, p. 446, **6**, p. 297 (1853); *Math. Papers*, **1**, pp. 609, 641.

† *Phil. Trans.*, **148** (1858), p. 17; *Coll. Papers*, **2**, p. 475.

‡ *Berlin Monatsb.*, 1873, p. 117; *Berlin Sitzungsab.*, 1890, p. 1081.

§ *Annales de la Fac. des Sc. de Toulouse*, **3** (1889), H.





These equations (14) can all be comprehended in the statement that the matrix

$$M = \begin{pmatrix} b & 1 & 0 & 0 & 0 & \dots \\ a_1 & b_1 & 1 & 0 & 0 & \dots \\ 0 & a_2 & b_2 & 1 & 0 & \dots \\ 0 & 0 & a_3 & b_3 & 1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

is the product of the two matrices

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ c_2 & 1 & 0 & 0 & \dots \\ 0 & c_4 & 1 & 0 & \dots \\ 0 & 0 & c_6 & 1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} c_1 & 1 & 0 & 0 & 0 & \dots \\ 0 & c_3 & 1 & 0 & 0 & \dots \\ 0 & 0 & c_5 & 1 & 0 & \dots \\ 0 & 0 & 0 & c_7 & 1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

and as Stieltjes' equations (9) and (10) are evidently formed of the substitutions which correspond to these latter matrices, the matrix  $M$  and its powers now enter the problem quite naturally, and the harmonisation of Stieltjes' formulæ with those of § 2 presents no further difficulty. Stieltjes' constants  $\alpha_{0k}, \alpha_{1k}, \alpha_{2k}, \dots$  are found to be simply the elements in the first row of  $M^k$ .

IV. RELATIONS INVOLVING PERSYMMETRIC DETERMINANTS.

The problem converse to that of § 2 is the conversion of a power-series

$$\frac{1}{x} - \frac{b^{(1)}}{x^2} + \frac{b^{(2)}}{x^3} - \frac{b^{(3)}}{x^4} + \dots$$

into a continued-fraction

$$\frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots$$

in such a way that the  $n^{\text{th}}$  convergent to the continued-fraction, when expanded as a power-series in  $1/x$ , shall agree with the above series as far as the  $2n^{\text{th}}$  term inclusive.\* This converse problem has been much more

\* This latter condition excludes the comparatively worthless method of converting infinite series into continued-fractions which is commonly given in elementary text-books on algebra, and which may be expressed by the equation

$$\frac{1}{a} - \frac{1}{b} + \frac{1}{c} - \frac{1}{d} + \dots = \frac{1/a}{1} + \frac{a/(b-a)}{1} + \frac{b^2/(b-a)(c-b)}{1} + \dots;$$

for in this equation the  $n^{\text{th}}$  convergent to the continued-fraction coincides with the series only so far as its  $n^{\text{th}}$  term, instead of as far as its  $2n^{\text{th}}$  term.

studied, and indeed was completely solved by Heilermann\* so long ago as 1845. His result is that the coefficients  $b, a_1, b_1, a_2, b_2, \dots$  in the continued-fraction are simply expressible in terms of determinants of the kind called by Sylvester *persymmetric*, namely,

$$\begin{vmatrix} 1 & b^{(1)} \\ b^{(1)} & b^{(2)} \end{vmatrix}, \quad \begin{vmatrix} 1 & b^{(1)} & b^{(2)} \\ b^{(1)} & b^{(2)} & b^{(3)} \\ b^{(2)} & b^{(3)} & b^{(4)} \end{vmatrix}, \quad \begin{vmatrix} b^{(1)} & b^{(2)} \\ b^{(2)} & b^{(3)} \end{vmatrix}, \quad \begin{vmatrix} b^{(1)} & b^{(2)} & b^{(3)} \\ b^{(2)} & b^{(3)} & b^{(4)} \\ b^{(3)} & b^{(4)} & b^{(5)} \end{vmatrix}, \text{ etc.,}$$

which are formed of the coefficients in the given power-series.

On comparing Heilermann's result with that of § 2, it is evident that a connection must exist between persymmetric determinants and the coefficients in powers of matrices of the form (4). This connection is easily shown by aid of Stieltjes' theorem. For if we denote the quadratic form (11) by  $A$ , we see that the persymmetric determinant

$$\begin{vmatrix} 1 & b^{(1)} & b^{(2)} \\ b^{(1)} & b^{(2)} & b^{(3)} \\ b^{(2)} & b^{(3)} & b^{(4)} \end{vmatrix}$$

is  $\frac{1}{8} \times$  the Jacobian of  $\frac{\partial A}{\partial X_0}, \frac{\partial A}{\partial X_1}, \frac{\partial A}{\partial X_2}$ , with respect to  $X_0, X_1, X_2$ . But by (12) and (14)  $A$  may be written in the form

$$A = P^2 + a_1 Q^2 + a_1 a_2 R^2 + a_1 a_2 a_3 S^2 + \dots$$

where

$$P = a_{00} X_0 + a_{01} X_1 + a_{02} X_2 + \dots$$

$$Q = a_{11} X_1 + a_{12} X_2 + a_{13} X_3 + \dots$$

$$R = a_{22} X_2 + a_{23} X_3 + \dots$$

and as  $Q$  does not contain  $X_0$ ,  $R$  does not contain  $X_0$  or  $X_1$ , etc., we see that the Jacobian has the value

$$\frac{\partial^2 A}{\partial P^2} \frac{\partial^2 A}{\partial Q^2} \frac{\partial^2 A}{\partial R^2} \left( \frac{\partial P}{\partial X_0} \right)^2 \left( \frac{\partial Q}{\partial X_1} \right)^2 \left( \frac{\partial R}{\partial X_2} \right)^2$$

or

$$8a_1^2 a_2.$$

Thus we have the result that *if  $b^{(1)}, b^{(2)}, b^{(3)}, b^{(4)}$  are the leading coefficients in the first, second, third, and fourth powers of the matrix*

\* *Journal für Math.*, **33** (1845), p. 174.

$$\begin{vmatrix} b & 1 & 0 & 0 & 0 & \dots \\ a_1 & b_1 & 1 & 0 & 0 & \dots \\ 0 & a_2 & b_2 & 1 & 0 & \dots \\ 0 & 0 & a_3 & b_3 & 1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

then

$$\begin{vmatrix} 1 & b^{(1)} & b^{(2)} \\ b^{(1)} & b^{(2)} & b^{(3)} \\ b^{(2)} & b^{(3)} & b^{(4)} \end{vmatrix} = a_1^2 a_2$$

and similarly, in general, the persymmetric determinant of the  $n^{\text{th}}$  order formed of  $1, b^{(1)}, b^{(2)}, b^{(3)}, b^{(4)}, \dots$  has the value  $a_1^{n-1} a_2^{n-2} \dots a_{n-1}$ .

It is easily shown in the same way that the persymmetric determinants formed of  $b^{(1)}, b^{(2)}, b^{(3)}, \dots$  can be evaluated in terms of  $b, b_1, b_2, b_3, \dots$ . Hence we see that if the leading elements in all the powers of a continuant matrix are given, the elements of the matrix itself can be obtained in terms of persymmetric determinants formed of these quantities.

## V. THE DIFFERENTIATION OF CONTINUED-FRACTIONS.

We shall now proceed to find the differential coefficient of a continued-fraction

$$\frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x}$$

with respect to  $x$ . So far as I am aware, no investigation on the subject has been published hitherto.

Denoting the continued-fraction by  $S$ , we know from § 2 that  $S$  is equal to the leading coefficient in the matrix  $\frac{1}{M+xE}$ , when  $E$  denotes the unit matrix and  $M$  denotes the matrix (4). So  $-\frac{dS}{dx}$  must be equal to the leading coefficient in the matrix  $\frac{1}{(M+xE)^2}$ ; that is to say, it must be equal to the leading coefficient in the matrix reciprocal to  $(M+xE)^2$ . We thus have immediately the theorem:

If

$$S = \frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x},$$

then  $-\frac{dS}{dx}$  is the leading element in the determinant reciprocal\* to the square of the determinant

$$\begin{vmatrix} b+x & c_1 & 0 & 0 & \dots & 0 & 0 \\ d_1 & b_1+x & c_2 & 0 & \dots & 0 & 0 \\ 0 & d_2 & b_2+x & c_3 & \dots & 0 & 0 \\ 0 & 0 & d_3 & b_3+x & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & d_n & b_n+x \end{vmatrix} \quad . \quad . \quad . \quad (15)$$

where  $c_1, c_2, \dots, d_1, d_2, \dots$  are any quantities such that  $c_1 d_1 = a_1, c_2 d_2 = a_2, \dots, c_n d_n = a_n$ .

We shall now proceed to deduce an explicit expression for  $-\frac{dS}{dx}$ . If we consider the equations

$$\left. \begin{aligned} Y_0 &= (b+x)z_0 + c_1 z_1 \\ Y_1 &= d_1 z_0 + (b_1+x)z_1 + c_2 z_2 \\ Y_2 &= d_2 z_1 + (b_2+x)z_2 + c_3 z_3 \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ Y_n &= d_n z_{n-1} + (b_n+x)z_n \\ 0 &= (b+x)y_0 + c_1 y_1 - z_0 \\ 0 &= d_1 y_0 + (b_1+x)y_1 + c_2 y_2 - z_1 \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ 0 &= d_n y_{n-1} + (b_n+x)y_n - z_n \end{aligned} \right\} \quad . \quad . \quad . \quad (16)$$

it is evident that  $(Y_0, Y_1, Y_2, \dots, Y_n)$  are derived from  $(y_0, y_1, y_2, \dots, y_n)$  by the substitution which corresponds to the square of the matrix

$$\begin{vmatrix} b+x & c_1 & 0 & 0 & \dots & 0 & 0 \\ d_1 & b_1+x & c_2 & 0 & \dots & 0 & 0 \\ 0 & d_2 & b_2+x & c_3 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & d_n & b_n+x \end{vmatrix} \quad . \quad . \quad . \quad (17)$$

and therefore the leading element in the reciprocal of the square of this matrix is the value of  $y_0/Y_0$  derived from equations (16) when  $Y_1, Y_2,$

\* If  $D = |a_{11}, a_{22}, a_{33}, \dots, a_{nn}|$  is any determinant, and if  $A_{rs}$  denotes the co-factor of  $a_{rs}$  in  $D$ , then the determinant  $|A_{11}, A_{22}, A_{33}, \dots, A_{nn}|$  is called the *adjugate* of  $D$ . This nomenclature follows that of Cauchy's original memoir, and is always used by Sir Thomas Muir in his extensive writings on determinants: some writers have improperly used the word *reciprocal* in the sense of adjugate. We use the word *reciprocal* to signify the determinant  $\left| \frac{A_{11}}{D}, \frac{A_{22}}{D}, \frac{A_{33}}{D}, \dots, \frac{A_{nn}}{D} \right|$ , which is the determinant of the matrix reciprocal to the matrix  $(a_{11}, a_{22}, \dots, a_{nn})$ .

$Y_3, \dots Y_n$  are replaced by zero. That is to say, the value of  $-\frac{dS}{dx}$  is equal to the value of  $y_0/Y_0$  derived from the equations

$$\left\{ \begin{array}{l} Y_0 = (b+x)z_0 + c_1z_1 \\ 0 = d_1z_0 + (b_1+x)z_1 + c_2z_2 \\ 0 = d_2z_1 + (b_2+x)z_2 + c_3z_3 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ 0 = d_nz_{n-1} + (b_n+x)z_n \\ 0 = (b+x)y_0 + c_1y_1 - z_0 \\ 0 = d_1y_0 + (b_1+x)y_1 + c_2y_2 - z_1 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ 0 = d_ny_{n-1} + (b_n+x)y_n - z_n \end{array} \right.$$

But these equations give at once

$$\left| \begin{array}{cccccccccccc} -Y_0/y_0 & 0 & 0 & \dots & 0 & 0 & b+x & c_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & d_1 & b_1+x & c_2 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & d_n & b_n+x \\ b+x & c_1 & 0 & \dots & 0 & 0 & -1 & 0 & 0 & \dots & 0 & 0 \\ d_1 & b_1+x & c_2 & \dots & 0 & 0 & 0 & -1 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \dots & d_n & b_n+x & 0 & 0 & 0 & \dots & 0 & -1 \end{array} \right| = 0$$

and therefore we have finally

$$\frac{dS}{dx} = \frac{\left| \begin{array}{cccccccccccc} 0 & 0 & 0 & \dots & 0 & 0 & d_1 & b_1+x & c_2 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & d_2 & b_2+x & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & d_n & b_n+x \\ c_1 & 0 & 0 & \dots & 0 & 0 & -1 & 0 & 0 & \dots & 0 & 0 \\ b_1+x & c_2 & 0 & \dots & 0 & 0 & 0 & -1 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \dots & d_n & b_n+x & 0 & 0 & 0 & \dots & 0 & -1 \end{array} \right|}{\left| \begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & \dots & 0 & 0 & b+x & c_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & d_1 & b_1+x & c_2 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & d_2 & b_2+x & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & d_n & b_n+x \\ b+x & c_1 & 0 & 0 & \dots & 0 & 0 & -1 & 0 & 0 & \dots & 0 & 0 \\ d_1 & b_1+x & c_2 & 0 & \dots & 0 & 0 & 0 & -1 & 0 & \dots & 0 & 0 \\ 0 & d_2 & b_2+x & c_3 & \dots & 0 & 0 & 0 & 0 & -1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & d_n & b_n+x & 0 & 0 & 0 & \dots & 0 & -1 \end{array} \right|} \quad (18)$$

*This formula gives the explicit expression of the differential coefficient of the continued-fraction*

$$S = \frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x}$$

where  $a_1 = c_1 d_1$ ,  $a_2 = c_2 d_2$ , . . .  $a_n = c_n d_n$ .

It will be noticed that the determinant in the numerator is the principal minor of the determinant in the denominator: and also that the latter determinant is the square of the determinant formed of the elements in its top right-hand quadrant. We may further remark that by reducing the order of the determinants in the same way as in a well-known proof of the multiplication-theorem for determinants, we can obtain the result in the form

$$-\frac{dS}{dx} = \frac{\begin{vmatrix} (x+b_1)^2 + c_1 d_1 + c_2 d_2 & c_2(2x+b_1+b_2) & c_2 c_3 & 0 & \dots \\ d_2(2x+b_1+b_2) & (x+b_2)^2 + c_2 d_2 + c_3 d_3 & c_3(2x+b_2+b_3) & c_3 c_4 & \dots \\ d_3 d_3 & d_3(2x+b_2+b_3) & (x+b_3)^2 + c_3 d_3 + c_4 d_4 & c_4(2x+b_3+b_4) & \dots \\ 0 & d_3 d_4 & d_4(2x+b_3+b_4) & (x+b_4)^2 + c_4 d_4 + c_5 d_5 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}{\begin{vmatrix} (x+b)^2 + c_1 d_1 & c_1(2x+b+b_1) & c_1 c_2 & 0 & \dots \\ d_1(2x+b+b_1) & (x+b_1)^2 + c_1 d_1 + c_2 d_2 & c_2(2x+b_1+b_2) & c_2 c_3 & \dots \\ d_1 d_2 & d_2(2x+b_1+b_2) & (x+b_2)^2 + c_2 d_2 + c_3 d_3 & c_3(2x+b_2+b_3) & \dots \\ 0 & d_2 d_3 & d_3(2x+b_2+b_3) & (x+b_3)^2 + c_3 d_3 + c_4 d_4 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}} \quad (19)$$

For the benefit of those who dislike or distrust symbolic-matrix proofs, the following proof by ordinary "mathematical induction" may be added.

First, the theorem may be verified by direct expansion in the simplest cases. Suppose then that formula (19) holds when there is some particular number  $n$  of quotients in the continued-fraction, say for the continued-fraction

$$T = \frac{1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x};$$

then we shall prove that it holds for the next higher number  $(n+1)$  of quotients, say for the continued-fraction

$$S = \frac{1}{b+x} - \frac{a_1}{b_1+x} - \frac{a_2}{b_2+x} - \dots - \frac{a_n}{b_n+x}.$$

For we have by hypothesis

$$-\frac{dT}{dx} = \frac{\begin{vmatrix} (x+b_2)^2+c_2d_2+c_3d_3 & c_3(2x+b_2+b_3) & c_3c_4 & 0 & \dots \\ d_3(2x+b_2+b_3) & (x+b_3)^2+c_3d_3+c_4d_4 & c_4(2x+b_3+b_4) & c_4c_5 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}{\begin{vmatrix} (x+b_1)^2+c_2d_2 & c_2(2x+b_1+b_2) & c_2c_3 & 0 & \dots \\ d_2(2x+b_1+b_2) & (x+b_2)^2+c_2d_2+c_3d_3 & c_3(2x+b_2+b_3) & c_3c_4 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}$$

so we may write

$$-a_1\frac{dT}{dx} = \frac{\begin{vmatrix} c_1d_1 & 0 & 0 & 0 & \dots \\ d_2(2x+b_1+b_2) & (x+b_2)^2+c_2d_2+c_3d_3 & c_3(2x+b_2+b_3) & c_3c_4 & \dots \\ d_2d_3 & d_3(2x+b_2+b_3) & (x+b_3)^2+c_3d_3+c_4d_4 & c_4(2x+b_3+b_4) & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}{\begin{vmatrix} (x+b_1)^2+c_2d_2 & c_2(2x+b_1+b_2) & c_2c_3 & 0 & \dots \\ d_2(2x+b_1+b_2) & (x+b_2)^2+c_2d_2+c_3d_3 & c_3(2x+b_2+b_3) & c_3c_4 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}$$

and therefore

$$1-a_1\frac{dT}{dx} = \frac{\begin{vmatrix} (x+b_1)^2+c_1d_1+c_2d_2 & c_2(2x+b_1+b_2) & c_2c_3 & 0 & \dots \\ d_2(2x+b_1+b_2) & (x+b_2)^2+c_2d_2+c_3d_3 & c_3(2x+b_2+b_3) & c_3c_4 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}{\begin{vmatrix} (x+b_1)^2+c_2d_2 & c_2(2x+b_1+b_2) & c_2c_3 & 0 & \dots \\ d_2(2x+b_1+b_2) & (x+b_2)^2+c_2d_2+c_3d_3 & c_3(2x+b_2+b_3) & c_3c_4 & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}$$

But the equation  $S = \frac{1}{b+x-a_1T}$  gives  $-\frac{dS}{dx} = S^2(1-a_1\frac{dT}{dx})$ . Substituting for  $1-a_1\frac{dT}{dx}$  the determinantal expression just found, and for  $S$  its expression as a quotient of two determinants given by Sylvester's theorem, we obtain formula (19) for  $\frac{dS}{dx}$ , and the theorem is thus established. Formula (18) may be derived by reversing the process by which (19) was derived from (18).

(Issued separately January 19, 1917.)

XVII.—The Ochil Earthquakes of the Years 1900–1914. By  
Charles Davison, Sc.D., F.G.S. *Communicated by THE*  
GENERAL SECRETARY.

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I. INTRODUCTION.

THE earthquakes of Scotland, as compared with those of England and Wales, are notable on several accounts:—(i) They are more numerous. During the twenty-six years from 1889 to 1914, 357 earthquakes originated within the area of Great Britain—51 in England, 27 in Wales, and 279 in Scotland. If area be taken into account, the inequality is still greater. For every shock occurring in England, there were during this period four in Wales and nine in Scotland. (ii) They are invariably simple earthquakes, twin earthquakes (which include the strongest of all British earthquakes) being confined to England and Wales. (iii) As simple earthquakes, they are due to movements along strike-faults, and, for the most part, along well-known faults, the majority of which belong to the Caledonian system, the only exceptions known to me being those connected with the Loch Broom earthquake of 1892 and the two Glasgow earthquakes of 1910. (iv) They occur usually in groups or series. For instance, most, if not all, of the Inverness earthquakes of 1890, and seventeen of the nineteen Inverness earthquakes of 1901, were due to slips along the great northern boundary-fault of the Highland district, which traverses Scotland in a south-westerly direction from Tarbat Ness; the well-known Comrie earthquakes, of which only three occurred between 1889 and 1914, are associated with the southern boundary-fault of the same district, which crosses Scotland from Stonehaven to the southern end of Loch Lomond; while as many as 186 shocks originated between 1900 and 1914 along the southern boundary-fault of the Ochil Hills.

Little is known with regard to the earthquakes of the last-mentioned district before the year 1900. So far as I know, only five shocks are recorded. On April 30 and May 1, 1736, there were earthquakes strong enough to damage houses; on July 10, 1842, a slight shock occurred; on August 8, 1872, there was an earthquake which, in intensity and extent of disturbed area, closely resembled the three strongest earthquakes of the present century; and lastly, on January 12, 1881, a smart shock was felt at Bridge of Allan and Menstrie. If we may judge from the recent



earthquakes, only a few of which are noticed in the newspaper press, it is probable that the five shocks mentioned above are only the more prominent of a long series of tremors and earth-sounds which have escaped record owing to the lack of interested observers.

My investigation of the recent earthquakes has been rendered possible through the kindness and courtesy of a large number of persons living in different parts of the area considered. More especially am I indebted to the following observers who have sent me frequent notices of the earthquakes felt by them:—Airthrey, Mr J. Dempster<sup>‡</sup> and Mr E. J. Sim; Alva, Dr W. L. Cunningham, Mr W. Havery, and Rev. J. Williamson; Blairlogie, the late Mr R. D. Taylor; Bridge of Allan, the late Surgeon-General Bidie; Dunblane, Dr James Barty and Miss C. H. M. Johnstone; Gogar, Mr J. M. Morries, D.L., J.P.; Greenloaning, Rev. T. Blackwood; Logie, Rev. M. Fergusson; Menstrie, Rev. J. Boyd, Mr T. J. H. Drysdale, and Mr W. H. Lindsay; Red Carr (Blairlogie), Mr T. B. Johnstone; Tillicoultry, Rev. J. Conn and Mr Alex. Scott, Jun.; Tullibody, Rev. A. Thom.\*

The series of earthquakes studied in this paper began in September 1900, and ended in December 1914. The numbers of earthquakes recorded in successive years are 4 in 1900, 1 in 1903, 10 in 1905, 19 in 1906, 13 in 1907, 17 in 1908, 18 in 1909, 19 in 1910, 8 in 1911, 74 in 1912, 2 in 1913, and 1 in 1914. The total number of shocks observed in 200 months is thus 186, or, on an average, very nearly one a month. Three earthquakes (those of September 21, 1905, October 20, 1908, and May 3, 1912) were of unusual strength and disturbed area, and are referred to in the following description as principal earthquakes. The intensity of the shocks is determined in terms of the well-known Rossi-Forel scale, or rather of the modification of it which I have adopted for use in this country.†

## II. DESCRIPTION OF THE EARTHQUAKES.‡

### (1) 1900, *September 17*, 3.30 *p.m.*

A slight shock, felt at Menstrie.

### (2) 1900, *September 17*, 10.5 *p.m.*

A slight shock, felt at Alva.

\* The expenses of the investigations were defrayed from a grant received from the Government Research Fund.

† *Geographical Journal*, vol. xlv, 1915, pp. 360-361.

‡ The descriptions of the earthquakes numbered 1-42 are abridged from a paper in the *Quart. Journ. Geol. Soc.*, vol. lxiii, 1907, pp. 362-374; those of the earthquakes numbered 43-82 from papers in the *Geological Magazine*, vol. v, 1908, pp. 296-309, and vol. vii, 1910, pp. 315-320.

(3) 1900, *September 17*, 10.15 *p.m.*

Intensity, 4; centre of isoseismal 4 in lat.  $56^{\circ} 11'5''$  N., long.  $3^{\circ} 50'5''$  W.; number of records, 56, from 26 places, and 13 negative records from 11 places (fig. 1).

The boundary of the disturbed area is an isoseismal line of intensity slightly less than 4. It is 15 miles long,  $9\frac{1}{2}$  miles wide, and 117 square

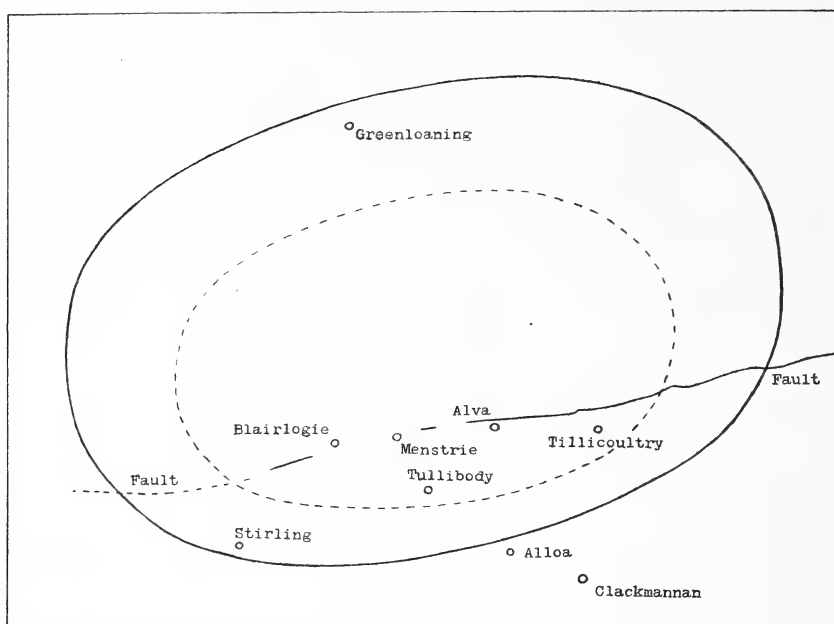


FIG. 1.—Earthquake of Sept. 17, 1900: Continuous line. Earthquake of Sept. 22: Broken line.

miles in area. The longer axis is directed E.  $13^{\circ}$  N., and the centre of the area is 3 miles N.  $8^{\circ}$  E. of Menstrie.

All over the disturbed area, the shock consisted of a single prominent vibration, followed by a tremor, such as would be caused by a heavy weight falling on the floor and making the building shake. The duration of the shock was not more than three seconds. The sound was heard by 94 per cent. of the observers, and was compared to passing waggons, etc., in 29 per cent. of the records, thunder in 10, wind in 5, the fall of a load of stones in 12, the fall of a heavy body in 17, blasting or explosions in 12, and miscellaneous sounds in 15 per cent.

(4) 1900, *September 22*, 4.30 *p.m.*

Intensity, 4; centre of isoseismal 4 in lat.  $56^{\circ} 10' 5''$  N., long.  $3^{\circ} 50' 4''$  W.; number of records, 20, from 13 places, and 17 negative records from 15 places (fig. 1).

The boundary of the disturbed area is an isoseismal line of intensity slightly less than 4. It is 11 miles long, 7 miles wide, and 60 square miles in area. The longer axis is directed E.  $13^{\circ}$  N., and the centre of the area is  $1\frac{3}{4}$  miles N.  $17^{\circ}$  E. of Menstrie.

The shock consisted of a single prominent vibration followed by a tremor. The sound was heard by 87 per cent. of the observers.

Slight shocks were also felt on September 18, at 2 a.m., at Alva, and at about 2.55 a.m. at Bridge of Allan, but, so far as known, by only one observer in each case.

(5) 1903, *May 15*, 6.15 *p.m.*

A distinct shock, felt generally at Menstrie.

(6) 1905, *April 23*, 12.15 *a.m.*

A distinct shock, felt at Red Carr (Blairlogie).

(7) 1905, *July 23*, 12.15 *a.m.*

Intensity, 5; centre of isoseismal 4 in lat.  $56^{\circ} 11' 3''$  N., long.  $3^{\circ} 47' 6''$  W.; number of records, 33, from 16 places, and 4 negative records from 4 places (fig. 2).

The boundary of the disturbed area is an isoseismal line of intensity 4. It is  $16\frac{1}{2}$  miles long,  $10\frac{1}{2}$  miles wide, and 136 square miles in area. Its centre is  $3\frac{1}{2}$  miles N.E. of Menstrie, and the direction of its longer axis E.  $27^{\circ}$  N. On the same map, the portion of the isoseismal 5 lying on the south side of the Ochil Hills is shown. Its distance from the isoseismal 4 is  $1\frac{1}{2}$  miles.

Within the isoseismal 5, the shock consisted of one or more prominent vibrations followed by a series of tremors. Its mean duration was three seconds. The sound-area coincided with the disturbed area in all directions except perhaps towards the east. The sound was heard by 89 per cent. of the observers, 17 per cent. of whom compared it to passing waggons, 10 to thunder, 27 to wind, 40 to the fall of a heavy body, 3 to explosions, and 3 to miscellaneous sounds.

(8) 1905, *July 26*, 6.3 *p.m.*

Intensity, 4; number of records, 9, from 7 places.

The only records of this shock come from places in the Hillsfoot

district, namely, Alva, Gogar, Logie, Menstrie, Sauchie, Tillicoultry, and Tullibody. The disturbed area was much smaller than that of the preceding shock, and probably it did not extend so far as Dunblane, Greenloaning, Blackford, and Glendevon, and must therefore have contained less than 80 square miles.

The shock was as if a very heavy body had fallen on the floor, followed by the quivering which such a fall would produce. Its duration was one

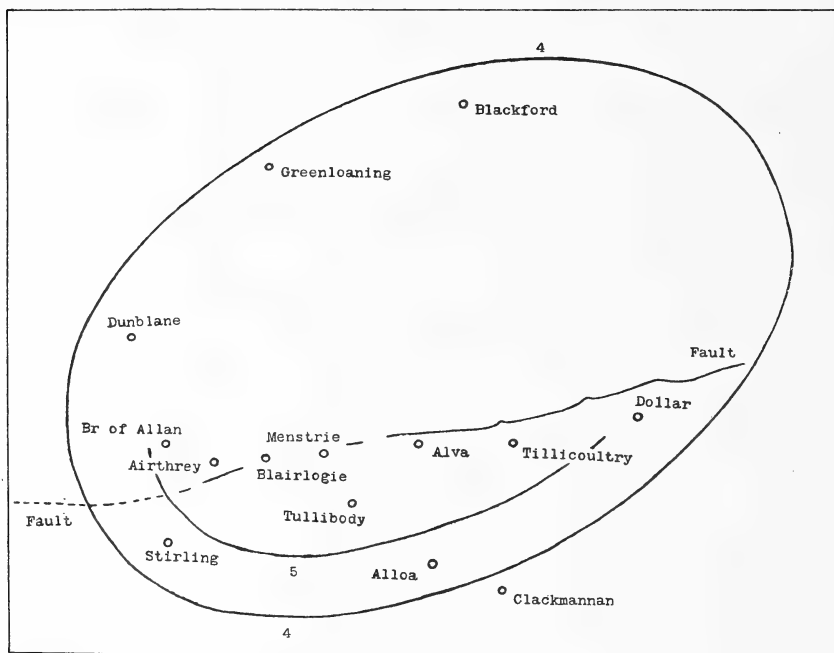


FIG. 2.—Earthquake of July 23, 1905.

and a half or two seconds. At Menstrie, the noise was as loud as thunder overhead.

(9) 1905, *August 3, about 6 p.m.*

A very perceptible shock, felt by several observers at Red Carr.

(10) 1905, *September 21, 11.33 p.m.* (*First Principal Earthquake.*)

Intensity, 6; centre of isoseismal 5 in lat.  $56^{\circ} 11' 8''$  N., long.  $3^{\circ} 49' 1''$  W.; number of records, 139, from 57 places, and 22 negative records from 19 places (fig. 3).

On the map of the earthquake are shown three isoseismal lines of intensities 6, 5, and 4. The isoseismal 6 is 13 miles long, 8 miles wide, and

82 square miles in area. Towards the north-east, the course of this curve is uncertain. The isoseismal 5, which is determined by a much larger number of observations, is  $20\frac{1}{2}$  miles long, 14 miles wide, and 227 square miles in area. Its centre is 3 miles N.  $25^\circ$  E. of Menstrie, and its longer axis is directed E.  $29^\circ$  N. The distance between the isoseismals 6 and 5 is  $3\frac{1}{4}$  miles on the north side and  $2\frac{3}{4}$  miles on the south. The isoseismal 4 is  $33\frac{1}{2}$  miles long,  $26\frac{1}{2}$  miles wide, and 700 square miles in area. Its longer

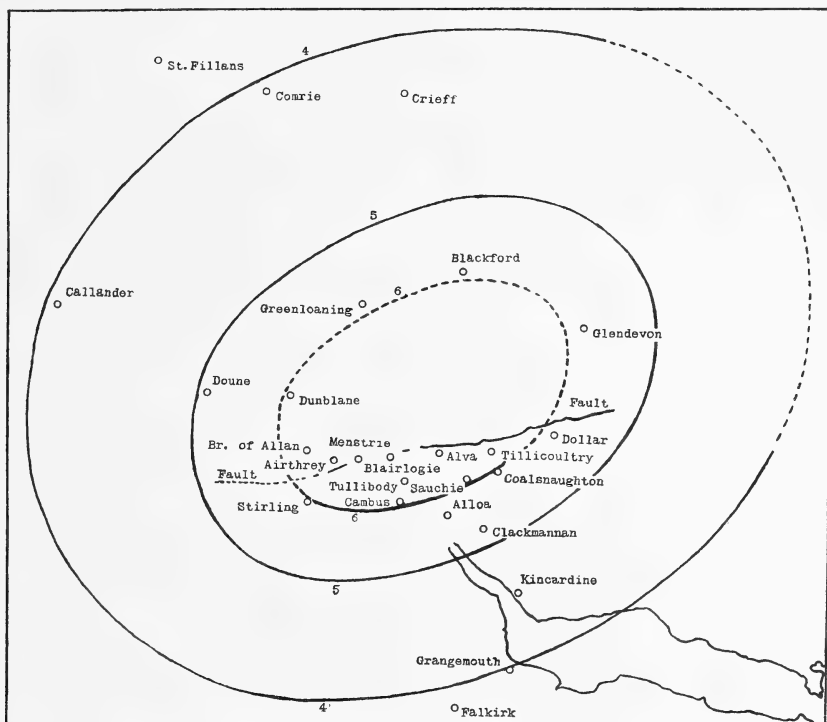


FIG. 3.—Earthquake of Sept. 21, 1905.

axis is directed E.  $27^\circ$  N. The distance between the isoseismals 5 and 4 is  $7\frac{1}{2}$  miles on the north side and 5 miles on the south. Outside this isoseismal, the shock was felt at St Fillans and Balmeanach, 3 and  $4\frac{1}{2}$  miles to the north-west, and at Falkirk, 1 mile to the south. The disturbed area must therefore contain about 1000 square miles.

The shock was similar to its predecessors, except that, near the epicentre, the principal vibrations were preceded, as well as followed, by tremors. The mean of 54 estimates of the duration of the shock is 3.4 seconds. The sound-area is co-extensive with the disturbed area. The percentage of audibility is 84 for the whole area, 89 within the isoseismal 6, 82 between the isoseismal lines 6 and 5, and 79 between the isoseismals 5 and 4.

Towards the latter isoseismal, however, there was a marked decline in audibility. The sound was compared to passing waggons, etc., in 24 per cent. of the records, thunder in 16, wind in 8, loads of stones falling in 10, the fall of a heavy body in 24, explosions in 16, and miscellaneous sounds in 2 per cent.

(11) 1905, *September 22, about 1.30 a.m.*

A tremor, felt at Alloa and Bridge of Allan.

(12) 1905, *September 25, early morning.*

A very slight tremor, felt at Alloa and Cambus.

(13) 1915, *September 30, 9.45 p.m.*

A slight shock, accompanied by a rumbling noise, felt at Menstrie.

(14) 1905, *October 29, 10.53 a.m.*

A sound, resembling distant thunder, without any accompanying tremor, heard at Menstrie.

(15) 1905, *December 22, 9.15 p.m.*

A slight shock, felt throughout the village of Menstrie.

(16) 1906, *July 3, 2.15 p.m.*

A slight shock (intensity 3), felt at Menstrie, accompanied by a sound like the fall of a heavy body.

(17) 1906, *July 4, 3.45 a.m.*

The shock (intensity 4) was felt at Airthrey, Alva, Blairlogie, Menstrie, Red Carr, and Tullibody, all in the Hillsfoot district. The shock is described as a quivering thud, strongest at the beginning, and lasting two seconds. The sound was heard by all the observers.

(18) 1906, *July 7, 5.29 a.m.*

The shock (intensity 4) was felt at Alva and Menstrie. The sound was heard by all the observers.

(19) 1906, *August 24, 5.25 p.m.*

A very slight shock, felt at Menstrie.

(20) 1906, *September 28, 12.25 p.m.*

A slight shock, with sound, observed at Blairlogie, Menstrie, and Red Carr.

(21) 1906, *October 3, 4.32 a.m.*

A shock, strong enough to awaken several observers, felt at Blairlogie and Menstrie. The accompanying sound was like a cannon-shot.

(22) 1906, *October 8, 7.24 a.m.*

Intensity, 5; centre of isoseismal 4 in lat.  $56^{\circ} 10' 9''$  N., long.  $3^{\circ} 51' 3''$  W.; number of records, 23, from 14 places, and 8 negative records from 6 places (fig. 4).

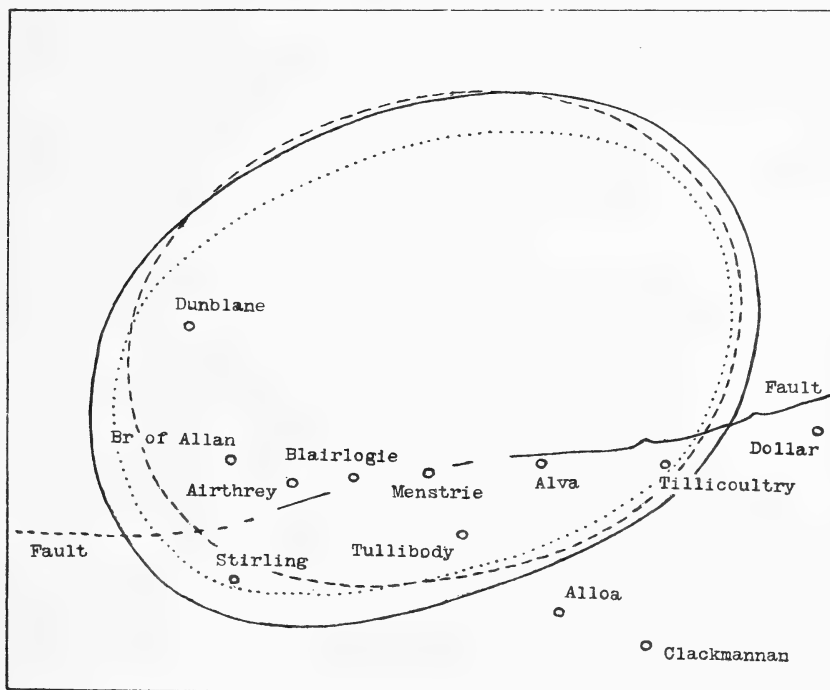


FIG. 4.—Earthquake of Oct. 8 (7.24 a.m.), 1906: Continuous line. Earthquake of Dec. 28, 1906: Broken line. Earthquake of Dec. 30 (4.15 p.m.), 1906: Dotted line.

The disturbed area is bounded by an isoseismal of intensity 4, which is 12 miles long,  $9\frac{1}{2}$  miles wide, and 90 square miles in area. Its centre is 2 miles north of Menstrie. At Logie, Menstrie, and Tillicoultry the shock consisted as a rule of one prominent vibration, its intensity being 5. The sound was heard by 86 per cent. of the observers, and in 57 per cent. of the records was compared to the dull concussion caused by the fall of a heavy body.

(23) 1906, *October 8*, 8.16 *a.m.*

A shock (intensity 4) was felt at Airthrey, Alva, Blairlogie, Bridge of Allan, Dunblane, Logie, Menstrie, and Red Carr. The disturbed area was probably about  $9\frac{1}{2}$  miles long, 6 miles wide, and 45 square miles in area. The shock consisted of two vibrations, lasting not more than two seconds, and was accompanied by a sound resembling that of a muffled explosion.

(24) 1906, *October 12*, 7.20 *a.m.*

A distinct shock, felt at Menstrie.

(25) 1906, *October 20*, 7.15 *a.m.*

A slight tremor, with sound, observed at Airthrey.

(26) 1906, *October 24*, 7.11 *p.m.*

A report, like that of a distant cannon, heard generally at Menstrie.

(27) 1906, *October 26*, 7.15 *p.m.*

A slight shock, felt at Airthrey and Menstrie; the shock consisted of two vibrations at Menstrie, and at Airthrey was accompanied by sound.

(28) 1906, *October 30*, 12.15 *p.m.*

A slight shock and sound, observed at Menstrie.

(29) 1906, *December 28*, 4.12 *p.m.*

Intensity, 6; centre of isoseismal 4 in lat.  $56^{\circ} 11' 3''$  N., long.  $3^{\circ} 51' 0''$  W.; number of records, 10, from 7 places, and 5 negative records from 5 places (fig. 4).

The shock was felt at Airthrey, Alva, Burnside Farm (Braco), Dunblane, Menstrie, Red Carr, and Tillicoultry. The disturbed area was probably about 11 miles long,  $8\frac{1}{2}$  miles wide, and about 74 square miles in area, its centre being  $2\frac{1}{2}$  miles north of Menstrie. The intensity of the shock was 6 at Menstrie. The sound was heard by 78 per cent. of the observers. From the rapid decline in the intensity of the shock outwards from Menstrie, it follows that the focus was situated at a slight depth.

(30) 1906, *December 29*, 1.30 *p.m.*

A slight shock, accompanied by a noise like the fall of a heavy body, felt at Airthrey.

(31) 1906, *December 30*, about 1 *a.m.*

A shock, felt at Menstrie.



(32) 1906, *December 30*, 2.10 *p.m.*

A shock (intensity 3), felt at Airthrey, Alva, Dunblane, Menstrie, and Red Carr. The sound was like the fall of a heavy body.

(33) 1906, *December 30*, 4.15 *p.m.*

Intensity, 6; centre of isoseismal 4 in lat.  $56^{\circ} 10' 8''$  N., long.  $3^{\circ} 51' 3''$  W.; numbers of records, 13, from 8 places, and 6 negative records from 5 places (fig. 4).

The disturbed area is about 11 miles long,  $8\frac{1}{2}$  miles wide, and about 82 square miles in area, its centre being about 2 miles north of Menstrie. The intensity of the shock was 6 at Menstrie. The sound was heard by all the observers.

(34) 1906, *December 31*, 1 *a.m.*

A shock, accompanied by sound, felt at Menstrie.

(35) 1907, *February 10*, 5.40 *p.m.*

A slight shock, felt at Menstrie.

(36) 1907, *March 19*, 7.33 *p.m.*

A shock (intensity 5), felt at Airthrey, Alva, Bridge of Allan, and Menstrie. It consisted of two prominent vibrations, and was accompanied by a loud report, like that of an explosion.

(37) 1907, *April 7*, 11.11 *p.m.*

A very slight shock, accompanied by sound, felt at Menstrie.

(38) 1907, *April 7*, 11.19 *p.m.*

A slight shock, consisting of two vibrations, felt at Menstrie.

(39) 1907, *April 8*, 6.45 *a.m.*

A slight shock, felt at Menstrie.

(40) 1907, *April 11*, 5.30 *a.m.*

A shock (intensity 4), accompanied by sound, observed at Menstrie and Red Carr.

(41) 1907, *April 11*, 5.40 *a.m.*

A shock (intensity 4), felt at Airthrey, Menstrie, and Red Carr.

(42) 1907, *April 11*, 6.5 *a.m.*

A shock, with very slight noise, observed at Menstrie.

(43) 1907, *June 14, 1.59 a.m.*

A slight shock, felt at Menstrie, consisting of two vibrations, each accompanied by sound.

(44) 1907, *June 20, 3.36 p.m.*

A shock (intensity 4), felt at Alva and Menstrie, preceded and accompanied by a rumbling sound.

(45) 1907, *July 5, 9.48 p.m.*

A slight shock, consisting of a single vibration, felt at Menstrie.

(46) 1907, *July 21 or 28, between 5 and 7.30 p.m.*

A slight shock, felt at Menstrie. The day of the shock was one of the last two Sundays in July.

(47) 1907, *September 18, about 5.30 p.m.*

A very slight shock, felt at Menstrie.

(48) 1908, *January 19, 1.27 a.m.*

A distinct shock, felt at Menstrie.

On February 9, 1908, at 4.6 a.m., a shock, stronger than the preceding, was felt at Menstrie, but only, so far as known, by one observer.

(49) 1908, *May 1, 6.54 p.m.*

A shock (intensity 4), felt at Airthrey, Alva, Dunblane, Menstrie, and Tillicoultry. The shock consisted of a single series of vibrations, with one maximum of intensity, and lasted two seconds. It was accompanied by a loud noise like a muffled explosion.

(50) 1908, *May 2, 7.5 a.m.*

A shock (intensity 4), lasting three seconds, felt at Airthrey, Alva, and Menstrie. At Airthrey it consisted of two concussions, with intervening tremor, the latter concussion being the stronger. The shock was preceded, accompanied, and followed by a rumbling sound.

(51) 1908, *May 10, 12.48 a.m.*

A shock (intensity 4), lasting about four seconds, felt at Airthrey, Alva, Menstrie, Tillicoultry, and Tullibody. At Airthrey and Menstrie it consisted of two concussions, each accompanied by a loud noise like an underground explosion, the former being much the stronger at Airthrey, and both of about the same intensity at Menstrie.

(52) 1908, *May* 10, 12.58 *a.m.*

A shock (intensity 4), slighter than the preceding, felt at Airthrey, Alva, and Menstrie. It was accompanied by a muffled sound like that of an explosion.

(53) 1908, *June* 21, 3 *a.m.*

A distinct shock, consisting of a single series of vibrations, felt at Alva and Menstrie, at the former place being accompanied by a loud noise.

(54) 1908, *June* 21, 4.20 *a.m.*

A slight but distinct shock, consisting of a single series of vibrations, felt at Menstrie.

(55) 1908, *July* 17, 5.27 *p.m.*

A slight but distinct tremor, felt at Alva and Menstrie, and accompanied by noise.

(56) 1908, *September* 2, 8.16 *a.m.*

A shock (intensity 3), consisting of a single series of vibrations, felt at Menstrie, accompanied by a rumbling noise.

(57) 1908, *September* 2, 8.51 *a.m.*

A shock (intensity 3), consisting of a single series of vibrations, felt at Menstrie, accompanied by a rumbling noise.

(58) 1908, *October* 16, 9.53 *p.m.*

A tremor (intensity 4), lasting two seconds, felt at Airthrey, Alva, Blair Ochil (Dunblane), Menstrie, and Red Carr. At Menstrie the shock consisted of two prominent vibrations. The sound was compared to an underground explosion, a clap of thunder, or the thud of falling rock.

(59) 1908, *October* 19, 9.18 *a.m.*

A slight tremor, accompanied by sound, felt at Airthrey. The shock was also felt at Blair Ochil (Dunblane).

(60) 1900, *October* 19, 9.39 *a.m.*

A noise, without any tremor, heard at Menstrie.

(61) 1908, *October* 20, 4.8 *p.m.* (*Second Principal Earthquake.*)

Intensity, 7 ; centre of isoseismal 6 in lat.  $56^{\circ} 11' 4''$  N. ; long.  $3^{\circ} 47' 3''$  W. ; number of records, 59, from 34 places, and 20 negative records from 18 places (fig. 5).

This shock was stronger than all the earlier recorded shocks in the Ochil district. The intensity was not less than 7 at Alva and Tillicoultry, while that of the earthquake of September 21, 1905, was at no place higher than 6.

The only isoseismal which it is possible to draw is that of intensity 6. This is indicated by the continuous line in fig. 5. The curve is 15 miles long,  $10\frac{1}{2}$  miles wide, and contains 123 square miles. Its centre is  $3\frac{1}{2}$  miles E.  $48^\circ$  N. of Menstrie, and the direction of its longer axis is E.  $25^\circ$  N.

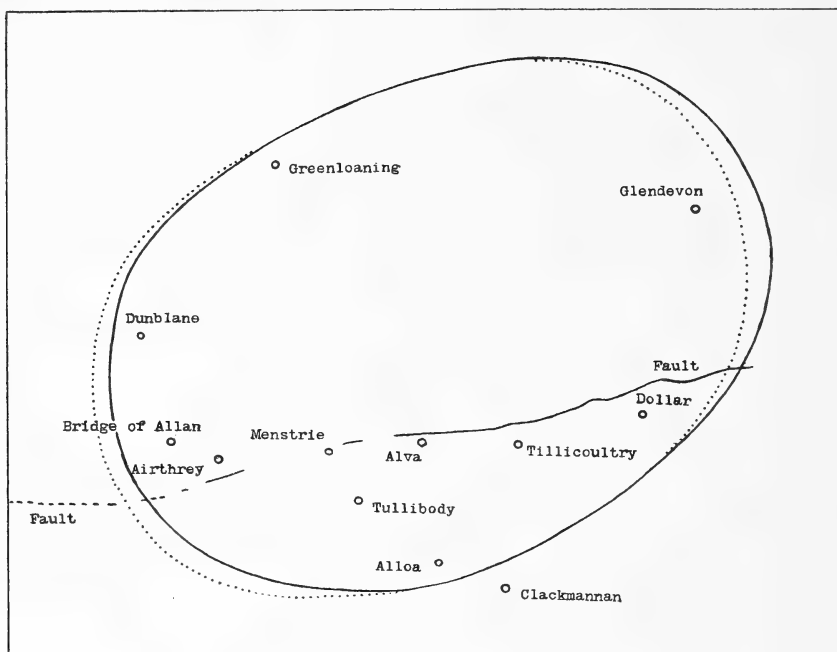


FIG. 5.—Earthquake of Oct. 20 (4.8 p.m.), 1908.

The boundary of the disturbed area, which cannot be drawn with any approach to accuracy, differs slightly from that of the earthquake of 1905, for the shock was felt one or two miles farther to the south, at Falkirk, Polmont, and Bo'ness, while it was not felt at Comrie, Crieff, and Monzie.

At several places within the isoseismal 6 (Alloa, Cambus, Greenloaning, Menstrie, and Tillicoultry), the shock consisted of two distinct parts, separated by an interval of about 3 seconds, the first part being much the stronger. At places outside this isoseismal, only one part was observed, the effect resembling that caused by the fall of a heavy weight, or the passage of a traction engine. The average of fifteen estimates of the duration of the shock is  $2\frac{3}{4}$  seconds.

The sound which accompanied the shock was heard by 94 per cent. of

the observers within the isoseismal 6, by 87 per cent. of those outside that isoseismal, and by 92 per cent. of those in the whole disturbed area. It is compared to passing waggons, etc., in 47 per cent. of the records, to thunder in 27, to the fall of a heavy body in 17, and to explosions in 10 per cent.

(62) 1908, *October 20*, 4.13 *p.m.*

Intensity, 5; centre of disturbed area in lat.  $56^{\circ} 11' 3''$  N., long.  $3^{\circ} 48' 0''$  W.; number of records, 18, from 13 places (fig. 5).

The boundary of the disturbed area (represented by the broken line in fig. 5) coincides nearly with the isoseismal 6 of the preceding earthquake, except that it is displaced about half a mile to the west-south-west. It is 15 miles long,  $10\frac{1}{2}$  miles wide, and contains about 123 square miles. Its centre is 3 miles E.  $50^{\circ}$  N. of Menstrie, and half a mile west-south-west of that of the isoseismal 6 of the preceding earthquake. Its longer axis is directed E.  $25^{\circ}$  N. The intensity of the shock was 5 at Tillicoultry. The shock was merely a brief tremor, lasting about  $1\frac{1}{2}$  seconds, without any prominent vibration or "thud." The sound was compared to thunder or an explosion.

(63) 1908, *October 20*, 9.26 *p.m.*

A very slight shock, felt at Airthrey and Menstrie. At Airthrey the sound and vibration began together and terminated simultaneously in a thud.

(64) 1908, *November 6*, 4.45 *p.m.*

A noise, heard at Menstrie.

(65) 1909, *January 19*, 5.28 *a.m.*

A shock (intensity 4), accompanied by a loud sound, felt at Airthrey, Alva, and Menstrie.

(66) 1909, *January 19*, 5.29 *a.m.*

A shock, felt at Menstrie. Two other tremors were also felt at Alva after 5.28 a.m., but the times were not recorded.

(67) 1909, *January 23*, 12.15 *p.m.*

A slight shock (intensity 3), accompanied by a noise like that of an underground explosion, felt at Airthrey and Menstrie.

(68) 1909, *January 24*, 12.15 *p.m.*

A shock (intensity 3), felt at Menstrie.

(69) 1909, *January 24*, 2.28 *p.m.*

A tremor, felt at Airthrey.

(70) 1909, *January 24*, 3.35 *p.m.*

A tremor, of about the same intensity as the preceding, felt at Airthrey.

(71) 1909, *January 27*, 1.40 *p.m.*

A slight shock, felt at Menstrie.

(72) 1909, *February 22*, 7.26 *p.m.*

A noise, heard at Menstrie.

(73) 1909, *March 19*, 9.35 *a.m.*

A shock, accompanied by noise, felt at Menstrie.

(74) 1909, *May 22*, 3.23 *p.m.*

A shock (intensity 5), accompanied by a noise like that of an explosion, felt at Airthrey and Menstrie.

(75) 1909, *May 22*, 5.1 *p.m.*

A shock (intensity 4), accompanied by a noise like that of a slight explosion, felt at Airthrey.

(76) 1909, *May 22*, 5.24 *p.m.*

A slight shock, felt at Menstrie.

(77) 1909, *May 22*, 8.23 *p.m.*

A shock (intensity 5), accompanied by a sound like that of an explosion, felt at Airthrey.

(78) 1909, *October 21*, 8.37 *a.m.*

A shock (intensity 4), followed by a sound like that of an explosion, felt at Airthrey.

(79) 1909, *October 21*, 9.53 *a.m.*

A slight shock, felt at Menstrie.

(80) 1909, *October 22*, 6.55 *a.m.*

A slight shock, felt at Menstrie.

(81) 1909, *October 22*, 7.57 *a.m.*

A shock (intensity 4), preceded and followed by a sound like that of a heavy body falling, felt at Airthrey.

(82) 1909, *October 22*, 9.8 *p.m.*

At Airthrey there were three concussions, separated by intervals of two and five seconds, preceded and followed by sounds like those of sharp explosions.

(83) 1910, *February 8*, 9.33 *a.m.*

A shock (intensity 4), felt at Airthrey, Bridge of Allan, Menstrie, and Red Carr. At Airthrey it consisted of two parts, separated by an interval of two seconds, the first part being the stronger. The sound resembled that of an explosion.

(84) 1910, *February 10*, 9.30 *a.m.*

A loud, rumbling noise, heard at Menstrie.

(85) 1910, *February 10*, 9.50 *a.m.*

A shock (intensity 4), felt at Airthrey, Bridge of Allan, and Menstrie. At Airthrey it consisted of two parts, separated by an interval of two seconds, the first part being the stronger. The sound resembled that of an explosion.

(86) 1910, *February 11*, 10.55 *p.m.*

A shock (intensity 4), consisting of a single series of vibrations, and accompanied by a sound like that of an explosion, was felt at Airthrey.

(87) 1910, *April 17*, 1.45 *a.m.*

A shock (intensity 4), consisting of a single series of vibrations, and accompanied by a sound like that of a heavy waggon passing, was felt at Airthrey.

(88) 1910, *May 19*, 9.45 *a.m.*

A bumping noise, as of a huge rock rolled over and over, heard at Menstrie.

(89) 1910, *May 20*, 2.40 *p.m.*

A shock (intensity 4), felt at Airthrey, Alva, and Menstrie. At Airthrey the sound, which preceded the shock, was compared to that of an explosion.

(90) 1910, *May 20*, 2.50 *p.m.*

A shock (intensity 4), preceded by a sound like that of an explosion, was felt at Airthrey and Alva.

(91) 1910, *May* 20, 3.5 *p.m.*

A shock (intensity 4), consisting of a single vibration, and preceded by a sound like that of an explosion, was felt at Airthrey and Menstrie.

(92) 1910, *May* 20, 3.15 *p.m.*

A bumping noise, heard at Menstrie.

(93) 1910, *May* 27, 12.30 *a.m.*

A shock (intensity 4), followed by a sound like the firing of heavy guns, was felt at Airthrey.

(94) 1910, *July* 10, 3.7 *p.m.*

A fairly sharp shock (intensity 4), preceded and followed by a sound like that of an explosion, was felt at Airthrey. The noise was also heard at Menstrie.

(95) 1910, *July* 11, 12.1 *a.m.*

A shock (intensity 4), accompanied by a sound like that of an explosion, was felt at Airthrey.

(96) 1910, *July* 22, 11.59 *p.m.*

A shock (intensity 4), accompanied by a sound like that of an explosion, was felt at Airthrey.

(97) 1910, *October* 25, 8.55 *p.m.*

A bumping noise, heard at Menstrie.

(98) 1910, *October* 29, 7.41 *p.m.*

A bumping noise, heard at Menstrie.

(99) 1910, *November* 20, 1.57 *p.m.*

A bumping noise, consisting of two loud reports, heard at Menstrie.

(100) 1910, *December* 7, 5.54 *p.m.*

A slight shock, with a bumping noise, was felt at Menstrie.

(101) 1910, *December* 8, 3.45 *p.m.*

A slight shock, with a bumping noise, was felt at Menstrie.

(102) 1911, *July* 27, 8.10 *a.m.*

A bumping noise, heard at Menstrie.



(103) 1911, *July 27, 5.15 p.m.*

A bumping noise, heard at Menstrie.

(104) 1911, *July 28, 7.46 p.m.*

A bumping noise, heard at Menstrie.

(105) 1911, *October 12, 1.50 a.m.*

A shock (intensity 4), accompanied by a loud sound, was felt at Alva and Menstrie.

(106) 1911, *October 12, about 2.20 a.m.*

A shock, slighter than the preceding, and accompanied by a loud sound, was felt about half an hour later at Alva.

(107) 1911, *October 12, 3.45 a.m.*

A shock, accompanied by a bumping noise, was felt generally in Menstrie.

(108) 1911, *October 17, 6.35 a.m.*

A slight shock, accompanied by a bumping noise, was felt generally in Menstrie.

(109) 1911, *October 21, 1.25 p.m.*

A shock (intensity 5), accompanied by a loud sound, was felt at Alva. The sound, consisting of three thuds, was heard at Menstrie.

(110) 1912, *January 20, about 4 a.m.*

A single slight vibration, lasting about two seconds, was felt at Bridge of Allan.

(111) 1912, *January 26, 3.59 a.m.*

A shock (intensity 5), consisting of a single series of vibrations, was felt at Alva, Dunblane, and Menstrie. The sound was compared to dull thunder, the fall of a heavy body, and an approaching train in a tunnel ending with a detonation as of a deep explosion.

(112) 1912, *January 28, 4 a.m.*

A shock (intensity 4) was felt at Airthrey, Coalsnaughton, Dunblane, Kinbuck, and Tillicoultry; but not at Alloa, Cambus, Greenloaning, and Tullibody. The disturbed area contained about 80 square miles. The shock consisted of a single series of vibrations lasting two seconds. The sound resembled that of a load of coals falling.

(113) 1912, *January* 28, 4.40 *p.m.*

A single series of vibrations, accompanied by sounds like those of wind and a heavy body falling, was felt at Red Carr.

(114) 1912, *January* 28, 5.25 *p.m.*

A rather strong shock, accompanied by a loud noise, was felt at Red Carr.

(115) 1912, *January* 30, 5.25 *p.m.*

A shock (intensity 4), consisting of a single series of vibrations lasting for two or three seconds, was felt at Menstrie. A sound like loud thunder followed the shock.

(116) 1912, *January* 31, 5.29 *p.m.*

A slight shock (intensity 3), accompanied by a noise like that of a prolonged explosion or the distant firing of a cannon, was felt at Dunblane. A very loud sound was heard at Menstrie.

(117) 1912, *February* 1, 5.25 *p.m.*

A shock (intensity 4), accompanied by a loud sound, was felt at Alva.

(118) 1912, *February* 9, 1.23 *p.m.*

Two vibrations (intensity 4), followed by tremulous motion, were felt at Alva. The sound was heard at Alva and Menstrie.

(119) 1912, *February* 9, 1.32 *p.m.*

The shock (intensity 5) was felt at Alva, Coalsnaughton, Dunblane, Logie, and Menstrie. The disturbed area towards the south and west was probably bounded by a curve coinciding nearly with the isoseismal 6 of the earthquake of September 21, 1905, but not extending quite as far to the east as that isoseismal. It must therefore have contained somewhat less than 80 square miles. The shock consisted of a single series of vibrations, and lasted two or three seconds. The sound was compared to dull thunder, a heavy body falling, and an explosion in a quarry.

(120) 1912, *February* 9, 1.39 *p.m.*

A very slight bumping noise, like an echo of that which accompanied the preceding shock, was heard by many people at Menstrie.

(121) 1912, *February* 11, 3.45 *a.m.*

A slight tremor (intensity 3), and accompanied by a sound like that of the fall of a heavy body, was felt at Alva.

(122) 1912, *February* 24, 3.15 *a.m.*

The shock, lasting about one second, was felt at Alva. The accompanying sound, like a double knock, was heard at Alva and Menstrie.

(123) 1912, *February* 24, 4.53 *a.m.*

A very loud bumping noise, heard at Menstrie.

(124) 1912, *March* 3, 2.24 *p.m.*

A slight noise, heard at Menstrie.

(125) 1912, *March* 20, 7.50 *a.m.*

A slight tremor, felt at Alva.

(126) 1912, *March* 21, 10.20 *p.m.*

A very distinct double tremor, felt at Alva.

(127) 1912, *March* 21, 10.25 *p.m.*

A slight tremor, felt at Alva.

(128) 1912, *March* 21-22, 12.0.

A slight tremor, felt at midnight at Alva.

(129) 1912, *March* 22, 5.30 *a.m.*

A slight tremor, felt at Alva.

(130) 1912, *March* 22, 7.50 *a.m.*

The shock (intensity 5) was felt at Airthrey, Alva, Dunblane, and Menstrie. The sound was compared to the fall of a load of stones, the fall of a heavy body, and the sound of cannon firing with reverberation.

(131) 1912, *March* 22, 10.15 *p.m.*

The shock (intensity 5), consisting of a single series of vibrations, felt at Airthrey, Alva, Dunblane, and Menstrie. The sound resembled thunder or an explosion.

(132) 1912, *March* 22, 11.15 *p.m.*

A noise, as of distant thunder, was heard at Dunblane and Menstrie.

(133) 1912, *March* 23, 5.33 *a.m.*

The shock (intensity 5), consisting of a single series of vibrations, accompanied by a noise like that of wind or heavy waggons passing, was felt at Airthrey, Alva, Dunblane, and Menstrie.

(134) 1912, *March 23*, 6.8 *a.m.*

A bumping noise, heard at Menstrie.

(135) 1912, *April 3*, 10.33 *a.m.*

A noise was heard at Airthrey and Menstrie, at the former place like that of an engine passing.

(136) 1912, *April 3*, 5.20 *p.m.*

A bumping noise, heard at Menstrie.

(137) 1912, *April 3*, 5.35 *p.m.*

A bumping noise, heard at Menstrie.

(138) 1912, *April 18*, 8.14 *p.m.*

A bumping noise, heard at Menstrie.

(139) 1912, *April 18*, 10.15 *p.m.*

A bumping noise, heard at Menstrie.

(140) 1912, *April 19*, 8.11 *p.m.*

A shock (intensity 4), increasing in strength to a maximum and then dying away, was felt at Airthrey, Alva, Doune, and Dunblane. The duration of the shock was about three seconds. The sound resembled a loud rumble of thunder.

(141) 1912, *April 19*, 9.10 *p.m.*

A rather strong shock, accompanied by sound and lasting four seconds, was felt at Airthrey.

(142) 1912, *April 19*, 10.10 *p.m.*

A shock, accompanied by the usual noise, and lasting about three seconds, was felt at Alva and Dunblane.

(143) 1912, *April 19*, 10.30 *p.m.*

A slight shock, felt at Doune and Tullibody.

(144) 1912, *April 19-20*, *midnight*.

A sound, heard at Doune.

(145) 1912, *April 20, 2 a.m.*

A rather strong shock, felt at Airthrey and Dunblane. At Airthrey it consisted of two parts, the first lasting four seconds, the second, two seconds, with an interval of two seconds between.

(146) 1912, *April 20, 2.5 a.m.*

A shock, of less intensity than the preceding, felt at Airthrey, Doune, and Tullibody.

(147) 1912, *April 20, 2.8 a.m.*

A shock (intensity 4), felt at Dunblane and in the surrounding district.

(148) 1912, *April 20, 2.10 a.m.*

A shock of less intensity than the preceding, but strong enough to awaken people, was felt at Alva, Doune, Dunblane, and Menstrie. A bumping noise was heard with it at Menstrie.

(149) 1912, *April 20, 2.12 a.m.*

A bumping noise, heard at Menstrie.

(150) 1912, *April 20, 2.14 a.m.*

A slight shock was felt at Dunblane, and a noise was heard at Menstrie.

(151) 1912, *April 20, 2.18 a.m.*

A bumping noise, heard at Menstrie.

(152) 1912, *April 20, 4.15 a.m.*

A bumping noise, heard at Menstrie.

(153) 1912, *April 21, 1.5 a.m.*

A slight tremor was felt at Alva, Dunblane, and Menstrie, and was accompanied by the usual bumping noise at Menstrie.

(154) 1912, *April 21, 2.47 p.m.*

A bumping noise, heard at Menstrie.

(155) 1912, *April 22, 7.17 a.m.*

A slight tremor, accompanied by the usual bumping noise, felt at Menstrie.

(156) 1912, *April 23*, 6.10 *a.m.*

The shock was felt as a thud (with sound) at Airthrey, and as a slight tremor at Alva.

(157) 1912, *April 26*, about 4 *a.m.*

A shock (intensity 4), felt at Bridge of Allan.

(158) 1912, *May 3*, 6.20 *a.m.*

A slight tremor, felt at Airthrey.

(159) 1912, *May 3*, about 2.15 *p.m.*

A slight shock, accompanied by a sound like distant thunder, was felt at Alva.

(160) 1912, *May 3*, 4.13 *p.m.* (*Third Principal Earthquake.*)

Intensity, 7; centre of isoseismal 6 in lat.  $56^{\circ} 10' 5''$  N., long.  $3^{\circ} 52' 5''$  W.; number of records, 91, from 53 places, and 11 negative records from 11 places (fig. 6).

Though the disturbed area of this earthquake is less than that of the earthquakes of September 21, 1905, and October 20, 1908, the shock within the central isoseismal was certainly stronger than that of any other felt during the present century. A small increase of intensity would probably have brought it within the range of destructive earthquakes.

On the map of the earthquake are shown four isoseismal lines, corresponding to intensities 7, 6, 5, and 4. Portions of three of these curves are somewhat doubtful owing to insufficient observations, and are indicated by broken lines. The isoseismal 7 is  $8\frac{1}{2}$  miles long,  $4\frac{3}{4}$  miles wide, and 32 square miles in area. The isoseismal 6, which is the most accurately drawn of the series, is 14 miles long,  $8\frac{1}{4}$  miles wide, and contains an area of 92 square miles. Its centre is about 2 miles N.N.W. of Menstrie, and the direction of its longer axis is E.  $7\frac{1}{2}^{\circ}$  N. The isoseismal 5 is 19 miles long,  $12\frac{1}{4}$  miles wide, and 182 square miles in area. The isoseismal 4, which may be regarded as the boundary of the disturbed area, is 30 miles long,  $29\frac{1}{2}$  miles wide, and contains 605 square miles. The distance between the isoseismals 7 and 6 is  $2\frac{1}{4}$  miles on the north side and  $1\frac{1}{4}$  miles on the south; between the isoseismals 6 and 5,  $2\frac{1}{2}$  miles on the north side and  $1\frac{1}{4}$  miles on the south; between the isoseismals 5 and 4, 9 miles on the north side and  $4\frac{1}{4}$  miles on the south.

Outside the isoseismal 4, the earthquake was felt by persons lying down at Dunfermline (17 miles from the centre of the isoseismal 6), Glasgow (26 miles), Rutherglen ( $27\frac{1}{2}$  miles), and Musselburgh (35 miles). If the disturbed area were regarded as bounded by an isoseismal of

intensity 2, in the form of a circle with a radius of 35 miles, its area would be about 3850 square miles.

The shock is usually described as a single series of about four prominent vibrations, followed, and occasionally preceded, by a tremulous motion. It thus only differed from the slighter shocks in possessing more than one prominent vibration or thud. As in the earthquake of October 20, 1908, a few observers (at Alloa, Blair Drummond, Greenloaning, Logie, and

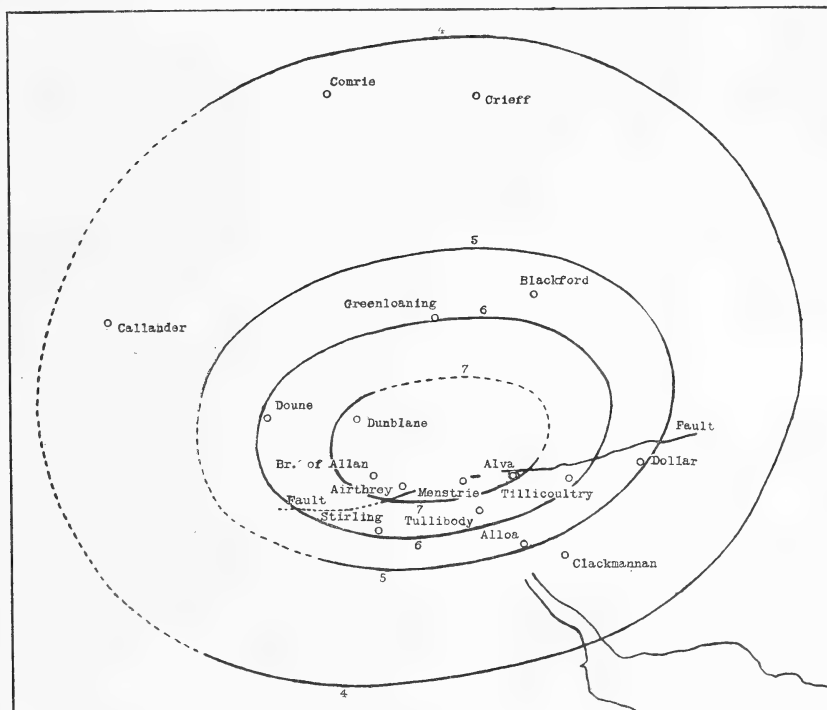


FIG. 6.—Earthquake of May 3 (4.13 p.m.), 1912.

Stirling) detected two series of vibrations, the first of which (except at Logie) was the stronger, the mean duration of the interval between the series being two seconds. The average of thirty-eight estimates of the total duration of the shock is 3.0 seconds.

The sound-area is co-extensive with the disturbed area, but towards the boundary there is a marked decline in audibility. While the percentage of audibility is 92 for the whole area, it is 100 within the isoseismal 7, 96 between the isoseismals 7 and 5, and 72 between the isoseismals 5 and 4. The sound is compared to passing waggons, etc., in 20 per cent. of the records, thunder in 22, wind in 4, loads of stones falling in 6, a heavy body falling in 26, and explosions in 22 per cent.

(161) 1912, *May 3*, 11.45 *p.m.*

A shock, accompanied by a loud thud, with rumbling before and after, was felt at Airthrey. A shock was also felt during the evening of May 3 at Tillicoultry.

(162) 1912, *May 4*, 12.10 *a.m.*

A slight shock, consisting of a single series of vibrations, was felt at Alva, Blackford, Dunblane, and Kinbuck. The sound was compared to that of a heavy body falling and an explosion.

(163) 1912, *May 4*, 2 *a.m.*

A shock, (intensity 3), accompanied by a noise like that of a heavy body falling, was felt at Alva.

A slight shock was felt (so far as I know, by one observer only) on May 4 or 5, at about 11.50 *p.m.*, at Auchendoune (Doune).

(164) 1912, *May 9*, 11.28 *p.m.*

The shock was felt at Airthrey, Alva, and Dunblane. At Airthrey it consisted of a thud (with noise), preceded and followed by tremors.

(165) 1912, *May 10*, 9.20 *p.m.*

The shock, which consisted of a thud, preceded and followed by tremors, was felt at Airthrey, Alva, Doune, and Dunblane. Its duration was about 2 or  $2\frac{1}{2}$  seconds. The sound resembled that of an explosion.

(166) 1912, *May 11*, about 11.23 *p.m.*

A slight vibration and noise, increasing to a maximum and then dying away, were observed at Dunblane.

(167) 1912, *May 14*, 6.54 *a.m.*

A slight shock (intensity 3), like that of a heavy weight falling with a thud, was felt at Airthrey, Alva, Dunblane, and Logie. The sound, as of the fall of a heavy body, was heard at Airthrey.

(168) 1912, *May 14*, 6.58 *a.m.*

A shock (intensity 3), similar to the preceding, and lasting two seconds, was felt at Airthrey, Alva, Dunblane, Greenloaning, and Logie.

(169) 1912, *May 18*, 4 *a.m.*

A slight shock, felt at Dunblane.



(170) 1912, *May* 19, 3.50 *a.m.*

A thud (with sound), preceded and followed by tremors, was felt at Airthrey.

(171) 1912, *May* 19, 11.50 *p.m.*

A slight shock (with sound) was felt at Airthrey, Bridge of Allan, and Dunblane.

A slight shock during the night of May 20-21 was felt by one observer at Bridge of Allan.

(172) 1912, *July* 5, 1.50 *a.m.*

A shock, followed after one or two seconds by one less pronounced, was felt at Greenloaning. Each part was accompanied by a sound as of a heavy body falling, the former sound being the more intense.

(173) 1912, *July* 6, 1.52 *a.m.*

A slight but distinct shock was felt at Alva, Doune, and Dunblane. At Alva, the sound resembled that of a falling body, and at Doune, that of an explosion.

(174) 1912, *July* 6, *about* 2.7 *a.m.*

A very slight tremor, felt at Dunblane.

(175) 1912, *July* 7, 10.20 *a.m.*

A slight vibration and rumbling noise, observed at Alva.

(176) 1912, *July* 9, 1.25 *p.m.*

A shock, lasting one or two seconds, and accompanied by very little noise, was felt at Bridge of Allan.

(177) 1912, *July* 17, 2.25 *a.m.*

A shock (intensity 4), consisting of a single series of vibrations, was felt at Greenloaning. It was accompanied by a sound like that of wind blowing on a door.

(178) 1912, *July* 17, *about* 3.20 *a.m.*

A distinct shock, lasting four or five seconds, and strong enough to awaken most people, was felt at Dunblane.

(179) 1912, *July* 18, 3.15 *a.m.*

A slight shock, accompanied by a noise like that of a falling body, was felt at Alva.

(180) 1912, *July 29, 9.22 a.m.*

A slight shock, accompanied by a noise like that of a falling body, was felt at Alva.

(181) 1912, *September —, about midnight.*

On a day about the middle of the month, a slight tremor was felt at Dunblane.

(182) 1912, *October 18, about 4.15 p.m.*

A slight but distinct shock, felt at Dunblane.

(183) 1912, *November 6, 5.15 a.m.*

A slight shock, accompanied by a noise like that of a falling body, felt at Alva.

(184) 1913, *August 27, 4.30 a.m.*

A thud, followed by a tremor, and succeeded immediately by a second thud and tremor, the whole lasting about four seconds, was felt at Airthrey.

(185) 1913, *October 28, 4.30 a.m.*

A shock (intensity 4), accompanied by a sound like that of a falling body, was felt at Alva.

(186) 1914, *December 18, 12.54 a.m.*

A slight shock was felt at Airthrey, Alva, Dunblane, and Menstrie. At Airthrey there were two distinct shocks in rapid succession, like two shots fired from a cannon in the immediate neighbourhood.

### III. CHARACTERISTICS OF THE OCHIL EARTHQUAKES.

The earthquakes felt in the Ochil district from 1900 to 1914 (186 in number) may be divided into three classes:—

(i) The first class contains the three principal earthquakes of September 21, 1905; October 20, 1908; and May 3, 1912, which were of intensities 6, 7, and 7, and disturbed areas of 1000, about 1000, and at least 605 square miles, respectively.

(ii) The second class contains 10 earthquakes, namely, those of September 17 (10.15 p.m.), 1900; July 23 and 26, 1905; October 8 (7.24 a.m.), December 28 and 30 (4.15 p.m.), 1906; October 20 (4.13 p.m.), 1908; January 28 (4 a.m.), February 9 (1.32 p.m.), and May 4 (12.10 a.m.), 1912. One of these earthquakes was of intensity 3, 3 of intensity 4, 4 of intensity 5, and 2 of intensity 6. The disturbed area ranges from 74 to 136 square miles, the average for all the earthquakes being about 94 square miles.

(iii) The third class contains the remaining 173 earthquakes. In 24 of these, it is doubtful whether any tremor was felt. Of the remainder,

8 were of intensity 5, 34 of intensity 4, 101 of intensity 3 or nearly so, while 6 were earth-sounds without any accompanying tremor. The disturbed area is generally very small, rarely, if ever, exceeding 60 square miles.

*Periodicity.*—The monthly distribution of the earthquakes is shown in Table I, the first half of the month being the first 14 days in February and the first 15 days in the other months.

TABLE I.

	J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.
First half . . .	0	10	1	9	16	1	11	1	2	7	2	2
Second half . . .	15	3	12	22	13	3	14	2	10	22	1	8

Taking account of the slightly unequal lengths of the halves of the months, and using the method of six-monthly means for the annual period and of six-half-monthly means for the semi-annual period, it would appear that both periods are well marked. In the annual period, the maximum epoch is about the end of April and the amplitude 0·38; in the semi-annual period, the maximum epochs are in April and October and the amplitude 0·39.\*

For the successive hours of the day (counting from midnight), the numbers of earthquakes are 8, 8, 9, 8, 7, 8, 9, 8, 5, 9, 3, 0, 4, 8, 8, 8, 8, 14, 3, 6, 4, 8, 9, 9. Thus, during the twelve night-hours (6 p.m. to 6 a.m.) there were 80 earthquakes; during the twelve day-hours (6 a.m. to 6 p.m.), 82.

*Nature of the Shock.*—When the shock is of intensity 3, it consists as a rule of a series of tremors, the separate vibrations of which do not vary much in strength. Sometimes, however, only one vibration is felt; and, in a few cases, there is one prominent vibration, usually called a “thud,” followed by slight tremulous motion, the effect being like that caused by the fall of a heavy weight on the floor, succeeded by the quivering which such a fall would produce. When the intensity of the shock attains the degree 4 or 5, the thud, followed by a tremor, is the normal form, series of vibrations of nearly uniform strength being less frequently felt. Shocks of intensity 6 or 7 contain from two to four prominent vibrations, preceded and followed by tremulous motion. In a few cases,

\* If any number (*n*) of earthquakes were to occur at random, Dr Schuster has shown that the amplitude should exceed  $\sqrt{(\pi/n)}$ , which in the present case is ·13 (*Roy. Soc. Proc.*, vol. lxi, 1897, pp. 455-465). \*Though the above figures for the amplitude are about three times this amount, it seems to me doubtful whether the results obtained above are of much value, owing to the marked occurrence of the earthquakes in a limited number of series.

the shock consists of two parts, separated by an interval of about two seconds, but there is no reason whatever for regarding such as twin earthquakes.

*Sound-Phenomena.*—In the earthquakes of the first class, the sound was heard by 88 per cent. of the observers; in those of the second class by 92 per cent.; and in those of the third class by 98 per cent. The increasing percentage as the intensity lessens is due to the fact that the sound is by far the more prominent feature in the slighter earthquakes.

The variation in the nature of the sound in the three classes of earthquakes is shown in Table II, in which the figures represent percentages for each class of earthquake to the different types of sound (Davison scale).

TABLE II.

	Waggons passing.	Thunder.	Wind.	Loads of Stones falling.	Fall of a Heavy Body.	Explo- sions.	Miscel- laneous.
Class i . .	28	19	5	7	24	15	2
Class ii . .	21	11	11	7	30	14	6
Class iii . .	9	13	5	5	25	42	1

Omitting the miscellaneous sounds in the last column, the first three types are of long, and the others of short, duration. In Class i, 47 per cent. of the comparisons are of short duration; in Class ii, 54 per cent.; and in Class iii, 73 per cent.

In Table III, the time-relations of the beginning and end, of the sound and shock are given, the figures in the columns headed *p*, *c*, and *f*, respectively, representing the percentage for each class in which the epoch of the sound preceded, coincided with, or followed, the corresponding epoch of the shock. Under the heading “relative duration,” the figures in the columns headed *g*, *e*, and *l*, respectively, represent the percentages for each class in which the duration of the sound was greater than, equal to, or less than, that of the shock.

TABLE III.

	Beginning.			End.			Relative Duration.		
	<i>p.</i>	<i>c.</i>	<i>f.</i>	<i>p.</i>	<i>c.</i>	<i>f.</i>	<i>g.</i>	<i>e.</i>	<i>l.</i>
Class i . .	50	42	8	19	51	30	41	47	12
Class ii . .	38	50	12	19	53	28	36	61	3
Class iii . .	37	46	17	9	51	40	29	67	4

*Length of Focus.*—Taking the difference between the longer and shorter axes of inner isoseismal lines as a measure of the length of the seismic focus, in earthquakes of the first class, the average length of the focus is 5 miles, and in those of the second class 4 miles. For the third class, the corresponding figures cannot be ascertained. It is, however, clear from the nature of the shock, the nature of the sound, and the time-relations of the sound and shock, that the focus in earthquakes of the third class was usually of very small dimensions.

#### IV. ORIGIN OF THE EARTHQUAKES.

The seismic evidence is sufficient to determine the approximate direction and the hade of the fault, and to trace roughly the position of the fault-line:—(i) The direction of the longer axes of the isoseismal lines is E.  $13^{\circ}$  N. in the earthquakes of September 17 and 22, 1900; E.  $27^{\circ}$  N. in that of July 23, 1905; E.  $29^{\circ}$  N. in that of September 21, 1905; E.  $18^{\circ}$  N. in those of October 8, December 28 and 30, 1906; E.  $25^{\circ}$  N. in the two earthquakes of October 20, 1908; and E.  $7\frac{1}{2}^{\circ}$  N. in the earthquake of May 3, 1912. The average of these ten directions is E.  $19^{\circ}$  N. (ii) The direction of the hade of the fault is indicated by the relative positions of the isoseismal lines of the earthquakes of September 21, 1905, and May 3, 1912. As these lines are farther apart to the north than to the south, the originating fault must hade to the north. (iii) The fault-line must therefore run approximately in the direction above indicated and pass through a point a short distance to the south of the centres of the inner isoseismal lines of these earthquakes. From the unusual intensity of some of the shocks at Alva, Menstrie, Airthrey, and Bridge of Allan, and from the fact that so many of the shocks are felt only at one or a few of these places, it may be inferred that the fault-line traverses the district in their immediate neighbourhood.

On the maps of the earthquakes is shown the course of the great Ochil Fault in the epicentral districts. Its mean direction there is E.  $13^{\circ}$  N.; its hade is unknown from geological evidence, and the fault-line passes through or near the Hillsfoot villages. As it satisfies two of the elements required by the seismic evidence, it may be concluded that the Ochil earthquakes are due to movements along this great fault.

Assuming this conclusion to be correct, there can be no doubt, from the evidence provided by the principal earthquakes of 1905 and 1912, that the fault hades to the north. This inference is supported, if it needs further support, by the evidence of the principal earthquake of 1908 and of all the earthquakes of the second class and of several of those of the third class.

The centres of the isoseismal lines of all these earthquakes lie on the north side of the fault, along a band which is roughly parallel to the fault-line.

In the great majority of the shocks, it is impossible to define the boundary of the disturbed area towards the north. The shocks were not strong enough to be felt beyond the Ochil Hills. In these cases, it may be inferred that the epicentres were as a rule not far from the fault-line. In the earthquakes of the first and second classes, however, the isoseismal lines or the boundary of the disturbed area can usually be drawn, and it is a significant fact that, in every one of these earthquakes, by far the larger part of the disturbed area lies on the north side of the fault-line. For instance, in the earthquake of May 3, 1912, the portion of the area within the isoseismal 7 on the north side was 10·3 times as great as that on the south side. For the isoseismals 6, 5, and 4, the corresponding figures were 4·4, 3·7, and 2·8. Now, in the Inverness earthquake of September 18, 1901, the areas within the isoseismals 8, 7, 6, 5, and 4 on the south-east side of the fault were respectively 1·9, 1·6, 2·1, 1·9, and 2·0 times the areas on the north-west side. From the unusual expansion of the disturbed areas of the stronger Ochil earthquakes on the north side of the fault, it may be inferred that the epicentral areas were of considerable magnitude in a horizontal direction at right angles to the fault-line.

Another important feature of the stronger earthquakes is their great intensity considering the smallness of the disturbed areas. Taking the three principal earthquakes, that of 1905 was of intensity 6 and disturbed an area of about 1000 square miles; those of 1908 and 1912 were of intensity 7 and disturbed areas of about 1000 and 605 square miles. Now, in other British earthquakes of intensity 7 since the year 1889, the disturbed area ranges from 12,000 to 63,600 square miles, the average being 31,000 square miles. Thus, the seismic foci of the stronger Ochil earthquakes must have been at a very slight depth below the surface.

From these two conclusions, it may be inferred that the originating fault is inclined at a very small angle to the horizon.

The portion of the fault affected by the recent movements extends from a mile or two east of Tillicoultry to a short distance west of Bridge of Allan—that is, over a length of about nine miles.

In order to study the distribution of the epicentres along the fault, it will be convenient to divide the portion affected into four regions. That in the neighbourhood of Dunblane, Bridge of Allan, and Airthrey may be called the western region; that between Airthrey and Menstrie the west-central region. The region between Alva and Tillicoultry may be called the eastern region; that from Menstrie to Alva the east-central region.

Of the earthquakes of the first class, that of 1905 originated in the east-central region, that of 1908 in the eastern region, and that of 1912 in the west-central region, the isoseismal centres of the two last being respectively 1 mile east and  $2\frac{1}{2}$  miles west of the centre of the first. One effect of the principal slip of 1905 was evidently to cause a sudden increase of stress within and just beyond the lateral margins of the focus, the stress near the eastern margin being relieved by the principal slip of 1908, and that near the western margin by the principal slip of 1912.

The distribution of the epicentres of the earthquakes of the second and third classes are shown in Table IV, the figures being percentages of the total number for each interval:—

TABLE IV.

	W.	W.C.	E.C.	E.
Before Sept. 21, 1905 . . . . .	33	...	67	...
Between Sept. 21, 1905, and Oct. 20, 1908 . . . . .	8	24	68	...
Between Oct. 20, 1908, and May 3, 1912 . . . . .	25	22	53	...
After May 3, 1912 . . . . .	50	19	31	...

Thus, during the first twelve and a half years, seismic activity predominated in the east-central region, and during the last two and a half years in the western region, no minor epicentre throughout the whole fifteen years being confined to the eastern region.

It is perhaps worthy of notice that, in the Inverness earthquakes of 1901, there was the same oscillation of epicentres in the central region, ending with a progression of activity towards the west. In the Inverness earthquakes of 1816 and 1890, the concluding after-shocks were marked by the same westerly migration beneath the bed of Loch Ness.

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XVIII.—The Trachytic and Allied Rocks of the Clyde Carboniferous Lava-Plateaus. By G. W. Tyrrell, A.R.C.Sc., F.G.S., Lecturer in Mineralogy and Petrology, University of Glasgow. *Communicated by* DR HORNE, F.R.S.

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1. INTRODUCTION.

THE lavas of the Scottish Carboniferous are predominantly basaltic. True andesites and rhyolites are conspicuous by their absence, whilst trachytes and allied rocks are present in quite subordinate quantity. This association of basalt and trachyte is in accordance with the fact that the basalts have a slight alkaline cast, which is evidenced by an alkali content greater than that of the average basalt, and by the occasional presence of nepheline and analcite amongst their constituents. The transition from basalts to more acid types is accomplished, not by way of the andesites, but through the mugearites, volcanic rocks with chemical affinities to the essexites. All transitions from the more felspathic types of basalts, the Jedburgh and Markle types, can be traced through mugearites to trachytes or allied rocks.

Hitherto the Scottish Carboniferous trachytes have been known chiefly from the East Lothian area, where they form a series of flows overlying the basalts of the Calciferous Sandstone, and also compose the rocks of the three massive plugs of Traprain Law, the Bass Rock, and North Berwick Law. The work of Hatch,\* and that of the Geological Survey of Scotland,† have made these types well known. Barron,‡ and recently Lady MacRobert,§ have described an interesting series of trachytic rocks of the same age from the Eildon Hills, near Melrose. The continuance of petrographical work upon the great Clyde plateau of Carboniferous basalts, so aptly named by Sir A. Geikie, has disclosed the fact that trachytes and allied rocks are widely distributed in the great masses of volcanic material stretching from Stirling, over the Clyde area, to South Bute and Campbeltown. These rocks occur generally as dykes, sills, and plugs, but rarely as distinct lava-flows; and they will probably be found to bulk quite as

\* F. H. Hatch, *Trans. Roy. Soc. Edinburgh*, vol. xxxvii, 1892, pp. 115–126.

† *Geol. Surv. Mem.: Geology of East Lothian*, 1910, pp. 127–133.

‡ T. Barron, *Geol. Mag.* (iv), iii, 1896, pp. 371–378.

§ Lady MacRobert, *Quart. Jour. Geol. Soc.*, vol. lxx, 1914, pp. 303–315.



largely in the Clyde plateau, especially in its south-western portion, as do similar rocks in the East Lothian area, although it must be remembered that they are always subordinate to the basaltic types. In the present paper it is desired to systematise our knowledge of the trachytes and allied rocks of the Clyde plateau, and also to record and describe some hitherto unrecognised occurrences.

## 2. CLASSIFICATION.

The rocks may be classified as follows:—

### (a) *Albite-bostonite, albite-trachyte, and albite-keratophyre.*

The essential character of this group is the predominance of a felspar near to albite. Many of the rocks also contain phenocrysts of sanidine, and have a little quartz in the groundmass. The terms bostonite, trachyte, and keratophyre are here used with textural significations for rocks which have essentially the same chemical and mineral composition. The bostonites are of comparatively coarse but equidimensional texture. The constituent feldspars generally have a more or less perfect fluxional arrangement. The trachytes are of much finer grain, and their groundmass is composed of minute felspar laths with a good fluxional or trachytic texture. In the keratophyres the groundmass is dense and felsitic, and its constituents are devoid of fluxional arrangement. These terms are used regardless of whether the rocks are of intrusive or extrusive origin.

This group contains the majority of the rocks described in this paper. They occur abundantly in the Great Cumbrae island, South Bute, the Misty Law district and Neilston Pad (Renfrewshire), and in the Campsie and Kilsyth Hills (Stirlingshire).

### (b) *Bostonite, trachyte, and keratophyre.*

In this group orthoclase felspar approximately equals the albite in quantity. The orthoclase molecule appears as phenocrysts of sanidine, and is also prominent in the groundmass. The textures of these rocks correspond generally with those of the above group. These types are not so abundant as the foregoing, and occur only in the Great Cumbrae and in the Misty Law district.

### (c) *Quartz-keratophyre and Felsite.*

These rocks, while corresponding generally with those of the first two groups in mineral composition, are considerably richer in quartz. Rocks with the bostonitic or trachytic textures have not been found—a fact to be correlated, perhaps, with the change in chemical composition. In the quartz-keratophyres the alkali-feldspars can be identified with fair certainty.

The rocks have been called felsites when the texture becomes excessively dense and cryptocrystalline, and when there is no possibility, apart from chemical analysis, of determining the nature of the principal mineral constituents.

These rocks are not so abundant as those of the first group. An excellent quartz-keratophyre occurs in the great Craigmushat sill at Gourrock (Renfrewshire), and others in the Great Cumbrae and the Misty Law district. Felsites occur in the Great Cumbrae and in the Campsie Fells.

(d) *Phonolite*.

Trachytic rocks carrying nepheline are at present definitely known from only one locality, Fintry (Stirlingshire), in the west of Scotland. A new locality will probably be found in North Ayrshire, as heaps of road-metal consisting of nepheline-bearing phonolitic trachyte have been noticed near Darvel and Galston.

### 3. PETROGRAPHY.

(a) *Albite-bostonite, albite-trachyte, and albite-keratophyre*.

In hand specimens these are compact, grey, yellowish, or dark-coloured rocks, exhibiting small flesh-coloured phenocrysts of felspar. They frequently have a platy structure, and in the more trachytic varieties show a resinous lustre on freshly broken surfaces, and have a sonorous ring under the hammer.

In thin section they show phenocrysts of sanidine, and occasionally albite and anorthoclase, in a groundmass which consists predominantly of laths of a felspar near albite in composition, with subordinate orthoclase. There is generally a diffused yellowish stain of limonite over phenocrysts and groundmass alike, and a widespread dissemination of minute chlorite scales. Sanidine forms by far the most abundant set of phenocrysts, and frequently occurs alone. It forms large, elongated, rectangular, simply-twinned crystals, orientated in the direction of flow, and frequently occurring in sporadic groups of two or three. Albite and anorthoclase occur as phenocrysts much less often than sanidine, and generally form much smaller crystals. Albite has only been noticed in nodules of albite-keratophyre enclosed in the bostonite dyke north of Keppel Pier, Great Cumbrae, in an albite-trachyte from the March Burn, Kilsyth Hills, and in an albite-bostonite from South Bute. The felspar identified as anorthoclase occurs in small crystals which have a peculiar mottled appearance in polarised light and frequently show a minute

microcline striation. It appears in the albite-bostonite of Uamh Capuill, South Bute, along with small microphenocrysts of albite which have broad marginal zones of orthoclase, and an occasional large phenocryst of albite. It also occurs in a few trachytic dykes from the March Burn, Kilsyth Hills, and from the Misty Law district of Renfrewshire.

The groundmass of these rocks generally consists of well-shaped felspar laths, most of which are zonal, or have multiple twinning, which in suitable sections give maximum extinctions ranging in different rocks from  $16^{\circ}$  to  $7^{\circ}$ . As the refractive index is well below that of Canada balsam, these figures indicate albite ranging in composition from  $Ab_1An_0$  to  $Ab_{95}An_5$ . There is always a subordinate quantity of non-zonal untwinned laths with straight extinction and refractive index less than that of albite, which are referred to orthoclase. When the orthoclase increases in amount so as to equal or exceed the albite, these rocks become true bostonite, trachyte, or keratophyre (see later). Quartz is a constant and sometimes abundant constituent of the groundmass. It frequently forms small plates which ophitically enclose the felspar laths.

Ferromagnesian constituents only occur in very small amounts, and rarely in a fresh condition. Magnetite in small specks is a constant constituent, but whatever ferromagnesian minerals have been present have almost invariably gone over to chlorite and limonite, rarely even leaving definite pseudomorphs. In some cases there may have been a little green pyroxene (Neilston Pad). Pseudomorphs in hæmatite after biotite are common in the Great Cumbrae rocks, but fresh biotite was met with in only one specimen, from the bostonite north of Keppel Pier.

In some of the keratophyric and trachytic types there is a little cryptocrystalline matter in the groundmass, and this is frequently in process of replacement by hæmatite. Sharply bounded hæmatised patches formed in this way occur in an albite-bostonite lava from east of St Blane's Hill, South Bute, and give the rock an appearance of having picked up angular inclusions. Occasionally the groundmass has been partly replaced by chalcedonic silica.

The texture of these rocks varies from the comparatively coarse bostonitic varieties to the dense, almost cryptocrystalline keratophyres. As regards the mode of arrangement of the feldspathic constituents, the bostonites generally show a rude fluxional arrangement which may occasionally become very perfect, as in the dyke from north of Keppel Pier, Great Cumbrae. On the other hand, the laths may be quite diverse and unorientated, thus giving the rocks a felted or pilotaxitic texture. The rocks designated as trachytes are of much finer grain than the

bostonites, and have a good fluxional or trachytic texture. The ground-mass of the keratophyres, on the other hand, whilst felspathic, is very dense, and the constituents are unorientated, giving a felsitic aspect to the rock.

Albite-bostonite is by far the most abundant variety. It forms many of the dykes, sills, and bosses of the Great Cumbrae, and several lavas intercalated with the Calciferous Sandstone flows of South Bute. It also appears in the Ayrshire mainland in the Misty Law district, forms the plug of Neilston Pad (Renfrewshire) piercing Calciferous Sandstone basalt lavas, and dykes in the March Burn district of the Kilsyth Hills. An alkali determination, by Mr J. V. Harrison, of a dyke contiguous to a quartz-dolerite dyke in the March Burn gave soda 5.85 per cent., and potash 2.71 per cent. The biotite-trachyte dykes of the Meikle Bin, Campsie Fells, probably belong here, as also do the albite-keratophyres of the same district mentioned in the *Glasgow Memoir* (p. 145).

The Great Cumbrae rocks have been briefly described as trachyte and bostonite(?).<sup>\*</sup> They were regarded as being prior to the Calciferous Sandstone lavas of South Bute and the Little Cumbrae, and consequently as the earliest of the Carboniferous igneous series. The evidence from other parts of the Clyde plateau, however, and also certain evidence provided by the Cumbrae trachytes themselves, point clearly to these rocks having been one of the later eruptives of the Lower Carboniferous suite.<sup>†</sup> They appear to be closely associated with the great tuff vents of the Meikle Bin type (Meikle Bin in the Campsie Fells, and Misty Law in the Renfrewshire Hills).<sup>‡</sup> The Great Cumbrae area of trachytic dykes lies ten miles south-west of Misty Law, and may be related to that great vent; but it is perhaps more probable that they are related to a vent of the Meikle Bin type now buried beneath the waters of the Firth of Clyde. Numerous sills, bosses, and dykes of trachytic rocks are to be found along the shore of the Firth between Ardrossan and Fairlie, but these have not yet received detailed attention. §

(b) *Bostonite, trachyte, and keratophyre.*

The albite-bostonites, albite-trachytes, and albite-keratophyres pass insensibly into true bostonites, trachytes, and keratophyres by a gradual

<sup>\*</sup> *Mem. Geol. Surv.: North Arran, South Bute, and the Cumbraes*, 1903, p. 177.

<sup>†</sup> *Mem. Geol. Surv.: Geol. of Glasgow District*, 1911, p. 110.

<sup>‡</sup> *Ibid.*, p. 104, 109.

§ *Summ. Prog. Geol. Surv.* for 1913 (1914), p. 58. Since the above was written, Mr G. V. Wilson, of the Scottish Geological Survey, has read a paper to the Glasgow Geological Society (March 9, 1916) on the vents of North Ayrshire, in which some of these trachytic rocks are described in detail. See "Preliminary Notes on Volcanic Necks in N.W. Ayrshire," *Trans. Geol. Soc. Glasgow*, vol. xvi, pt. 1 (1915-16), 1916, pp. 86-99.

increase in the amount of potash felspar until it equals or exceeds that of the soda felspars. Augmentation of the potash felspar may take place by an increase in the number of sanidine phenocrysts, or by an increase in the amount of orthoclase in the groundmass, or in both ways simultaneously. In the chemical analyses of the albite-bostonites there is a decided excess of soda over potash, whilst the bostonites proper show approximate equality between these constituents.

It is unnecessary to give a full petrographical description of these rocks, as it would merely duplicate that given in the previous section, with the single qualification that orthoclase is relatively more abundant both among the phenocrysts and as a constituent of the groundmass.

The rock of the Barbay sill in the centre of Great Cumbrae island is a typical bostonite of this group. Other dykes and sills in the Cumbrae may also be referred to this group, notably one from the Hawk's Nest, north of Farland Point, in which Dr J. J. H. Teall found on analysis 11 per cent. of alkalies, principally potash, whilst the percentages of lime and iron were very low.\* Mr J. V. Harrison found 8.75 per cent. of alkalies in the Barbay sill, divided up almost equally between soda and potash (see Table I, 5).

Trachytes belonging to this group occur in the March Burn, Kilsyth Hills, and in the Misty Law area. A dyke from the latter district shows clusters of sanidine and anorthoclase (?) phenocrysts in a very dense groundmass consisting of fluxionally arranged felspars which are mainly orthoclase. The only other constituents are diffused areas of chlorite and hæmatite, with a little magnetite. These types pass over gradually into those in which the groundmass becomes dense, felsitic, and devoid of fluxional arrangement. A keratophyre dyke from the Misty Law area illustrates this group. It consists of numerous microphenocrysts of anorthoclase in a dense quartzo-felspathic groundmass containing a few prisms of green soda-pyroxene. Similar rocks are mentioned as occurring in the Meikle Bin area.† The rocks of this group may perhaps be correlated with the porphyritic and non-porphyritic trachytes of East Lothian,‡ and with the sanidine trachytes of the Eildon Hills.§

#### (c) *Quartz-keratophyre and Felsite.*

This group is more highly siliceous than either of the foregoing. The mineralogical expression of this chemical fact is the abundance of

\* *Mem. Geol. Surv. : North Arran, South Bute, and the Cumbraes*, 1903, p. 62.

† *Mem. Geol. Surv. : Geol. of Glasgow District*, 1911, p. 145.

‡ *Mem. Geol. Surv. : Geol. of East Lothian*, 1910, p. 132.

§ Lady MacRobert, *Q.J.G.S.*, vol. lxx, 1914, p. 305.

quartz. A typical quartz-keratophyre forms the rock of a great sill at Craigmushat Quarry, Gourrock (Renfrewshire). This contains numerous small, euhedral phenocrysts of feldspar belonging to at least two groups. Some show multiple twinning in a zone surrounding an irregular core of untwinned feldspar. From the extinction the external zone may be referred to albite-oligoclase ( $\text{Ab}_{85}\text{An}_{15}$ ). Other phenocrysts with a square cross-section are simply-twinned, and have a mottled appearance due to an intergrowth of albite, or to albitisation. There are also elongated laths which also show simple twinning. These are referred to orthoclase, probably a soda-bearing variety. The feldspar phenocrysts are embedded in a groundmass consisting of large and small quartz areas rendered curiously ragged and irregular in outline owing to their indentation by the terminations of small feldspar laths. The latter are euhedral, especially where partly or wholly enveloped by quartz. Their determination is difficult, for the crystals are very turbid, and the groundmass is full of diffused secondary chloritic material. They appear to be untwinned and to have straight extinction. Hence they probably belong to orthoclase.

Mention should also be made of the large druses in this rock, which are lined with an assemblage of beautifully crystallised minerals, including quartz, calcite, fluor-spar, and others. In thin section small cavities lined with beautiful euhedral quartz crystals may be seen, the remainder of the space being filled with calcite. The only ferro-magnesian constituents in the rock are the diffused chloritic alteration products in the groundmass, and sporadic clusters of large irregular grains of titaniferous magnetite. There is also some apatite. The texture is very dense, as the minute constituents of the groundmass are closely felted together, and present an almost felsitic appearance.

A quartz-keratophyre from the Burnho Burn, Campsie Fells, has been collected by Mr J. V. Harrison. This rock contains little clusters of sanidine phenocrysts in a dense, felsitic, quartzo-feldspathic groundmass. Mr Harrison has also obtained a quartz-keratophyre from one of the numerous acid dykes of the Misty Law area. In this rock there are a few small phenocrysts of soda-orthoclase in a thoroughly felsitic groundmass in which much quartz is visible. The only ferro-magnesian constituents are a few small specks of magnetite, and pseudomorphs in magnetite after biotite.

The *felsites*, which are very abundant as dykes in the Meikle Bin area of the Campsie Fells, the Misty Law area of the Renfrewshire hills, and on the eastern coast of Great Cumbrae island, are probably not very

different from the quartz-keratophyres in chemical composition. They differ from them by containing a large proportion of dense crypto-crystalline matter, and in their distinctively felsitic texture. They are usually rich in quartz.

A group of dykes north of Downcraig Ferry, on the east coast of the Great Cumbrae, well illustrates the characters of the felsites. One of these dykes provides a most striking rock in hand specimens, as it is banded in broad stripes of yellow and purple. The groundmass of these rocks is usually quartzo-felspathic, and may either be minutely crystalline or with a good deal of crypto-crystalline material in it. Occasionally hair-like microlites apparently of some iron-ore are numerous in the groundmass. Some of the dykes are non-porphyrific; others have felspar phenocrysts which have been entirely replaced by chalcedonic silica, without loss of crystalline form. They are all stained yellow and red with limonite or hæmatite, but the only indications of original ferro-magnesian constituents are occasional pseudomorphs after biotite in magnetite. These rocks are probably identical with a felsite from the Meikle Bin vént, of which an analysis is given later (Table I, 10).

#### (d) *Phonolites.*

The Fintry occurrence, which, with one unlocated exception, is the only phonolitic rock yet discovered in the Clyde area, is fully described in the *Glasgow Memoir* (p. 144). Its clear mineralogical and chemical resemblance to the rock of Traprain Law is there illustrated and commented upon.

An excellent phonolite has also been found by Miss A. T. Neilson in heaps of road-metal in the Galston district of Ayrshire. This rock should be called a phonolitic-trachyte, perhaps, rather than nepheline-phonolite, as nepheline occurs only in insignificant quantity. In thin section the rock has a trachytic groundmass consisting of flow-orientated orthoclase laths interspersed with grains of ægirine-augite and magnetite. In this are numerous phenocrysts of orthoclase in which euhedral and somewhat altered nepheline is embedded or intergrown. Microphenocrysts of ægirine-augite are numerous; and olivine, invariably altered to iddingsite, also occurs in the same way but in smaller amount. This rock is much more mafic than the nepheline-phonolite of Fintry, and closely resembles the olivine-augite-trachyte or orthophyre of the Eildon Hills, described by Lady MacRobert.\*

\* *Op. cit.*, p. 309.

## 4. QUANTITATIVE CHEMICAL AND MINERAL CHARACTERS.

In the accompanying table are listed four full analyses (Nos. 1, 2, 6, 10) and two partial analyses (Nos. 3, 5) of the series of rocks treated in this paper. For comparison are added analyses of albite-trachyte of Skomer Island, Pembrokeshire (No. 4), and of three bostonitic rocks from the Kristiania district of Norway (Nos. 7, 8, 9). Each of the four groups into which the series has been divided are represented. The albite-bostonites are represented by two complete analyses of lavas from South Bute; the bostonites by a partial analysis of a sill from Great Cumbrae island; the quartz-keratophyres and felsites by the analysis of a felsite from the Campsie Fells, near the Meikle Bin vent; and the phonolites by an analysis of the well-known rock of Fintry. The arrangement of the analyses is roughly from a great excess of soda over potash towards equality of these constituents or slight dominance of the potash.

The principal features of all the analyses are relatively high silica and high alkalis, with insignificant ferrous iron, magnesia, and lime. This of course expresses the fact that the rocks are highly felspathic. The alumina diminishes in the albite-bostonites (Nos. 1, 2), and in the quartz-rich rocks (Nos. 9, 10), relative to the bostonites and phonolites (Nos. 6, 7, 8). With regard to the relations of soda and potash, the albite-bostonites show a decided predominance of soda—that is, in the terminology of the American Quantitative Classification, the rocks are *dosodic*; but in the other rocks the alkalic oxides are approximately equal—that is, the rocks are *sodio-potassic*. These facts are perhaps better expressed in the consideration of the norms. In many of the rocks ferric oxide is considerably in excess of ferrous oxide, a fact probably due to hæmatisation of the mafic constituents.

*The Norms.*—The norms, as calculated by the methods of the American Quantitative Classification, give results that express quite nearly the actual mineral composition or mode of the rocks. Table II gives the norms, with the rocks arranged in the same order as in Table I.

A general consideration of the norms shows that the rocks are highly felsic, as is shown by the figures expressing the ratio between the salic and femic minerals. All the rocks thus fall on or near the border-line between Classes I and II in the American Quantitative Classification. This is expressed by the frequency of bracketed numbers in the first figure of the symbols for the rocks (see Table I). Quartz is relatively abundant in the albite-bostonites, and in the felsitic rocks (1, 2, 4, 9, 10); but there is a deficiency of silica in the phonolites and bostonites, so



TABLE I.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
SiO <sub>2</sub>	61.19	66.60	...	58.47	...	57.98	60.11	62.30	69.00	69.48
TiO <sub>2</sub>	1.83	.55	...	2.17	...	.20	.96	tr.	.35	.14
Al <sub>2</sub> O <sub>3</sub>	14.20	12.95	...	18.60	...	17.92	19.01	17.05	13.95	11.99
Fe <sub>2</sub> O <sub>3</sub>	8.08	7.25	...	1.92	...	4.23	4.63	1.30	1.56	2.54
FeO	2.49	.83	...	4.77	...	2.72	.37	2.46	2.38	2.46
MnO	n.d.	n.d.	...	.19	...	.21	tr.	...	.55	.20
MgO	.50	.76	...	.94	...	.34	.23	.57	.14	1.16
CaO	1.74	1.41	...	.99	...	2.12	.66	1.20	.49	1.72
Na <sub>2</sub> O	6.64	6.14	5.85	5.52	4.42	6.99	6.53	5.14	5.67	3.33
K <sub>2</sub> O	1.80	3.10	2.71	3.30	4.33	5.24	5.36	6.18	5.11	4.01
H <sub>2</sub> O +	} 1.58	.46	...	2.19	...	1.76	} 1.37	} .45	} .70	1.56
H <sub>2</sub> O -		.49	...	.50	...	.64				.32
CO <sub>2</sub>	...	...	...	.04	...	n.f.	.84	2.65	...	1.34
P <sub>2</sub> O <sub>5</sub>	.08	...	...	.45	...	.08	...	...	...	0.8
(BaSr)O.	...	...	...	.04	...	n.f.	...	...	...	n.f.
Cl	...	...	...	.02	...	...	...	...	...	...
Li <sub>2</sub> O	...	...	...	tr.	...	? tr.	...	...	...	tr.
FeS	...	...	...	...	...	...	...	.43	...	.05
(CoNi)O.	...	...	...	...	...	...	...	...	...	.03
	100.13	100.54	...	100.11	...	100.43	100.07	99.73	99.95	100.41

1. Albite-bostonite (*Umptekose-Pantellerose*, 'II. 4(5). 1'. 4(5)), lava, lenticle enclosed in Tertiary composite sill, Uamh Capuill, South Bute. W. R. Smellie, "The Igneous Rocks of Bute," *Trans. Geol. Soc. Glasgow*, xv, pt. iii, 1916, p. 359. Analyst, A. Stevens, M.A., B.Sc.
2. Albite-bostonite (*Kallerudose-Pantellerose*, (I)II. 4. 1. 4), lava, flow near Loch-na-Leighe, South Bute. Smellie, *op. cit.*, p. 359. Analyst, J. V. Harrison, M.A., B.Sc.
3. Albite-bostonite, dyke, March Burn, Kilsyth Hills. J. V. Harrison, "Geology of the East Kilsyth Hills," *Trans. Geol. Soc. Glasgow*, xv, pt. iii, 1916, p. 326. Analyst, J. V. Harrison.
4. Albite-trachyte (*Umptekose-Pantellerose*, (I)II. 4(5). 1. 4), lava, north cliff of Skomer Island, Pembrokeshire. H. H. Thomas, "The Skomer Volcanic Series," *Q.J.G.S.*, vol. lxxvii, 1911, p. 192. Analyst, E. G. Radley.
5. Bostonite, sill, Barbay, Great Cumbræ, Firth of Clyde. Analyst, J. V. Harrison. Analysis hitherto unpublished.
6. Nepheline-phonolite (*Nordmarkose-Umptekose*, (I)II. 5'. 1. '4), intrusion, Newtown of Fintry, Stirlingshire. *Mem. Geol. Surv.: The Geol. of the Glasgow District*, 1911, p. 144. Analyst, E. G. Radley.
7. Bostonite (*Phlegrose-Nordmarkose*, I'. 5. 1. (3)4), Tutvet, Hedrum, Norway. W. C. Brögger, *Des Ganggefolge des Laurdalits*, 1897, p. 243.
8. Lindöite (*Pulaskose-Phlegrose*, I'. 5. 1(2). 3), Gjefsen, Kirchspiel Gran, Kristiania, Norway. W. C. Brögger, *Die Gesteine des Grorudit-Tinguait Series*, 1894, p. 131. Analyst, Schmelk.
9. Quartz-lindöite (*Kallerudose-Liparose*, I(II). 4'. 1. 3(4)), Frön, Kristiania, Norway. W. C. Brögger, *ibid.*, p. 139. Analyst, Schmelk.
10. Felsite (*Adamellose-Toscanose*, I(II). '4. '2. 3), dyke, 700 yards S.S.E. of cairn on Lairs, 1½ miles north of Lennoxton, Stirlingshire. *Mem. Geol. Surv.: Geol. of Glasgow District*, 1911, p. 145. Analyst, E. G. Radley.

much so in the Fintry phonolite that nepheline to the extent of 7·4 per cent. is developed in place of quartz. All the rocks are highly felspathic, and, of the various feldspar molecules, albite is the most abundant, orthoclase is developed in subordinate quantity, but anorthite in very insignificant amount. This corresponds to the observed abundance of albite and soda-orthoclase in the rocks described. The development of quartz, feldspars, and feldspathoids in igneous rocks is largely controlled by the ratio between silica and alkalis in the magma; and this relation is expressed in the American Quantitative Classification by the ratio between quartz or feldspathoids, and feldspars (see Table II). The rocks in this series fall

TABLE II.

	1.	2.	4.	6.	7.	8.	9.	10.
Quartz . . . . .	12·5	17·0	9·7	...	·4	3·8	15·0	28·8
Orthoclase . . . . .	10·6	18·4	19·4	30·6	31·7	36·7	30·0	23·9
Albite . . . . .	56·1	49·2	46·6	45·6	55·0	43·5	43·0	28·3
Anorthite . . . . .	3·6	...	2·2	1·9	1·4	5·3	...	5·8
Nepheline . . . . .	...	...	...	7·4	...	...	...	...
Corundum . . . . .	...	...	5·1	...	1·9	...	...	...
Acmite . . . . .	...	2·3	...	...	...	...	4·2	...
Diopside . . . . .	3·0	4·1	...	4·7	...	·5	2·2	1·4
Hypersthene . . . . .	...	...	6·2	...	·6	4·7	3·8	4·7
Wollastonite . . . . .	...	·7	...	·9	...	...	...	...
Magnetite . . . . .	2·8	1·2	2·8	6·0	...	1·9	·2	3·7
Ilmenite . . . . .	3·5	1·0	4·1	·5	1·1	...	·6	·3
Hæmatite . . . . .	6·2	5·6	...	...	4·6	...	...	...
Apatite . . . . .	·3	...	1·0	·3	...	...	...	·3
Titanite . . . . .	...	...	...	...	·4	...	...	...
Class, Sal/Fem ratio . . .	5·2	6·1	5·8	6·8	11·8	12·6	8·1	8·3
Order, <u>Quartz or Lenad</u> . .	·17	·25	·14	·09	·005	·04	·20	·49
Feldspar								
Rang, $K_2O + Na_2O / CaO$ . .	9·7	∞	15·5	24·0	32·4	7·8	∞	4·6
Subrang, $K_2O / Na_2O$ . . .	·17	·35	·39	·49	·54	·80	·66	·80

in this respect within the fourth and fifth orders of the system. The relative abundance of alkali-feldspars (orthoclase and albite), with respect to lime-feldspar (anorthite), is expressed by the ratio of  $K_2O + Na_2O$  to  $CaO$  used in making feldspars and feldspathoids, and is given by the figures for this ratio in Table II. The rocks are all *peralkalic* with the exception of the Lennoxtown felsite, the ratio of alkalis to lime being over 7 to 1. Hence, with the above exception, all the rocks fall into Rang 1 of the American Quantitative Classification. The ratio between albite and orthoclase (expressed by the ratio between  $Na_2O$  and  $K_2O$ ) is used as the basis of the subrangs of the system. As shown by the ratios in Table II the rocks range from *dosodic* (the albite-bostonites) to sodi-

potassic (bostonites, phonolites, felsites), and therefore fall into subranges 3 and 4 in the American Quantitative Classification.

The norms are less satisfactory in indicating the nature of the mafic constituents of this series of rocks. As a matter of fact, the nature of the original ferromagnesian minerals can rarely be determined, at least in the Scottish rocks, from microscopic examination. A green sodapyroxene occurs but rarely; but it is probable that it was originally present in most of the Scottish rocks described. Titaniferous magnetite occurs to the extent of about 5 per cent.; and the abundance of ferric oxide as an alteration product or staining material is indicated by the presence of hæmatite in the norm.

I am much indebted to W. R. Smellie, M.A., B.Sc., and J. V. Harrison, B.Sc., for the loan of slides, and the use of chemical analyses prepared for or by them in furtherance of their own work, to which acknowledgment has been made in the course of this paper.

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XIX.—On Systems of Partial Differential Equations and the Transformation of Spherical Harmonics. By H. Bateman, Ph.D.  
(Communicated by PROFESSOR WHITTAKER.)

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§ 1. IT is well known that Laplace's equation  $\nabla^2 V = 0$  possesses certain classes of solutions of the form  $V = F(X, Y)$  where  $F$  satisfies a partial differential equation with  $X$  and  $Y$  as independent variables and  $X$  and  $Y$  are *real* functions of  $x, y$ , and  $z$ . Such solutions have been called *binary potentials*.\* They are of some interest in the problem of finding cases in which the equations of motion of an electron in a steady magnetic field are readily integrable.† There are also classes of solutions of the same type, except that  $X$  and  $Y$  are complex quantities and the coefficients in the partial differential equation for  $F$  may also be complex. In particular, there are solutions of the form  $V = Yf(X)$  where  $f$  is an arbitrary function with continuous second derivative.‡

Passing on to the case of four independent variables, we find that the equation of wave-motion

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} \quad . \quad . \quad . \quad . \quad (1)$$

possesses classes of solutions of the form  $V = F(X, Y, Z)$  where  $F$  satisfies a partial differential equation§ and  $X, Y, Z$  are *real* functions of  $x, y, z, t$ . It also possesses solutions of a similar type in which  $X, Y$ , and  $Z$  are not necessarily real, and in particular it possesses solutions of type  $V = ZF(X, Y)$  where  $F$  is an arbitrary function of its two arguments. ||

\* Cf. V. Volterra, *Ann. Sc. Normale di Pisa* (1883). T. Levi Civita, *Turin Memoirs*, t. 49 (1899), p. 105, classifies the binary potentials.

† Cf. C. Störmer, *Comptes Rendus*, Paris, t. 146 (1908), pp. 462, 623.

‡ These solutions have been discussed by Jacobi, Forsyth, Levi Civita, and the author. See the author's *Electrical and Optical Wave Motion*, pp. 114, 136, 153. Also *Annals of Mathematics*, vol. xiv (1912), p. 51. Solutions of type  $V = ZF(X, Y)$ , where  $F$  satisfies a partial differential equation, have been discussed by Amaldi, *Rend. Palermo*, t. 16 (1902), pp. 1–45. See also Hantzschel, *Reduction der Potentialgleichung*.

§ These solutions are of interest in the problem of transforming a special type of electromagnetic field into an electrostatic field. One solution of this problem is derived by setting up a Lorentzian transformation between  $X, Y, Z, T$  and  $x, y, z, t$ . As far as I know, a solution of this problem is not necessarily associated with a set of functions of type  $X, Y, Z$ ; for instance, I have not yet succeeded in determining the special type of electromagnetic field associated with the functions  $X = x \cos \omega t - y \sin \omega t$ ,  $Y = x \sin \omega t + y \cos \omega t$ ,  $Z = z - vt$ , where  $v$  and  $\omega$  are constants.

|| *Messenger of Mathematics* (1914), p. 164. Solutions of type  $V = WF(X, Y, Z)$ , where  $F$  satisfies a partial differential equation, have been discussed in a paper by the author, *Cambr. Phil. Trans.* (1910), vol. xxi, p. 257.

Since the equation of wave-motion is closely associated with the partial differential equations of Maxwell's electromagnetic theory, it is natural to inquire whether the partial differential equations

$$\left. \begin{array}{l} \text{rot } \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad \text{rot } \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \\ \text{div } \mathbf{E} = 0, \quad \text{div } \mathbf{H} = 0 \end{array} \right\} \quad (2)$$

possess classes of solutions analogous to those already mentioned. This question has already been answered in the affirmative in the case of solutions of the last type, for it is known that a complex set of solutions can be found in which \*

$$\mathbf{E} = i\mathbf{H} = Z\mathbf{F}(X, Y),$$

where  $\mathbf{F}$  is a vector function of  $X$  and  $Y$ : one component can be taken to be an arbitrary function of  $X$  and  $Y$  and the other two are then determined. The quantities  $X, Y, Z$  are certain functions of  $x, y, z$ , and  $t$ .

We shall now show that it is also possible to find classes of complex solutions of type

$$\mathbf{E} = i\mathbf{H} = \mathbf{F}(X, Y, Z),$$

where each component of the vector  $\mathbf{F}$  satisfies a partial differential equation in  $X, Y$ , and  $Z$ . Let us write  $s = ict$ , then  $\mathbf{E}$  must satisfy the set of equations

$$\text{rot } \mathbf{E} + \frac{\partial \mathbf{E}}{\partial s} = 0, \quad \text{div } \mathbf{E} = 0 \quad (3)$$

These equations will be satisfied in consequence of the equations

$$\text{rot } \mathbf{E} = 0, \quad \text{div } \mathbf{E} = 0 \quad (4)$$

which may be regarded as the fundamental equations of electrostatics in the variables  $X, Y, Z$ , if these quantities depend on  $x, y, z$ , and  $s$  in such a way that

$$\frac{\partial X}{\partial x} = \frac{\partial Y}{\partial y} = \frac{\partial Z}{\partial z}, \quad \frac{\partial Y}{\partial z} + \frac{\partial Z}{\partial y} = 0, \quad \frac{\partial Z}{\partial x} + \frac{\partial X}{\partial z} = 0, \quad \frac{\partial X}{\partial y} + \frac{\partial Y}{\partial x} = 0 \quad (5)$$

$$\frac{\partial Y}{\partial x} = \frac{\partial Z}{\partial s}, \quad \frac{\partial X}{\partial z} = \frac{\partial Y}{\partial s}, \quad \frac{\partial Z}{\partial y} = \frac{\partial X}{\partial s} \quad (6)$$

The equations (4) are of course satisfied by  $\mathbf{E} = \text{grad } V$ , where  $V$  is a solution of Laplace's equation

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} = 0.$$

Each component of  $\mathbf{E}$  will then be a solution of Laplace's equation, and so if equations (5) and (6) are also satisfied we shall have a solution of (3)

\* *Electrical and Optical Wave Motion*, pp. 124-127. The theorem is given here in a slightly different form.

in which each component of  $\mathbf{E}$  depends only on  $X$ ,  $Y$ , and  $Z$  and is a solution of Laplace's equation in these variables.

To solve the equations (5) and (6), we notice that if  $\epsilon$  is a small quantity whose square may be neglected, equations (5) imply that the transformation

$$x' = x + \epsilon X, \quad y' = y + \epsilon Y, \quad z' = z + \epsilon Z$$

is an infinitesimal conformal transformation of a space of three dimensions, for we have a relation of type

$$dx'^2 + dy'^2 + dz'^2 = (1 + 2\epsilon\lambda)(dx^2 + dy^2 + dz^2),$$

where  $s$  is kept constant. By a well-known result we may write \*

$$\begin{aligned} X &= a(x^2 - y^2 - z^2) + 2\beta xy + 2\gamma xz + px - ny + mz + u, \\ Y &= 2\alpha xy + \beta(y^2 - z^2 - x^2) + 2\gamma yz + nx + py - lz + v, \\ Z &= 2\alpha xz + 2\beta yz + \gamma(z^2 - x^2 - y^2) - mx + ly + pz + w, \end{aligned}$$

where  $a, \beta, \gamma, l, m, n, p, u, v, w$  are functions of  $s$ . With these values of  $X, Y$ , and  $Z$  equations (6) are satisfied if  $a, \beta, \gamma$ , and  $p$  are constants, and if

$$\frac{dl}{ds} = 2a, \quad \frac{dm}{ds} = 2\beta, \quad \frac{du}{ds} = 2\gamma, \quad \frac{dv}{ds} = l, \quad \frac{dw}{ds} = m, \quad \frac{dw}{ds} = n.$$

These equations give

$$\begin{aligned} l &= l_0 + 2as, & m &= m_0 + 2\beta s, & n &= n_0 + 2\gamma s, \\ u &= u_0 + l_0 s + as^2, & v &= v_0 + m_0 s + \beta s^2, & w &= w_0 + n_0 s + \gamma s^2, \end{aligned}$$

where  $l_0, m_0, n_0, u_0, v_0, w_0$  are arbitrary constants. The expressions for  $X, Y$ , and  $Z$  now take the form

$$\begin{aligned} X &= a(x^2 + s^2 - y^2 - z^2) + 2\beta(xy + zs) + 2\gamma(xz - ys) + px - n_0 y + m_0 z + l_0 s + u_0, \\ Y &= 2a(xy - zs) + \beta(y^2 + s^2 - z^2 - x^2) + 2\gamma(yz + xs) + n_0 x + py - l_0 z + m_0 s + v_0, \\ Z &= 2a(xz + ys) + 2\beta(yz - xs) + \gamma(z^2 + s^2 - x^2 - y^2) - m_0 x + l_0 y + pz + n_0 s + w_0, \end{aligned}$$

and we have the result that *a solution of Laplace's equation in the variables  $X, Y, Z$  is a solution of the wave-equation in the variables  $x, y, z, t$ .*

§ 2. A particular case of the last theorem is of special interest. If in the preceding equations we put  $l_0 = m_0 = n_0 = u_0 = v_0 = w_0 = p = 0$ , the equations resemble the formulæ of Rodrigues† for a transformation of rectangular co-ordinates from  $X, Y, Z$  to  $a, \beta, \gamma$ , and it is evident that a solution of Laplace's equation in the variables  $X, Y, Z$  is also a solution of Laplace's equation in the variables  $a, \beta, \gamma$ . This result may be used to express a standard set of spherical harmonics for one system of polar co-ordinates  $(\theta', \phi')$  in terms of a standard set of spherical harmonics for another set of polar co-ordinates  $(\theta, \phi)$ . The formulæ of transformation have already been

\* Sophus Lie, *Theorie der Transformationsgruppen*, Bd. ii, p. 460.

† See Whittaker's *Analytical Dynamics*, p. 8.

obtained by A. H. Leahy\* and Ad. Schmidt,† but the fact that the coefficients occurring in the formulæ can be regarded as wave-functions does not appear to have been noticed previously.

We easily find that

$$X \cos \omega + Y \sin \omega + iZ = a[(\xi^2 - \eta^2) \cos \omega + 2\xi\eta \sin \omega] + \beta[(\xi^2 - \eta^2) \sin \omega - 2\xi\eta \cos \omega] - i\gamma(\xi^2 + \eta^2),$$

where

$$\xi = x \cos \omega + y \sin \omega + iz, \quad \eta = x \sin \omega - y \cos \omega + is.$$

Let us now write

$$a = r \sin \theta \cos \phi, \quad \beta = r \sin \theta \sin \phi, \quad \gamma = r \cos \theta,$$

and make use of Jacobi's expansion‡

$$(\gamma + ia \cos \psi + i\beta \sin \psi)^n = \sum_{m=-n}^{+n} \frac{n!}{(n+m)!} i^m r^m \Gamma_n^m(\cos \theta) e^{im(\psi-\phi)}.$$

Then if we write  $\xi = \sigma \cos \nu$ ,  $\eta = \sigma \sin \nu$ , we find that

$$(Z - iX \cos \omega - iY \sin \omega)^n = \sum_{m=-n}^{m=+n} H_n^m(x, y, z, s) r^m \Gamma_n^m(\cos \theta) e^{-im\phi}. \quad (7)$$

where

$$\begin{aligned} H_n^m &= (-1)^n i^m \frac{n!}{(n+m)!} \sigma^{2m} e^{im(\omega-2\nu)} \\ &= (-1)^n i^m \frac{n!}{(n+m)!} e^{im\omega} [(x-iy)e^{i\omega} + i(z+is)]^{n-m} [(x+iy)e^{-i\omega} + i(z-is)]^{n+m}. \end{aligned}$$

Introducing polar co-ordinates  $X = R \sin \theta' \cos \phi'$ ,  $Y = R \sin \theta' \sin \phi'$ ,  $Z = R \cos \theta'$ , and expanding the left-hand side of (7) in powers of  $e^{i\omega}$ , we get

$$R^n P_n^k(\cos \theta') e^{-ik\phi'} = \sum_{m=-n}^{m=+n} L_n^{k,m}(x, y, z, s) r^n P_n^m(\cos \theta) e^{-im\phi},$$

where  $L_n^{k,m}$  is the coefficient of  $e^{ik\omega}$  in the expansion of the function

$$(-1)^n i^{m+k} \frac{(n+k)!}{(n+m)!} e^{im\omega} [(x-iy)e^{i\omega} + i(z+is)]^{n-m} [(x+iy)e^{-i\omega} + i(z-is)]^{n+m}$$

in ascending and descending powers of  $e^{i\omega}$ . This coefficient is evidently a polynomial of degree  $2n$  in  $x, y, z, s$ , and satisfies the equation of wave-motion; it may, in fact, be regarded as a standard homogeneous polynomial solution of this equation. §

\* *Proc. Roy. Soc. London*, vol. lvi (1894), p. 46; "Papers printed to commemorate the Incorporation of the University College of Sheffield" (1897), pp. 60-88. The formula which I have used recently (*Terrestrial Magnetism*, Sept. 1915) for the mean value of a function round a circle can be deduced immediately from a result given in the first of these papers.

† *Zeitschr. für Math. u. Phys.*, Bd. xlv (1899), p. 327.

‡ Cf. E. W. Hobson, *Encyclopædia Britannica*, vol. xxv, p. 651.

§ See the author's *Electrical and Optical Wave Motion*, p. 112.

Introducing the Eulerian angles  $\Theta, \Phi, \Psi$ , we may write \*

$$x = \rho \sin \frac{\Theta}{2} \sin \frac{\Psi - \Phi}{2}, \quad y = \rho \sin \frac{\Theta}{2} \cos \frac{\Psi - \Phi}{2}, \quad z = \rho \cos \frac{\Theta}{2} \sin \frac{\Psi + \Phi}{2}, \quad s = \rho \cos \frac{\Theta}{2} \cos \frac{\Psi + \Phi}{2}.$$

We then find that if  $\Theta = 2u$  and  $k \geq m$  we have †

$$\begin{aligned} L_n^{k,m} &= (-1)^{k+m} \frac{(n+k)! (n-m)! \rho^{2n}}{(n+m)! (n-k)! (k-m)!} e^{+ik\Psi + im\Phi} (\sin u)^{k-m} (\cos u)^{2n+m-k} \\ &\quad F(-n-m, k-n, k-m+1, -\tan^2 u) \\ &= (-1)^n \frac{(n+k)! \rho^{2n}}{(n-k)! (m+k)!} e^{+ik\Psi + im\Phi} (\sin u)^{2n-m-k} (\cos u)^{m+k} \\ &\quad F(m-n, k-n, k+m+1, -\cot^2 u). \end{aligned}$$

The second expression requires modification if  $m+k$  is negative. Other expressions may be obtained by using the identities

$$\begin{aligned} F(-n-m, k-n, k-m+1, -\tan^2 u) &= (\sec u)^{2n+2m} F(-n-m, n-m+1, k-m+1, \sin^2 u) \\ F(m-n, k-n, k+m+1, -\cot^2 u) &= (\operatorname{cosec} u)^{2n-2m} F(m-n, n+m+1, k+m+1, \cos^2 u) \end{aligned}$$

On the other hand, if  $m \geq k$  we have

$$L_n^{k,m} = \frac{\rho^{2n}}{(m-k)!} e^{+ik\Psi + im\Phi} (\sin u)^{m-k} (\cos u)^{2n+k-m} F(m-n, -k-n, m-k+1, -\tan^2 u),$$

while the second expression is the same as before. It should be noticed that

$$L_n^{k,m}(x, y, z, s) = (-1)^{k+m} \frac{(n+k)! (n-m)!}{(n+m)! (n-k)!} L_n^{m,k}(-x, +y, z, s).$$

This relation corresponds to the law of reciprocity discovered by Leahy and Schmidt. We also have the relation

$$L_n^{-k,-m}(-x, +y, -z, +s) = (-1)^{m+k} \frac{(n-k)! (n+m)!}{(n+k)! (n-m)!} L_n^{k,m}(x, y, z, s),$$

which becomes of interest when combined with the equation of Rodrigues,‡

$$P_n^{-k}(\cos \theta') = (-1)^k \frac{(n-k)!}{(n+k)!} P_n^k(\cos \theta).$$

In the particular case considered by Schmidt the transformation of co-ordinates is

$$X = \gamma \sin \sigma - \alpha \cos \sigma, \quad Y = \beta, \quad Z = \alpha \sin \sigma + \gamma \cos \sigma, \quad R = r,$$

or

$$\begin{aligned} \sin \theta' \cos \phi' &= \sin \sigma \cos \theta - \sin \theta \cos \sigma \cos \phi, & \sin \theta' \sin \phi' &= \sin \theta \sin \phi \\ \cos \theta' &= \cos \theta \cos \sigma + \sin \theta \sin \sigma \cos \phi. \end{aligned}$$

\* Whittaker's *Analytical Dynamics*, p. 11.

† This formula was needed in an investigation on the Diurnal Variation of Terrestrial Magnetism, made during the summer of 1915 at the Department of Terrestrial Magnetism of the Carnegie Institution of Washington.

‡ *Mémoire sur l'attraction des sphéroïdes*, Paris (1816).



We thus have  $\Phi=0$ ,  $\Psi=\pi$ ,  $\rho=1$ ,  $\sigma=\Theta$ . A comparison of the results leads to the formula

$$\begin{aligned}\frac{d^m}{dc^m}\left[(1+c)^k\frac{d^k}{dc^k}P_n(c)\right] &= \frac{(-1)^{n+m}(n+m)!}{2^m(n-m)!(m-k)!}F\left(n+m+1, m-n, m-k+1, \frac{1+c}{2}\right)_{m\geq k} \\ &= \frac{(-1)^{n+k}(n+k)!}{2^k(n-k)!(k-m)!}(1+c)^{k-m}F\left(n+k+1, k-n, k-m+1, \frac{1+c}{2}\right)_{m\leq k}\end{aligned}$$

which can be verified without difficulty by differentiation.

In the general case it appears from our results that Leahy's functions\*  $L(n, m, r, p)$  and  $K(n, m, r, p)$  can be expressed in terms of Jacobi's polynomial,† which is defined by the equation

$$F(-p, a+p, c, x) = \frac{x^{1-c}(1-x)^{c-a}}{c(c+1)\dots(c+p-1)} \frac{d^p}{dx^p}\{x^{c+p-1}(1-x)^{a+p-c}\} \quad (8)$$

where  $p$  is a positive integer.

§ 3. An interesting property of Jacobi's polynomial may be deduced from the fact that if we write

$$\rho^2 = 1 - \xi - \eta, \quad \cos^2 u = \frac{\xi\eta}{\xi + \eta - 1},$$

the equation of wave-motion is satisfied by‡

$$(\xi\eta)^{\frac{m-k}{2}}[(1-\xi)(1-\eta)]^{\frac{m+k}{2}}e^{-ik\Phi-im\Psi}F(m-n, m+n+1, m-k+1, \xi)F(m-n, m+n+1, m-k+1, \eta).$$

Expressing the solution  $I_n^{m,k}(x, y, z, s)$  in terms of the solutions of the above type, and vice versa, we find that there are two equations of the following kind:—

$$\begin{aligned}(1-\xi-\eta)^p F\left(-p, a+p, c, \frac{\xi\eta}{\xi+\eta-1}\right) &= \sum_{s=0}^p A_s F(-s, a+s, c, \xi)F(-s, a+s, c, \eta), \\ F(-s, a+s, c, \xi)F(-s, a+s, c, \eta) &= \sum_{p=0}^s B_p (1-\xi-\eta)^p F\left(-p, a+p, c, \frac{\xi\eta}{\xi+\eta-1}\right).\end{aligned}$$

The first expansion is already known under a slightly different form.§ The coefficient  $A_s$  is given by the equation

$$A_s = (a+2s) \frac{\Gamma(c+s)\Gamma(p+1)\Gamma(p+a-c+1)\Gamma(a+s)}{\Gamma(a-c+s+1)\Gamma(p-s+1)\Gamma(s+1)\Gamma(c)\Gamma(p+a+s+1)}.$$

To determine the coefficient  $B_p$ , we put  $\eta=0$ , then

$$F(-s, a+s, c, \xi) = \sum_{p=0}^s B_p (1-\xi)^p.$$

\* To pass from Leahy's notation to ours we must put  $2u=p$ .

† *Crelle's Journal*, Bd. lvi (1859), p. 156; *Werke*, Bd. vi, p. 191.

‡ See the author's *Electrical and Optical Wave Motion*, p. 109, Ex. 1.

§ H. Bateman, *Proc. London Math. Soc.*, ser. 2, vol. iii (1905), p. 123.

Now it follows at once from (8) that

$$F(-s, a+s, c, \xi) = (-1)^s \frac{(1+a-c)(2+a-c) \dots (s+a-c)}{c(c+1) \dots (c+s-1)} F(-s, a+s, 1+a-c, 1-\xi).$$

$$\therefore B_p = (-1)^{s+p} \frac{\Gamma(s+1)\Gamma(a+s+p)\Gamma(s+a-c+1)\Gamma(c)}{\Gamma(p+1)\Gamma(s-p+1)\Gamma(a+s)\Gamma(p+a-c+1)\Gamma(c+s)}.$$

An interesting expansion is obtained by writing  $\eta = 1 - \xi$ .

It should be noticed that the above theorem enables us to transform a series of type

$$\sum_{s=0}^{\infty} C_s F(-s, a+s, c, \xi) F(-s, a+s, c, \eta)$$

into a series of type

$$\sum_{p=0}^{\infty} D_p (1-\xi-\eta)^p F\left(-p, a+p, c, \frac{\xi\eta}{\xi+\eta-1}\right).$$

The problem of expanding an analytic function  $f(\xi)$  in a series of type

$$f(\xi) = \sum_{p=0}^{\infty} a_p F(-p, a+p, c, \xi)$$

has been discussed by H. A. Webb.\* He finds that if the real parts of  $c$  and  $a-c+1$  are positive, the series may represent the function within a region in the complex plane bounded by an ellipse whose foci are at the points 0, 1.

§ 4. The result obtained at the end of § 1 may be generalised as follows:—Let us consider the transformation

$$\begin{aligned} X &= (x_1x_2 + s_1s_2 = y_1y_2 - z_1z_2)x + (x_1y_2 + x_2y_1 + z_1s_2 + z_2s_1)y + (z_1x_2 + z_2x_1 - y_1s_2 - y_2s_1)z \\ &\quad - (y_1z_2 - y_2z_1 + x_1s_2 - x_2s_1)s, \\ Y &= (x_1y_2 + x_2y_1 - z_1s_2 - z_2s_1)x + (y_1y_2 + s_1s_2 - x_1x_2 - z_1z_2)y + (y_1z_2 + y_2z_1 + x_1s_2 + x_2s_1)z \\ &\quad - (z_1x_2 - z_2x_1 + y_1s_2 - y_2s_1)s, \\ Z &= (z_1x_2 + z_2x_1 + y_1s_2 + y_2s_1)x + (y_1z_2 + y_2z_1 - x_1s_2 - x_2s_1)y + (z_1z_2 + s_1s_2 - x_1x_2 - y_1y_2)z \\ &\quad - (x_1y_2 - x_2y_1 + z_1s_2 - z_2s_1)s. \end{aligned}$$

Then, if  $s_1 = -ict_1$ ,  $s_2 = -ict_2$ , it can be proved that if  $V = F(X, Y, Z)$  is a solution of Laplace's equation in the variables  $X, Y, Z$ , it is a solution of the wave-equation in each of the three sets of variables  $x, y, z, t$ ;  $x_1, y_1, z_1, t_1$ ,  $x_2, y_2, z_2, t_2$ .

Adopting Whittaker's expression † for a solution of Laplace's equation,

$$V = \int_0^{2\pi} f(X \cos \theta + Y \sin \theta + iZ, \theta) d\theta,$$

\* *Phil. Trans.*, A, vol. cciv (1904), p. 481.

† *Monthly Notices of the Royal Astron. Soc.*, vol. lxii (1902), p. 617; *Math. Ann.*, Bd. lviii (1903), p. 333.

and noticing that

$$X \cos \theta + Y \sin \theta + iZ = x[(\xi_1 \xi_2 - \eta_1 \eta_2) \cos \theta + (\xi_1 \eta_2 + \xi_2 \eta_1) \sin \theta] \\ + y[(\xi_1 \xi_2 - \eta_1 \eta_2) \sin \theta - (\xi_1 \eta_2 + \xi_2 \eta_1) \cos \theta] - iz(\xi_1 \xi_2 + \eta_1 \eta_2) + is(\xi_1 \eta_2 - \xi_2 \eta_1),$$

where

$$\xi_p = x_p \cos \theta + y_p \sin \theta + iz, \quad \eta_p = x_p \sin \theta - y_p \cos \theta + ct_p \quad (p = 1, 2),$$

we see that  $V$  can be expressed in the form

$$V = \int_0^{2\pi} \Phi(\xi_1, \xi_2; \eta_1, \eta_2; \theta) d\theta.$$

It follows then that  $V$  is a *right-handed double wave-function* \* of the variables  $x_1, y_1, z_1, t_1; x_2, y_2, z_2, t_2$ .

Again, if we use  $C(\theta)$  to denote the vector with components  $(\cos \theta, \sin \theta, i)$  it is easy to see that

$$\text{grad } V = \int_0^{2\pi} \Psi(\xi_1, \xi_2; \eta_1, \eta_2; \theta) C(\theta) d\theta = \mathbf{M} \text{ say,}$$

and that this complex vector  $\mathbf{M}$  satisfies the two sets of partial differential equations

$$\text{rot}_1 \mathbf{M} = -\frac{i}{c} \frac{\partial \mathbf{M}}{\partial t_1}, \quad \text{div}_1 \mathbf{M} = 0,$$

$$\text{rot}_2 \mathbf{M} = -\frac{i}{c} \frac{\partial \mathbf{M}}{\partial t_2}, \quad \text{div}_2 \mathbf{M} = 0,$$

where the suffixes indicate the set of variables with respect to which the differentiations are made. Writing  $\mathbf{M} = \mathbf{H} + i\mathbf{E}$  where  $\mathbf{E}$  and  $\mathbf{H}$  are real vectors, we find that these vectors satisfy Maxwell's equations in each the two sets of four variables  $x, y, z, t$  ( $p = 1, 2$ ).

The case in which

$$X = c(x_1 t_2 - x_2 t_1) + i(y_1 z_2 - y_2 z_1),$$

$$Y = c(y_1 t_2 - y_2 t_1) + i(z_1 x_2 - z_2 x_1),$$

$$Z = c(z_1 t_2 - z_2 t_1) + i(x_1 y_2 - x_2 y_1),$$

$$R^2 = (x_1 x_2 + y_1 y_2 + z_1 z_2 - c^2 t_1 t_2)^2 - (x_1^2 + y_1^2 + z_1^2 - c^2 t_1^2)(x_2^2 + y_2^2 + z_2^2 - c^2 t_2^2)$$

is of special interest. If in particular we write  $V = \frac{i}{R}$ , so that the components of  $\mathbf{E}$  and  $\mathbf{H}$  are given by expressions of type

$$E_x - iH_x = -iM_x = \frac{X}{R^3},$$

we obtain a specification of a vector field with some interesting properties.

The quantity  $R^2$  vanishes and  $\mathbf{M}$  becomes infinite when the equation

$$(x_1 + \lambda x_2)^2 + (y_1 + \lambda y_2)^2 + (z_1 + \lambda z_2)^2 = c^2(t_1 + \lambda t_2)^2$$

gives two equal values of  $\lambda$ . This happens when the common tangent

\* For a definition of this function see a paper by the author, *Bulletin of the American Mathematical Society*, April (1916).

of two directed spheres with Lie co-ordinates  $(x_1, y_1, z_1, ct_1)$ ,  $(x_2, y_2, z_2, ct_2)$  passes through the origin. If the origin of co-ordinates lies within the second of these spheres, the common tangent cone cannot pass through the origin unless the cone is imaginary and has its vertex at the origin. Hence, if  $x_2^2 + y_2^2 + z_2^2 < c^2 t_2^2$ , the only singularities of the field occur when  $x_1, y_1, z_1, t_1$  satisfy the equations

$$x_1 t_2 - x_2 t_1 = 0, \quad y_1 t_2 - y_2 t_1 = 0, \quad z_1 t_2 - z_2 t_1 = 0.$$

If we regard  $(x_1, y_1, z_1, t_1)$  as current co-ordinates, our field can be interpreted as the electromagnetic field due to an electric pole which passes through the point  $x_2, y_2, z_2$  at time  $t_2$  and moves with uniform velocity less than  $c$  along a straight line through the origin. The interpretation when  $(x_2, y_2, z_2, t_2)$  are taken as current co-ordinates is similar. The partial symmetry of the expressions for the vectors  $\mathbf{E}$  and  $\mathbf{H}$  in the two sets of co-ordinates  $(x_1, y_1, z_1, t_1)$ ,  $(x_2, y_2, z_2, t_2)$  is worthy of note.

§ 5. One of the theorems of § 4 may be generalised as follows:—

*If a complex vector  $\mathbf{M} = \mathbf{H} + i\mathbf{E}$  satisfies the system of partial differential equations*

$$\text{rot}_p \mathbf{M} = -\frac{i}{c} \frac{\partial \mathbf{M}}{\partial t_p}, \quad \text{div}_p \mathbf{M} = 0 \quad (p = 1, 2, \dots, n)$$

*and possesses continuous second derivatives, it is a right-handed multiple wave-function.\** This is easily verified by differentiation.

A solution of the above system of equations is given by

$$\mathbf{M} = \int_0^{2\pi} \Phi(\xi, \xi_2, \dots, \xi_n; \eta_1, \eta_2, \dots, \eta_n; \theta) \mathbf{C}(\theta) d\theta. \quad . \quad . \quad (9)$$

where  $\xi_p, \eta_p, \mathbf{C}(\theta)$  have the same meaning as in § 4. A solution may also be obtained by writing

$$\begin{aligned} \mathbf{M} &= i \text{rot}_p \mathbf{L}_q \equiv \frac{i}{c} \frac{\partial \mathbf{L}_q}{\partial t_p} + \text{grad}_p \Lambda_q, \\ \mathbf{L}_q &= \frac{i}{c} \frac{\partial \mathbf{G}}{\partial t_q} + i \text{rot}_q \mathbf{G} + \text{grad}_q K, \quad \Lambda_q = -\text{div}_q \mathbf{G} - \frac{i}{c} \frac{\partial K}{\partial t_q}, \end{aligned}$$

where the scalar  $K$  and each component of the vector  $\mathbf{G}$  is a right-handed multiple wave-function. If we write

$$\begin{aligned} \mathbf{G} &= \int_0^{2\pi} \mathbf{G}^*(\xi_1, \xi_2, \dots, \xi_n; \eta_1, \eta_2, \dots, \eta_n; \theta) d\theta, \\ K &= \int_0^{2\pi} K^*(\xi_1, \xi_2, \dots, \xi_n; \eta_1, \eta_2, \dots, \eta_n; \theta) d\theta, \end{aligned}$$

\* This means that the equations  $\text{div}_p \text{grad}_q V = \frac{1}{c^2} \frac{\partial^2 V}{\partial t_p \partial t_q}$ ,  $c \text{rot}_p \text{grad}_q V = i \text{grad}_p \frac{\partial V}{\partial t_q} - i \text{grad}_q \frac{\partial V}{\partial t_p}$  ( $p, q = 1, 2, \dots, n$ ) are satisfied by each component of  $\mathbf{M}$ .

we find that the present solution can be thrown into the form (9), where

$$\begin{aligned} \Phi = \cos \theta \left[ \frac{\partial^2 G_x^*}{\partial \eta_p \partial \eta_q} - \frac{\partial^2 G_x^*}{\partial \xi_p \partial \xi_q} + \frac{\partial^2 G_y^*}{\partial \xi_p \partial \eta_q} + \frac{\partial^2 G_y^*}{\partial \xi_q \partial \eta_p} \right] - \sin \theta \left[ \frac{\partial^2 G_x^*}{\partial \xi_p \partial \eta_q} + \frac{\partial^2 G_x^*}{\partial \xi_q \partial \eta_p} + \frac{\partial^2 G_y^*}{\partial \xi_p \partial \xi_q} - \frac{\partial^2 G_y^*}{\partial \eta_p \partial \eta_q} \right] \\ - i \left[ \frac{\partial^2 G_z^*}{\partial \xi_p \partial \xi_q} + \frac{\partial^2 G_z^*}{\partial \eta_p \partial \eta_q} \right] + \frac{\partial^2 K^*}{\partial \xi_q \partial \eta_p} - \frac{\partial^2 K^*}{\partial \xi_p \partial \eta_q}. \end{aligned}$$

Writing the last equation in the form  $\Phi = (G) + (K)$ , where the first term depends only on  $G$  and the second only on  $K$ , we may regard the equation  $(G) = (K)$  as a partial differential equation to determine  $K$  when  $G$  is given and to determine  $G$  when  $K$  is given. If  $p \neq q$  the equation is soluble in both cases, hence the part of  $M$  depending on the vector  $G$  can also be expressed by means of a scalar of type  $K$ , and similarly the part depending on  $K$  can also be expressed by means of a vector of type  $G$ . If  $p = q$  the part depending on  $K$  vanishes altogether. It should be noticed that the same value of  $M$  is obtained by writing

$$\begin{aligned} M = i \operatorname{rot}_q L_p \equiv \frac{1}{c} \frac{\partial L_p}{\partial t_q} + \operatorname{grad}_q \Lambda_p, \\ L_p = \frac{1}{c} \frac{\partial G}{\partial t_p} + i \operatorname{rot}_p G - \operatorname{grad}_p K, \quad \Lambda_p = -\operatorname{div}_p G + \frac{1}{c} \frac{\partial K}{\partial t_p}. \end{aligned}$$

The lack of symmetry in the expressions for  $L_p$ ,  $\Lambda_p$  and  $L_q$ ,  $\Lambda_q$  in terms of  $G$  and  $K$  is worthy of notice.

The theorem enunciated at the beginning of this paragraph is a particular case of the following theorem, which may be regarded as a generalisation of a theorem given by Appell.\*

*If a vector  $L$  and a scalar  $\Lambda$  satisfy the system of equations*

$$\begin{aligned} i \operatorname{rot}_p L = \frac{1}{c} \frac{\partial L}{\partial t_p} + \operatorname{grad}_p \Lambda, \\ \operatorname{div}_p L + \frac{1}{c} \frac{\partial \Lambda}{\partial t_p} = 0, \end{aligned} \quad (p = 1, 2, \dots, n) \quad (10)$$

*then  $L$  and  $\Lambda$  are right-handed multiple wave-functions.†* To obtain Appell's case we must put  $n = 1$

A solution of the above equations may be obtained by writing

$$L = \frac{1}{c} \frac{\partial G}{\partial t_q} + i \operatorname{rot}_q G + \operatorname{grad}_q K, \quad \Lambda = -\operatorname{div}_q G - \frac{1}{c} \frac{\partial K}{\partial t_q},$$

where the vector  $G$  and the scalar  $K$  are right-handed multiple wave-functions. If we adopt the expressions used previously for  $G$  and  $K$  and

\* *Bulletin de la Société mathématique de France*, t. 19, p. 68.

† This means that each component of  $L$  is a right-handed multiple wave-function.

use  $\mathbf{S}$  to denote the vector with components  $(\sin \theta, -\cos \theta, 0)$ , we find that

$$\mathbf{L} = \int_0^{2\pi} \mathbf{L}^*(\xi_1, \xi_2, \dots, \xi_n; \eta_1, \eta_2, \dots, \eta_n; \theta) d\theta$$

$$\Lambda = \int_0^{2\pi} \Lambda^*(\xi_1, \xi_2, \dots, \xi_n; \eta_1, \eta_2, \dots, \eta_n; \theta) d\theta$$

where

$$\begin{aligned} \mathbf{L}^* &= \frac{\partial \mathbf{G}^*}{\partial \eta_q} + i \left[ \mathbf{C} \frac{\partial \mathbf{G}^*}{\partial \xi_q} \right] + i \left[ \mathbf{S} \frac{\partial \mathbf{G}^*}{\partial \eta_q} \right] + \mathbf{C} \frac{\partial \mathbf{K}^*}{\partial \xi_q} + \mathbf{S} \frac{\partial \mathbf{K}^*}{\partial \eta_q} \\ \Lambda^* &= - \left( \mathbf{C} \frac{\partial \mathbf{G}^*}{\partial \xi_q} \right) - \left( \mathbf{S} \frac{\partial \mathbf{G}^*}{\partial \eta_q} \right) - \frac{\partial \mathbf{K}^*}{\partial \eta_q}. \end{aligned}$$

It should be noticed that

$$\begin{aligned} (\mathbf{C}\mathbf{L}^*) &= 0, & (\mathbf{S}\mathbf{L}^*) + \Lambda^* &= 0, \\ [\mathbf{C}\mathbf{L}^*] &= i \mathbf{C}\Lambda^*, & i[\mathbf{S}\mathbf{L}^*] &= \mathbf{L}^* + \mathbf{S}\Lambda^*, \end{aligned}$$

and that if  $\mathbf{L}^*$  and  $\Lambda^*$  satisfy these conditions then equations (10) are satisfied.

The question now arises whether the parts of  $\mathbf{L}^*$  and  $\Lambda^*$  depending on the vector  $\mathbf{G}^*$  can be expressed in terms of a scalar of type  $\mathbf{K}^*$ . This is not generally the case, because if we had

$$\begin{aligned} \frac{\partial \mathbf{G}^*}{\partial \eta_q} + i \left[ \mathbf{C} \frac{\partial \mathbf{G}^*}{\partial \xi_q} \right] + i \left[ \mathbf{S} \frac{\partial \mathbf{G}^*}{\partial \eta_q} \right] &= \mathbf{C} \frac{\partial \mathbf{K}^*}{\partial \xi_q} + \mathbf{S} \frac{\partial \mathbf{K}^*}{\partial \eta_q} \\ \left( \mathbf{C} \frac{\partial \mathbf{G}^*}{\partial \xi_q} \right) + \left( \mathbf{S} \frac{\partial \mathbf{G}^*}{\partial \eta_q} \right) &= \frac{\partial \mathbf{K}^*}{\partial \eta_q}, \end{aligned}$$

we should find on eliminating  $\mathbf{K}^*$  that

$$\frac{\partial^2 \mathbf{G}^*}{\partial \eta_q^2} + i \left[ \mathbf{C} \frac{\partial^2 \mathbf{G}^*}{\partial \xi_q \partial \eta_q} \right] + i \left[ \mathbf{S} \frac{\partial^2 \mathbf{G}^*}{\partial \eta_q^2} \right] - \mathbf{S} \left( \mathbf{C} \frac{\partial^2 \mathbf{G}^*}{\partial \xi_q \partial \eta_q} \right) - \mathbf{S} \left( \mathbf{S} \frac{\partial^2 \mathbf{G}^*}{\partial \eta_q^2} \right) = \mathbf{C} \left( \mathbf{C} \frac{\partial^2 \mathbf{G}^*}{\partial \xi_q^2} \right) + \mathbf{C} \left( \mathbf{S} \frac{\partial^2 \mathbf{G}^*}{\partial \xi_q \partial \eta_q} \right)$$

or

$$\left( \mathbf{C} \frac{\partial^2 \mathbf{G}^*}{\partial \xi_q^2} \right) + 2 \left( \mathbf{S} \frac{\partial^2 \mathbf{G}^*}{\partial \xi_q \partial \eta_q} \right) - \left( \mathbf{C} \frac{\partial^2 \mathbf{G}^*}{\partial \eta_q^2} \right) = 0,$$

where  $\overline{\mathbf{C}}$  is the vector with components  $(\cos \theta, \sin \theta, -i)$ . This last equation is not generally satisfied, and so the above statement is justified.

§ 6. We shall now consider the question whether it is possible to find a set of functions

$$\begin{aligned} x_p &= X_p(x, y, z, t), & y_p &= Y_p(x, y, z, t), & z_p &= Z_p(x, y, z, t), & t_p &= T_p(x, y, z, t) \\ & & p &= 1, 2, \dots, n \end{aligned}$$

such that a multiple wave-function of the  $n$  sets of four quantities  $(x_p, y_p, z_p, t_p)$  is an ordinary wave-function when considered as a function of  $x, y, z$ , and  $t$ . A partial answer to this question has already been given,\*

\* *Bulletin of the American Mathematical Society*, April (1916).

for the requirements are satisfied by putting  $x_p = x$ ,  $y_p = y$ ,  $z_p = z$ ,  $t_p = t$  ( $p = 1, 2, \dots n$ ).

Let us see if this is the only solution. We easily find that

$$\begin{aligned} \frac{\partial^2 V}{\partial x^2} &= \sum_{p=1}^n \frac{\partial V}{\partial x_p} \frac{\partial^2 x_p}{\partial x^2} + \sum_{p=1}^n \frac{\partial V}{\partial y_p} \frac{\partial^2 y_p}{\partial x^2} + \sum_{p=1}^n \frac{\partial V}{\partial z_p} \frac{\partial^2 z_p}{\partial x^2} + \sum_{p=1}^n \frac{\partial V}{\partial t_p} \frac{\partial^2 t_p}{\partial x^2} \\ &+ \sum_{p=1}^n \sum_{q=1}^n \frac{\partial^2 V}{\partial x_p \partial x_q} \frac{\partial x_p}{\partial x} \frac{\partial x_q}{\partial x} + \dots \\ &+ \sum_{p=1}^n \sum_{q=1}^n \frac{\partial^2 V}{\partial x_p \partial y_q} \frac{\partial x_p}{\partial x} \frac{\partial y_q}{\partial x} + \dots \end{aligned}$$

If the equation of wave-motion is to be satisfied when  $V$  is an arbitrary multiple wave-function, all the sets of equations of type

$$\frac{\partial x_p}{\partial x} \frac{\partial y_q}{\partial x} + \frac{\partial x_p}{\partial y} \frac{\partial y_q}{\partial y} + \frac{\partial x_p}{\partial z} \frac{\partial y_q}{\partial z} = \frac{1}{c^2} \frac{\partial x_p}{\partial t} \frac{\partial y_q}{\partial t}$$

must be satisfied ( $p, q = 1, 2, \dots n$ ), and these seem to imply that the variables  $x_q, y_q, z_q, t_q$  differ from constant multiples of the variables  $x_p, y_p, z_p, t_p$  by arbitrary constants. This solution of the problem is not of much interest. If, however, we limit  $V$  to be a *completely neutral multiple wave-function*, that is, a function which satisfies all the equations of type

$$\frac{\partial^2 V}{\partial x_p \partial y_q} = \frac{\partial^2 V}{\partial x_q \partial y_p}$$

in addition to the equation

$$\frac{\partial^2 V}{\partial x_p \partial x_q} + \frac{\partial^2 V}{\partial y_p \partial y_q} + \frac{\partial^2 V}{\partial z_p \partial z_q} = \frac{1}{c^2} \frac{\partial^2 V}{\partial t_p \partial t_q},$$

then the conditions to be satisfied by the variables  $x_p, y_p, z_p, t_p$  are not so numerous. We have, for instance, to satisfy equations of type

$$\frac{\partial x_p}{\partial x} \frac{\partial y_q}{\partial x} + \frac{\partial x_p}{\partial y} \frac{\partial y_q}{\partial y} + \frac{\partial x_p}{\partial z} \frac{\partial y_q}{\partial z} - \frac{1}{c^2} \frac{\partial x_p}{\partial t} \frac{\partial y_q}{\partial t} + \frac{\partial x_q}{\partial x} \frac{\partial y_p}{\partial x} + \frac{\partial x_q}{\partial y} \frac{\partial y_p}{\partial y} + \frac{\partial x_q}{\partial z} \frac{\partial y_p}{\partial z} - \frac{1}{c^2} \frac{\partial x_q}{\partial t} \frac{\partial y_p}{\partial t} = 0.$$

Adopting linear expressions in  $x, y, z$ , and  $t$  for each of the variables  $x_p, y_p, z_p, t_p$ , we are led to the following expressions:—

$$\left. \begin{aligned} x_p &= \mu_p x - h_p y + g_p z + i f_p c t + \xi_p \\ y_p &= h_p x + \mu_p y - f_p z + i g_p c t + \eta_p \\ z_p &= -g_p x + f_p y + \mu_p z + i h_p c t + \zeta_p \\ c t_p &= i f_p x + i g_p y + i h_p z + \mu_p c t + \tau_p \end{aligned} \right\} \dots \dots \dots (11)$$

where  $\mu_p, f_p, g_p, h_p, \xi_p, \eta_p, \zeta_p, \tau_p$  are arbitrary constants. With these expressions for  $x_p, y_p, z_p, t_p$  the wave-equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2}$$

is satisfied whenever  $V$  is a *completely neutral multiple wave-function*.

To verify this we may first generalise Whittaker's expression\* for a wave-function and obtain the following expression for a multiple wave-function which is completely neutral:

$$V = \int_0^\pi \int_0^{2\pi} F(a_1, a_2, \dots a_n; \theta, \phi) d\theta d\phi$$

where

$$\alpha_p = x_p \sin \theta \cos \phi + y_p \sin \theta \sin \phi + z_p \cos \theta - ct_p.$$

Substituting the expressions for  $x_p, y_p, z_p, t_p$ , we obtain an expression of the form

$$V = \int_0^\pi \int_0^{2\pi} \Phi(\alpha, \beta, \gamma, \delta, \theta, \phi) d\theta d\phi$$

where

$$\begin{aligned} \alpha &= x \sin \theta \cos \phi + y \sin \theta \sin \phi + z \cos \theta - ct, \\ \beta &= x \sin \theta \sin \phi - y \sin \theta \cos \phi - iz + ict \cos \theta, \\ \gamma &= x \cos \theta + iy - z \sin \theta \cos \phi - ict \sin \theta \sin \phi, \\ \delta &= x + iy \cos \theta - iz \sin \theta \sin \phi - ct \sin \theta \cos \phi. \end{aligned}$$

It is easy to verify that the above expression is a wave-function. If we regard  $(x, y, z, ict)$  as the rectangular co-ordinates of a point in a space of four dimensions, the formulæ (11) represent a particular type of transformation of rectangular axes.

\* *Math. Ann.*, Bd. lvii (1903).



XX.—The Structure and Life-History of *Bracon* sp.: a Study in Parasitism. By James W. Munro, B.Sc. (Agr.) and B.Sc. (For.), Board of Agriculture Research Scholar. *Communicated* by Dr R. STEWART MACDOUGALL. (With Two Plates.)

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#### INTRODUCTORY.

IN July 1913, while collecting the larvæ of the large Pine Weevil (*Hylobius abietis*), I discovered a number of tiny larvæ feeding on some of them. These I succeeded in rearing, and they proved to be the larvæ of a *Bracon* which seemed to be *B. hylobii*. In July 1914 I recorded my experiments and observations.\* In this record I stated that my identification of the species was based on Ratzeburg's description.† All the specimens I examined—and I have examined many—agree, for the most part, with his description of *B. hylobii*. In April 1914, specimens were sent for determination to Dr Szeplegeti, the authority for the family, in Budapest, but so far no reply has been received.

*Bracon hylobii* had never previously been recorded in this country. In 1911 Dr Stewart MacDougall received from the late Dr Nisbet a Braconid, parasitic on *Hylobius*, but in such a state that determination of the species was impossible. It was, however, definitely identified as a Braconid, after comparison with specimens in the British Museum.

The earliest record of *B. hylobii* I have been able to find is in Ratzeburg's *Ichneumoniden der Forstinsekten* (*loc. cit.*), published in 1848. He there describes the species, and quotes Nordlinger's observations on it. One other reference is that of Brischke, who includes *B. hylobii* in his list of Ichneumonidæ of West and East Prussia.‡ Neither of these authorities gives any account of the *Bracon*'s life-history. Recently my attention has been drawn to a Paper, by Forstmeister Dolles,§ the

\* *Annals of Applied Biology*, vol. i, No. 2, pp. 170-175. In this Paper the insect was named *B. hylobii*, but as there is some doubt yet, it has been thought better, throughout this new Paper, to describe the species as *Bracon* sp.

† *Ichneumoniden der Forstinsekten*, Band ii, p. 38.

‡ G. A. Brischke, "Die Ichneumoniden der Provinzen West- und Ost-Preussen," *Schrift. naturfors. Gesell.*, Danzig, Neue Folge, 1882, p. 135.

§ "Der Nutzender Braconiden in forstlichen Haushalte," *Forstlich-Naturwissenschaftlichen Zeitschrift*, 1897.

biological details of which concerning *B. hylobii* accentuate the doubt as to the real name of my species.

#### PROBLEMS INVOLVED.

The Pine Weevil (*Hylobius abietis*), measuring 8–14 mm. (Plate II, fig. 12), is the worst insect pest of forestry in this country. It is an enemy of newly formed coniferous plantations. The adult weevil alone is harmful. It attacks newly planted trees, from three to seven years old, during its swarming periods, gnawing the tender bark of stem and branch, and by reducing or altogether stopping the sap flow causes these young trees to wither and die. It prefers conifers, especially larch and Scots pine, but on occasion attacks hardwoods such as birch, beech, and oak.

The damage it can do is enormous. For instance, in a small plantation, about thirty acres in extent, in Kincardineshire, not a single young plant was left, all being killed by the weevil. In one estate in Perthshire, so serious was the damage done by *Hylobius* that the forester now delays planting for four years after felling, in the hope that by that time the weevil will no longer be breeding in the area.

#### LIFE-HISTORY OF HYLOBIUS.

The female weevil deposits her eggs in or under the bark of the stumps of trees which have been recently felled, confining herself to conifers, and especially to Scots pine. The larva on hatching bores and feeds in the bark until full grown, when it pupates at the end of its tunnel, in a cavity cut out in the wood or in the dry bark. The larva, from the nature of its feeding-place, does no harm. On emergence the adult attacks the young trees in the neighbourhood of the larval feeding-ground.

Unfortunately, the system of forestry in vogue in this country favours the breeding of *Hylobius*. Whole woods are cut down in one felling, thus providing extensive breeding areas for *Hylobius*. These areas are then replanted, and the weevil finds abundant food immediately on emerging. Very rarely are efforts made to arrange annual fellings at some distance from one another to avoid extensive clearings. For this reason the problem of *Hylobius* is a serious one, and any methods which aid in the reduction of the pest are worthy of serious attention.

Hitherto the methods employed against *Hylobius* have been entirely mechanical, and accordingly biological methods are naturally of interest. The object of this paper is to describe the life-history of this *Bracon* species, its distribution and the degree of its parasitism; and further, to discover the best means of using it to control the Pine Weevil.

LIFE-HISTORY OF *BRACON* SP.

Living a large part of its life in the tunnels of the *Hylobius* grub as it does, this *Bracon* is not easily studied in the field. In order to find it the bark must be removed from the stumps, and the larvæ or cocoons exposed, and there is an end to field observations on that particular brood. The life-history of the individual cannot well be studied in the field, and accordingly it was worked out in the laboratory.

The best means of obtaining the *Bracon* is to collect its cocoons just before spring sets in, during March and April. As the adults emerge from their cocoons, they are paired off and provided with pieces of bark containing the grubs of *Hylobius*. My method was as follows:—*Hylobius* grubs still feeding in the bark were brought in from the field. A large piece of thick bark was cut into small pieces. In each piece a hole the size of the larval tunnel was bored, and into each hole a healthy uninfected grub was placed. The grubs continued boring in the bark so provided, and the method proved quite successful. Each pair of *Bracons* was provided with one or more grubs in this way.

The following account of the life-history is based on observations in the laboratory:—

*Pairing.*

Both sexes are sexually mature on emergence, and pairing takes place almost immediately. The act is of short duration, and, so far as has been observed, is not repeated on the part of the females.

*Oviposition.*

Egg-laying may take place within a few hours of pairing, or it may be delayed for several days.

While my knowledge of *Bracon* was restricted to field observations the problem of oviposition was for long a puzzle to me. On the 3rd of June 1914 I had an opportunity for observing a *Bracon* in the act of egg-laying. The female in question had paired on 1st June. On the morning of the 3rd she appeared very restless, and towards mid-day she was busily engaged digging away the frass from the entrance of a weevil grub's tunnel in the bark supplied her. She stood firmly on her middle and hind legs, and, working excitedly with her fore legs, raked back the debris in front of her. Meanwhile her antennæ, continually vibrating and moving to and fro, seemed to direct her towards her prey. After some minutes' work she reached the chamber in which the grub was lying, and inserted first one antenna and then the other. Apparently satisfied, she retired a little, and then, turning round slowly, she backed towards the entrance she

had cleared and inserted her ovipositor, first raising over her back its two outer sheaths. She remained thus for nearly seven minutes, when she again turned and renewed her excavation, and once more turned round and inserted her ovipositor. After a short time she crawled away.

During all her digging she exposed only a small portion of her prey, and on examining it in the afternoon eleven eggs had been deposited, all of them on the hard chitin of the dorsal surface of the prothorax. In all the grubs examined this part of the body has invariably been selected by the *Bracon* for her eggs. Naturally, they are sometimes displaced by the wriggling of the host, but never to any extent.

In a previous paper I stated that the weevil grubs were attacked only in their resting stage. This has not been confirmed in the laboratory, both half and full-grown grubs being deposited on.

The act of oviposition causes no inconvenience or discomfort to the weevil grub. Its skin is not in any way injured, and indeed the female *Bracon* selects a strongly chitinated portion of her prey's body, and so eliminates most of the risk of disturbance which the act of oviposition might cause. (Plate II, fig. 13.)

The number of eggs deposited by the *Bracon* on a single grub varies considerably. In some cases it is as few as eight; in others I have counted twenty-two; the average was seventeen. The eggs are laid in clusters.

#### *The Egg.*

The egg of the *Bracon* is long and spindle-shaped. Its external surface is white, glistening, and unsculptured. The micropyle is not distinguishable. The egg measures .9 mm. in length and .15 mm. in diameter at the middle. (Plate I, fig. 3.)

#### *Hatching.*

Examination of the empty egg-shells shows that on hatching the egg splits along its dorsal surface. Hatching takes place two to four days after oviposition.

#### *The First-Stage Larva.*

The first-stage larva of this *Bracon* is a tiny grub .8 mm. in length. It is legless, and only two regions are recognisable, viz. head and tail. This tail, comprising the thoracic and abdominal regions, consists of thirteen segments, which are well defined. Each segment bears a row of tiny bristles over its whole surface. The first segment is the largest, and the rest become smaller in succession. The head is well defined, and bears a pair of simple (unjointed) horn-like antennæ. The mouth parts are most difficult to distinguish, but they consist of a pair of minute, sharp-hooked mandibles,

and below these two regions representing the maxillæ and labrum. Eyes are wanting.

The larvæ is active, and crawls rapidly about over its host. It is not in any way attached to it except when feeding, when it buries its head in the folds of the skin of the *Hyllobius* larva.

At this stage there are no signs of tracheæ or of spiracles, but the alimentary *sac* appears as a greyish mass shining through the skin.

The first instar lasts for one or two days. (Plate I, fig. 4.)

#### *The Second-Stage Larva.*

At the first moult the larva undergoes a marked change: the tiny hairs or bristles, that were so striking a feature of it, disappear. The head is much smaller proportionately, and the body no longer tapers posteriorly. The larva is now broadest at its middle, tapering anteriorly and posteriorly. Its length is 1.5 mm. A marked feature of the larva is a number of whitish spots shining through the skin. Newport\* in his paper on *Paniscus* comments on a similar phenomenon.

The second instar lasts one or two days.

#### *The Third-Stage Larva.*

This differs in no way from the second stage except that it is larger, 3.2 mm. long. For some time I failed to recognise this third stage, until I found in three broods the cast skins of this second change.

The third instar lasts a day, or a day and a half. (Plate I, fig. 5.)

#### *The Fourth-Stage Larva.*

At this period the larva measures 5 mm. in length. It is markedly different from the preceding stage. The head is now sunk in the first segment of the body and almost hidden by it. The antennæ are much less prominent. The tracheal system is clearly seen, and the spiracles are distinct. There are nine pairs of spiracles, viz. a pair on the first segment and a pair on the fourth and on each of the seven following segments. The whole of the integument is covered with very short reddish hairs. Hitherto the segmentation had been extremely simple, consisting of the head and thirteen consecutive rings. These body rings now show on their dorsal surfaces an additional fold corresponding to the prescutal fold in certain Coleopterous larvæ. The white spots have now disappeared. In general appearance the larva is now a curved maggot, tapering towards each end, the head end being slightly broader than the tail.

The mouth parts of the fourth-stage larva are very similar to those

\* Newport, *Trans. Linn. Soc.*, 1862, xxi, p. 63.

described by Westwood\* as typical of the Ichneumonid larvæ: "two obliquely deflexed, horny mandibles, very small, slender, and acute, beneath which is a curved fleshy lobe, formed by the union of the dilated maxillæ and labrum."

The fourth instar lasts one day. (Plate I, fig. 6.)

*The Fifth-Stage Larva.*

This differs in general shape from the fourth-stage larva. The abdominal region is now more swollen and the head region more pointed. This is probably accounted for by the fact that the alimentary canal is closed behind. The dorsal (prescutal) folds are also more marked and stand out in relief. The larva measures 6·5 mm. Towards the end of the fifth stage, in the second day of its duration, the larva becomes yellowish in colour and commences to spin its cocoon. (Plate I, fig. 7.)

*The Cocoon.*

When the parasites are full fed their host is reduced to an empty sac, and the parasites now fill the cavity in the bark their host previously occupied. It is here they spin their cocoons.

Just prior to spinning the larva becomes yellowish in colour. The first process in spinning is the enclosing and protecting of all the brood on the *Hylobius* larva. Each larva invests itself and its neighbours in a fine mesh of silk. Even isolated larvæ spin this loose shawl-like covering before beginning the cocoon proper. This covering, however, does not completely enclose the larvæ, for opposite the head of each a circular gap is left in the web. Each gap indicates the position of one end of the cocoon. The first covering is completed in a few hours, and in a few more the cocoons are so far constructed as to prevent further observation.

In spinning, the larva sways its head to and fro, but the abdominal region remains stationary. Often the larvæ are wedged between one another, but this in no way prevents each being enclosed in its own cocoon after the first covering is completed. The whole process rarely lasts twenty-four hours, and it is often completed in twelve.

The cocoons are elongated and oat-shaped, blunt at both ends. The outer coating is of coarser texture and more hairy than the interior lining, and often is interwoven with tiny bits of grass or other material that may be lying in the tunnel cavity. At first the cocoons are silvery white, but in a day or so they become creamy, and finally yellowish, or even brown. (Plate II, fig. 15.)

Inside the cocoon the larva remains quiescent until pupation. This

\* Westwood, *Introduction to the Classification of Insects*, vol. ii, p. 147.

period of quiescence varies considerably under different conditions, and its long duration is a feature of the Braconid's life-history. In the winter it lasts from four to six months, and in summer from three to six weeks.

#### *The Pupa.*

There is nothing specially noteworthy in the pupa of this *Bracon*. Viewed from the side, the antennæ are seen to extend almost the whole length of the body. They lie beneath the legs. The thorax is well defined, and in its relation to the head gives the pupa a hump-backed appearance. The abdomen is distended slightly. In the female the ovipositor is directed under the abdomen backwards. The wing-sheaths are very short, scarcely extending below the thorax. (Plate I, fig. 8.)

#### *The Adult.*

Ratzeburg's \* description of *Bracon hylobii* is as follows:—

“Inner discoidal cell completely closed. Nervus parallelus not interstitial. Discoidal cells equally long or nearly so. Metathoracic plates granulate.

“*B. hylobii*.  $1\frac{1}{2}$ –2 mm. long. Outer and inner discoidal cells equally long. Second cubital cell a little longer than the first. Antennæ, in the female, 31-jointed. Ovipositor as long as the abdomen, curved slightly upwards.

“All the legs and the greater part of the abdomen reddish-brown; of the latter, in the female, only the middle plate of the first ring, in the male, almost the whole of the first ring and the last half of the abdomen, quite black.

“Thorax and head mostly black, except in the female, where part of the metathorax is brown and the margin of the eyes is shimmering. Mouth and palps dull brown.”

My specimens agree generally with this description, except that there is a marked discrepancy in size and length between Ratzeburg's insect and mine. The average size in my specimens is 4 to 5 mm., and quite a number of females measured as much as 7 mm. Altogether I have handled over a thousand specimens, and I have always considered a 3-mm. specimen to be a small one.

Occasionally the black colour which, in Ratzeburg's description, is restricted to the middle plate of the first abdominal segment extends to the second and sometimes to the third ring. (Plate I, figs. 1 and 2, male and female of *Bracon* sp.).

The ovipositor of *Bracon* consists of two outer sheaths enclosing the borer. These sheaths cover the borer for its whole length. They are

\* Ratzeburg, *loc. cit.*

flexible, broader at their basal end, and covered with tiny hairs. The borer consists of three pieces, the borer proper and two dart-like rods, the spiculæ. The spiculæ lie inside the borer, whose edges overlap them. Together the borer and the two spiculæ form a tube along which the egg passes. These spiculæ are barbed at their apices, and are capable of sliding up and down in the channel formed by the borer. In oviposition the outer sheaths play no part, being raised over the back of the insect. They are also slightly raised during pairing.

#### HABITS OF *BRACON* SP.

##### *The Larva.*

The host of the larva is, as already mentioned, the grub of the large Pine Weevil (*Hylobius abietis*). This grub, when full-grown, is a large creature measuring 20–25 mm. long. Like all weevil grubs, it has a legless, fleshy, curved body, a well-marked brown and strongly chitinated head, and strong biting jaws. The whole of its life is spent tunnelling in the bark of the Scots pine stump on which it feeds.

The Braconid larvæ, on hatching, crawl all over their host's body. Soon they begin feeding (Plate II, fig. 14). They do not pierce the skin of their host, and show no preference for any part of its body, except that during their first and second instars they feed in the folds of the body. This would seem to be a precaution against falling off or being pushed off by the rubbing of their host against its tunnel. Occasionally the parasites do fall off, but they soon crawl on to their host again and resume feeding. By the time that the Braconid larvæ have reached their third moult the host has ceased feeding, although apparently still healthy and, though quiescent, if roused by the touch of a pencil or twig still capable of action. This quiescence begins four to six days after the eggs have been laid. During the next day or two the grub becomes flaccid. There is, however, not the slightest sign of decay or putrefaction.

By this time the parasitic larvæ are nearly full-grown. Their rate of growth is extraordinary. They double their bulk in a day's time, and increase to eight times their original size in from five to six days. They may feed for some time at one spot and then for no apparent reason leave it and resume elsewhere, yet no trace or sign of puncturing is visible on the skin of their host.

##### *The Adult.*

In emerging from the cocoons the Braconids bite an oval-shaped hole in the end of the cocoon. The long axis of this opening lies along the length of the cocoon.



The adults are extremely active, crawling and flying about immediately on emergence. They are strongly attracted to light, and if they escape from a dark breeding-cage immediately fly to the nearest window. This proved a fortunate habit in that on one occasion when three hundred escaped all were recovered from the main window of the laboratory.

#### FIELD EXPERIMENTS.

Before the value of *Bracon* as a factor in controlling the Pine Weevil could be demonstrated it was necessary to observe it at work in an area hitherto free from the parasite. It was found that such areas were not common. The experimental area finally chosen was a recent felling on Drumshoreland estate, near Edinburgh. It had the following advantages:—It was not far distant from Edinburgh, and easily accessible by train. The felling was of recent date, and *Hylobius* larvæ were plentiful. At the time of selection no *Bracon* had been found on the area. Later, *Bracon* cocoons were found in a wood near the selected area, and on the area itself one batch of fresh cocoons was obtained near the time of the experiment. The locality was visited on February 17 and 19, and several stumps were examined. These contained several weevil grubs.

Two methods of introducing the parasite to the area were available: to transport the unopened cocoons and distribute them here and there over the area; or to set free the adult parasites as they emerged. The first method was the easier, but, in view of the fact that nothing was known regarding the proportion of the sexes, it was decided to liberate the adult parasites after emergence.

The period during which the parasites were liberated extended from May 12 to June 7. They were liberated as follows:—

Date.	Males.	Females.	Total.
May 12, 1915.	5	9	14
" 19, "	72	88	160
" 20, "	27	7	34
" 21, "	73	63	136
" 24, "	156	226	382
" 26, "	23	25	48
" 28, "	42	45	87
" 31, "	18	38	56
June 3, "	34	14	48
" 7, "	23	12	35
Total	473	527	1000

From this table it will be seen that the numbers emerging increased up to May 24 and then gradually fell off again. The cocoons from which the parasites emerged were kept in a dark wooden cage. In one side of this cage two holes about three-quarters of an inch in diameter were bored, and into these two tightly fitting collecting-tubes were fitted with their open ends inside the box. The Braconids being positively phototropic, entered these tubes immediately on emerging, and were then removed and counted before being set free. The numbers of the sexes were practically equal—47 per cent. males and 53 per cent. females. Among these and several others which emerged later no hyper-parasites were observed.

During May and the first week of June, when the *Bracons* were liberated on Drumshoreland, that area was almost continually swept by a cold east wind. When released the parasites made no efforts to fly, but crawled over the soil and the little twigs and pine-needles covering it, while numbers of them hid in the bark of stumps. Some flew a few feet, but alighted again. On May 26 and 28 the weather was warm and sunny, but the Braconids were no more active than on dull days.

June 7 marked the close of the introduction of parasites, and Drumshoreland was not visited again till June 14. On that date I examined the whole area for adult *Bracons*, but I observed none. They were all set free in the centre of the chosen area, with the view that even if they spread in all directions they would still be in the neighbourhood of their host the Pine Weevil grub.

Two stumps were examined on June 18, and on one of them two batches of fourth-stage Braconid larvæ were found, and on another one batch of seventeen larvæ just about to spin.

On July 8 the area was visited and three stumps carefully examined. Larvæ and pupæ of *Hylobius* were found, but no *Bracon*. The month of July was very wet, and except for casual visits no systematic examination was begun until the 22nd. Unfortunately, very few cocoons were found, but *Hylobius* pupæ occurred in large numbers.

It had been intended to examine every stump in the area, but the wet weather and the large number of stumps which yielded no cocoons were the cause of a change of method. The centre of the area was thoroughly examined and the stumps along eight radii from it. This should have given a fair idea of the extent of the parasitism. Altogether only ten batches of cocoons were found.

On August 9 three grubs were found to be attacked by second-stage *Bracon* larvæ, and another by fourth-stage larvæ of *Bracon*. The felled

areas in the surrounding district were now examined, on the supposition that the liberated *Bracon* had gone beyond the area of liberation. All of them, three in number, proved of recent date, and contained no *Hylobius*.

#### CONCLUSION AND SUMMARY OF FIELD EXPERIMENT.

The field experiment had a double purpose: (1) to try the effect of liberating large numbers of *Bracon*; (2) if this had been successful, to obtain a mass of material for an experiment on a much larger scale.

The non-success of the field trial at Drumshoreland, as contrasted with the success of the laboratory experiments, can be explained partly by the unfavourable nature of the weather during the period of liberation of the Braconid parasites, and partly by the fact that the area for experiment was too circumscribed, so that the liberated *Bracons* wandered.

#### LABORATORY EXPERIMENTS.

The material used in the laboratory was collected during October 1914 and May 1915 in Aberdeenshire. As the adults emerged from the cocoons males and females were taken and placed in breeding-cages. These consisted either of a wooden cage with a glass front and fine-mesh gauze sides, or simply of a glass jar with a double fold of fine muslin over the opening. The bottom of each of these cages was covered with a layer of moss, which was kept constantly moist. *Hylobius* grubs were supplied in small pieces of Scots Pine bark in which they had begun to tunnel. This method proved quite satisfactory, and approached natural conditions fairly closely. It ensured that no grubs which were already parasites were introduced, and that the grubs were healthy and active. They were also easily examined.

##### *Series 1.*

My first experiments were carried out with cocoons collected in October 1914 and kept in the laboratory through the winter. The following are the records of them:—

##### *Expt. No. 1.*

1915.

- March 1. Newly emerged male and female placed in cage with bark containing a *Hylobius* grub.
- „ 17. 2 Braconid larvæ hatched; they measured 2 mm. in length, and were feeding on this grub, which was soft and flabby.
- „ 20. *Hylobius* grubs putrefied. Braconid larvæ dead.

*Expt. No. 2.*

- March 2. 1 male and 1 female placed in cage with bark containing  
*Hylobius* grub.  
 „ 25. *Hylobius* grub with 10 well-grown larvæ of *Bracon* feeding on it.  
 April 1. Braconid larvæ now yellowish. The *Hylobius* grub is sucked  
 empty.  
 „ 3. All Braconid larvæ, except 3, spun up.  
 „ 5. All Braconid larvæ spun up.  
 „ 6. One Braconid larva removed from its cocoon to see if it would  
 spin again.  
 „ 9. This larva shrivelled up.  
 May 13. Cocoons still intact; they were examined, but only shrivelled  
 larvæ were found in them.

*Expt. No. 3.*

- March 20. Male and female *Bracons* put in cage with bark containing a  
*Hylobius* grub.  
 May 6. 14 *Bracon* cocoons and the head of the parasitised *Hylobius* grub.  
 „ 24. The cocoons are empty, all the emerged *Bracons* being males.

As will be observed on comparison with later experiments, this series was subjected to as little disturbance as was necessary, in order to avoid failure through tampering with the larvæ.

Unfortunately, the first two experiments failed and the third produced males only, but they showed that *Bracon* could be reared in the laboratory. The failure of the second experiment demonstrated the need for uniformly moist conditions.

*Series 2.*

These experiments were carried out with cocoons collected in April 1915. They were subject to more frequent and particular examination than those of the first series.

*Expt. No. 1.*

- May 27. 1 male and 1 female *Bracon* placed in cage with bark containing  
 a *Hylobius* grub.  
 „ 28. Cage placed in the dark.  
 „ 31. Eggs of *Bracon* on *Hylobius* grub.  
 June 1. 8 *Bracon* larvæ and 5 eggs on grub.  
 „ 2. All eggs hatched.  
 „ 3. Larvæ in second stage.  
 „ 4. Larvæ in second stage 1.5 mm. long. Grub now quiescent.

- June 5. Larvæ now 1·8 mm. long, markedly spotted.  
 „ 6. Larvæ on grub have moulted a third time. The larvæ measure 5 mm.; they are now covered with fine reddish hairs. The spiracles are visible, and through the skin the tracheæ.  
 „ 8. Larvæ measure 6·5 mm.  
 „ 10. Larvæ spinning.  
 „ 11. Larvæ are spun up.  
 „ 20. Opened 2 cocoons; they contained pupæ.  
 „ 30. 1 male and 1 female *Bracon* emerged from the cocoons.  
 July 20. 4 females emerged.  
 August 2. 1 male emerged.  
 „ 6. 2 females and 5 males emerged.

*Expt.* No. 2.

- May 27. 1 male and 1 female *Bracon* placed in cage with *Hylobius* grub.  
 June 4. *Bracon* eggs on *Hylobius* grub.  
 „ 5. No progress.  
 „ 8. No progress.  
 „ 11. Eggs still unhatched; grub discoloured; experiment closed.

*Expt.* No. 3.

- May 27. 1 male and 1 female *Bracon* placed in cage with bark containing a *Hylobius* grub.  
 „ 28. Cage placed in the dark.  
 „ 31. 6 eggs of *Bracon* laid on *Hylobius* grub.  
 June 3. The eggs have hatched.  
 „ 5. *Bracon* larvæ have moulted and are now in second instar; they are markedly spotted.  
 „ 9. A third moult has taken place.  
 „ 10. Larvæ measure 5 mm. long.  
 „ 11. A fourth moult has taken place; the larvæ are now in the final stage.  
 „ 14. Larvæ spinning.  
 „ 15. Larvæ spun up.  
 July 5. 2 cocoons were examined and contained pupæ.  
 August 6. 2 males and 1 female emerged.

Information on the larval moults and on the general life-history were obtained in the course of these experiments. In the first experiment of this series a marked disparity in the dates of emergence will be noticed, the first adults emerging on June 30 and the last on August 6, more than

a month later. In these observations a moult escaped notice, which was observed, however, in the next series, where I refer to it.

Considering the amount of disturbance the larvæ in these experiments were subjected to, the results were very successful.

*Series 3.*

This series was conducted with Braconids reared in the laboratory from the Series 2 experiments. They were limited in number owing to the weevil grubs pupating when brought in from the field.

*Expt. No. 1.*

- August 3. 1 male and 1 female put in cage with bark containing 2 weevil grubs.
- „ 6. 22 *Bracon* eggs on the prothorax of one of the grubs; this grub was transferred to another cage, and is referred to as grub *a*. The male *Bracon* was unfortunately destroyed; it was replaced by another. No further pairing was observed.
- „ 7. Female *Bracon* active in gallery of second grub, later referred to as grub *b*.
- „ 10. Eggs on grub *a* hatched. Larvæ in first instar. Grub *b* deposited on.
- „ 11. Brood *a* larvæ still in first instar. Newly hatched larvæ crawling on grub *b*.
- „ 12. Larvæ of both broods in second instar.
- „ 13. Larvæ of both broods spotted.
- „ 14. Larvæ of brood *a* have moulted a *third* time, but are still exactly similar to the preceding on second-stage larvæ. This gives a moult between the second and so-called third stage of the previous experiments.
- „ 15. Larvæ of brood *a* in fourth instar. Larvæ of brood *b*, some in third and some in fourth. A third *Hylobius* grub (referred to as *c*) was introduced to the male and female *Bracon* from which broods *a* and *b* have been removed.
- „ 16. Larvæ of brood *a* in fifth stage, of brood *b* in fourth stage. Numerous *Bracon* eggs on the prothorax of grub *c*.
- „ 18. Brood *a* larvæ spun up; brood *b* larvæ spinning. Eggs on grub *c* hatched, the larvæ being in the first stage.
- „ 21. Brood *c* larvæ in third stage; other broods spun up.
- „ 23. Brood *c* larvæ, some in fourth and some in fifth stage.

August 24. All brood *c* larvæ in fifth stage.

„      25. Brood *c* larvæ spinning.

„      27. Brood *c* spun up.

September 8. 2 cocoons of brood *a* empty, both emerged *Bracons* being males. 1 female has emerged from brood *b*.

„      11. No more adults emerged.

*Expt.* No. 2.

August 10. Male and female *Bracon* put in cage with bark containing *Hylobius* grub.

„      13. Weevil grub infested with eelworms.

„      14. Weevil grub dead; replaced by another in fresh bark.

„      18. This new grub deposited on.

„      19. A few of the eggs have hatched, and the *Bracon* larvæ are feeding.

„      20. All the eggs have hatched, and all the larvæ are in the first stage.

„      21. Some of the larvæ are in the first stage, some in the second.

„      23. Larvæ all in third stage.

„      25. Some of the larvæ in fourth stage, others in fifth.

„      30. All the larvæ are spun up.

September 9. 3 females emerged.

„      11. No more adults emerged.

*Expt.* No. 3.

August 11. Male and female *Bracon* placed in cage with bark containing *Hylobius* grub.

„      13. Weevil grub pupated.

„      17. No eggs of *Bracon* on this pupa.

„      20. Experiment closed.

The small number of adults obtained in these experiments is accounted for by the fact that several larvæ in each brood were sacrificed for purposes of description, and others were greatly disturbed and so failed to attain the adult state.

#### ROOF EXPERIMENT.

This experiment was carried out on the roof of the laboratory. The *Bracons* were supplied with bark containing *Hylobius* grubs and placed in a large box with muslin lid. The bark was placed in moss to retain moisture as far as possible. The insects were exposed to atmospheric conditions and the experiments arranged to be as natural as possible.

- May    27. 6 males and 6 females put in cage with several *Hylobius* grubs in five pieces of bark.
- June   12. 2 pieces of bark examined showed they contained weevil grubs attacked by second-stage Braconid larvæ.
- „      14. These larvæ in the third stage.
- „      24. Larvæ of both broods mentioned above spun up.
- August 13. 4 batches of *Bracon* cocoons; 2 *Hylobius* pupæ. Of one batch of cocoons, two cocoons had yielded adults; in another batch, one cocoon was empty; the others were intact and contained pupæ. Unfortunately, they were damaged in removal.

#### SUMMARY OF LABORATORY EXPERIMENTS.

The laboratory and roof experiments have provided the material for the description of the structure of the egg, larva, pupa, and cocoon of the *Bracon*. They have also afforded opportunity for observations, and in the third series for the checking and confirming of my first observations. They show that at least three broods may be reared in the laboratory during the summer. During the whole of my experiments I have observed no hyper-parasitism.

#### FIELD OBSERVATIONS.

Since my discovery of this *Bracon* in 1911, I have taken every opportunity of observing it in the field. The habitat of *Bracon*, as already mentioned, precludes regular and systematic observation of its habit, but by examining suitable areas in different localities I have endeavoured to obtain some knowledge of its life-history in the field. With two exceptions, adults have never been caught in the open.

The methods adopted in looking for *Bracon* material were as follows:—All recently felled areas—that is, of woods felled during the past four years—in the locality were examined. The stumps were barked and also the roots, and a careful look-out kept for *Bracon* cocoons and weevil grubs.

The most useful tools for making this examination were found to be: a small gardener's spade for clearing away soil and heathy turf, a fern trowel such as is used by botanists, and a chisel for barking the stumps.

In examining a stump the blade of the trowel was inserted between the wood and the bark and the bark prised off. In this way the whole of the stumps were barked. The roots were then barked in the same way, after having been laid bare of soil by the spade. The chisel was useful for removing pieces of bark not accessible to the trowel. The bark so removed was carefully examined, then the wood of the stump itself and the soil surrounding.



The *Bracon* cocoons may be mistaken for pieces of fungus mycelium, but a little practice soon enables one to recognise them.

The pine stump, as is well known, affords a breeding-place for many other insects than this *Bracon* and *Hylobius abietis*. Of these the bark beetles are of most interest. They comprise three species—*Myelophilus piniperda*, *Hylastes ater*, and *Hylastes palliatus*. Another pine weevil, *Pissodes pini*, also inhabits the Scots pine stump. Its larvæ resembles a half-grown *Hylobius* grub, but it lacks the furrows on the epicranium (Plate II, figs. 9 and 10). A further distinction is afforded by the spiracles: in *Pissodes* these are circular, in *Hylobius* elliptical. Two longicorn beetles and their larvæ are occasionally found—*Rhagium bifasciatum* and *R. indagator*.

I have obtained only one other insect enemy of *Hylobius*, viz. the Ichneumon *Ephialtes tuberculatus*. At Kingswells, Aberdeenshire, I found two large brown cocoons, to each of which was attached the head of a *Hylobius* larvæ. These proved to be the cocoons of *Ephialtes tuberculatus*. From one of them a female emerged; the other contained a dried male pupa. I have identified the species from Morley's description of it. *E. tuberculatus* has not hitherto been recorded as parasitic on *Hylobius* in this country. Ratzeburg records it on *Hylobius* in Germany.

## FIELD OBSERVATIONS.

Date.	County and Locality.	Date of Felling.	Bracon.	Other Insects.	Remarks.
1914. Sept.	Woodlands, Banchory Devenick estate, Kin- cardineshire	1911	Old cocoons plentiful	Old galleries of <i>M. piniperda</i>	Area examined in 1913, only a few stumps being left. The area consists of pure Scots pine.
"	Woodlands, E., adjoining the above	1912	Two batches of cocoons	Old galleries of <i>M. piniperda</i>	Only four stumps in this area yielded weevil grubs, the remaining stumps being attacked by fungus.
"	Woodlands, E., near the above	1914	One batch of cocoons	Adults and lar- væ of <i>H. ater</i> and <i>H. palliatus</i>	Only one stump yielded <i>Hylobius</i> larvæ, the area being freshly felled.
"	Hazelhead estate, Aber- deenshire	1913-14	Three batches of fresh cocoons	Adults and lar- væ of <i>H. ater</i> and <i>M. pini- perda</i>	The area consists of a mixed wood, chiefly beech and Scots pine, in which occasional trees have been cut. The stumps are widely scattered.

## FIELD OBSERVATIONS—continued.

Date.	County and Locality.	Date of Felling.	Bracon.	Other Insects.	Remarks.
1914. Oct. 5	Skene, Aberdeenshire	1912-13	.....	Adults of <i>H. ater</i> and <i>H. palliatus</i>	Stumps widely scattered, spruce predominating.
„ 5	Kintore, Aberdeenshire	1913-14	Cocoons numerous	<i>H. ater</i> adults and larvæ, <i>H. palliatus</i> adults	The wood was composed of Scots pine and larch, Scots pine predominating. The area has been irregularly felled, so that stumps yielding cocoons are scattered.
„ 7	Ballogie, Aberdeenshire	„	.....	<i>H. ater</i> adults and larvæ, <i>Rhagium indagator</i> adults and larvæ	<i>Bracon</i> is here entirely absent. The wood is pure Scots pine. <i>Rhagium</i> larvæ were as plentiful as <i>Hylobius</i> grubs—an unusual occurrence.
„ 10	Kintore, Aberdeenshire	„	Cocoons and two adult females	<i>H. ater</i> and <i>H. palliatus</i> larvæ and adults	The occurrence of adult <i>Bracons</i> at this time of year is remarkable.
„ 14	Kintore	„	Cocoons	„	The cocoons on this occasion all occurred on the older and larger roots.
„ 16	Kingswells, Countesswells estate, Aberdeenshire	1912-14	Cocoons	<i>M. piniperda</i>	The wood is a mixture of larch and Scots pine. <i>Bracon</i> cocoons were very plentiful.
1915. Feb. 17	Drumshoreland, West Lothian	1913-14	One batch of fresh cocoons	Larvæ and adults of <i>M. piniperda</i>	These cocoons were found on an isolated stump in a standing wood some distance from a cleared area on which none were found.
„ 19	„	1914	.....	<i>H. palliatus</i> and <i>H. ater</i> larvæ and adults	The <i>Hylobius</i> larvæ were found only on the older felled stumps, the bark beetles on the fresher stumps.
Apr. 7	Kingshill, Aberdeenshire	1912-14	Fresh cocoons plentiful	Galleries of <i>M. piniperda</i>	The cocoons were most numerous on the older stumps, and especially in the moister portions of the area.

FIELD OBSERVATIONS—*continued*.

Date.	County and Locality.	Date of Felling.	Bracon.	Other Insects.	Remarks.
1915. Apr. 11	Kingshill, Aberdeen-shire	1912-14	Fresh cocoons plentiful	Two cocoons of the <i>Ichneumon Ephialtes tuberculatus</i>	The cocoons of <i>Bracon</i> occurred chiefly on the stumps themselves, and less on the smaller roots.
„ 12	Kintore	„	Fresh cocoons	Adult and larvæ of <i>Rhagium indagator</i> , adult <i>R. inquisitor</i> in older stumps, one adult <i>Acanthocinus ædilis</i>	The cocoons of <i>Bracon</i> occur chiefly on the smaller roots, and less on the stumps.

COMPARISON OF FIELD AND LABORATORY EXPERIMENTS, SHOWING OCCURRENCE OF *BRACON* IN THE VARIOUS MONTHS.

Month.	Field.	Laboratory.
Jan.	Cocoons containing larvæ.	Cocoons containing larvæ.
Feb.	Cocoons containing larvæ.	Cocoons containing larvæ and pupæ.
Mar.	Cocoons containing larvæ.	Cocoons containing pupæ, adults.
Apr.	Cocoons containing pupæ.	Pupæ, adults, and eggs.
May	Cocoons containing pupæ, adults.	All stages.
June	Adults, eggs, larvæ.	All stages.
July	Adults, eggs, larvæ, cocoons containing larvæ.	All stages.
Aug.	Adults, cocoons containing larvæ, larvæ.	All stages.
Sept.	Larvæ in cocoons, adults.	Adults, larvæ and pupæ in cocoons.
Oct.	Larvæ in cocoons, adults.	Larvæ and pupæ in cocoons.
Nov.	Larvæ in cocoons.	Adults, larvæ in cocoons.
Dec.	Larvæ in cocoons.	Larvæ in cocoons.

*Hylobius abietis* is to be found in all stages throughout the year.

## SUMMARY OF FIELD NOTES.

It will be seen from the foregoing table that during the autumn, winter, and spring months the *Bracon* is almost invariably to be found in the cocoon, either as a resting larva or a pupa. So far, unfortunately, observations on the adult are almost entirely wanting. These can be

made only by frequent and continued examination of an infected area, and in any case will always be difficult. The adult is small, and, except on close examination, is readily confused with the numerous Hymenoptera which are to be seen on a summer's day.

So far as the insects inhabiting the Scots pine stumps are concerned, *Hylobius abietis* appears to be the only one which *Bracon* attacks. In all cases where its cocoons were collected they were invariably accompanied by the head of the grub of *Hylobius*. Observations also show that freshly felled stumps, stumps less than one year cut, may harbour *Hylobius* grubs, but that it is in the second year of felling that *Bracon* cocoons are first to be found. The *Bracon* persists as long as the weevil larvæ, and fresh unopened cocoons may be found when no weevil larvæ are present, indicating that *Bracon* has accounted for them.

On the whole, *Bracon* is extremely hardy, and can endure similar conditions to its host.

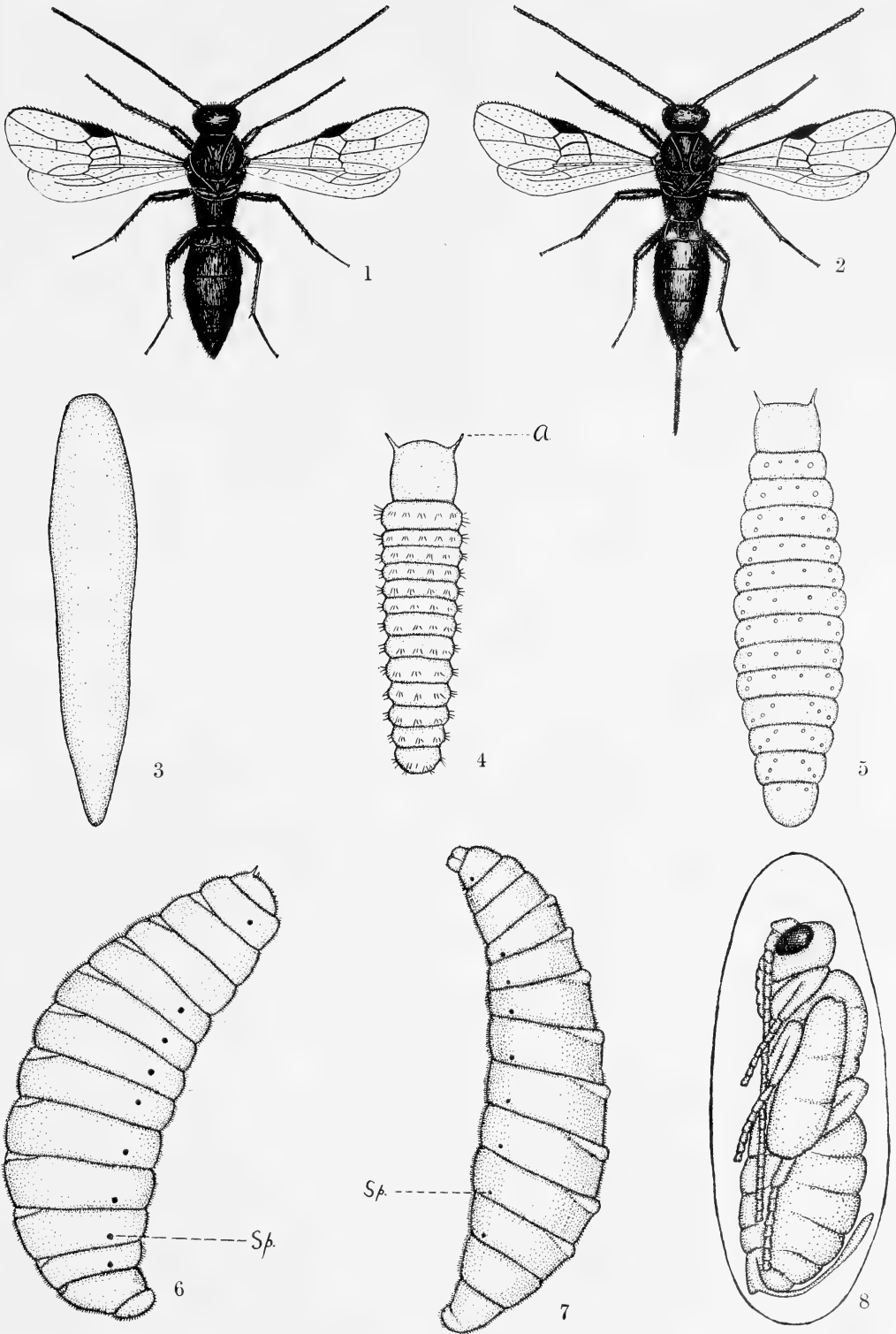
#### THE DISTRIBUTION OF *BRACON* SP.

Two methods of determining the distribution of the *Bracon* have been adopted. Suitable areas have been examined personally, and pieces of roots known to contain *Hylobius* grubs have been obtained from areas which were not readily accessible. Through the courtesy of the Board of Agriculture for Scotland a memorandum was sent to landowners and foresters throughout the country asking them to send in roots for examination from suitable areas. The following is a list of the counties in which the *Bracon* has been found to occur in Scotland:—

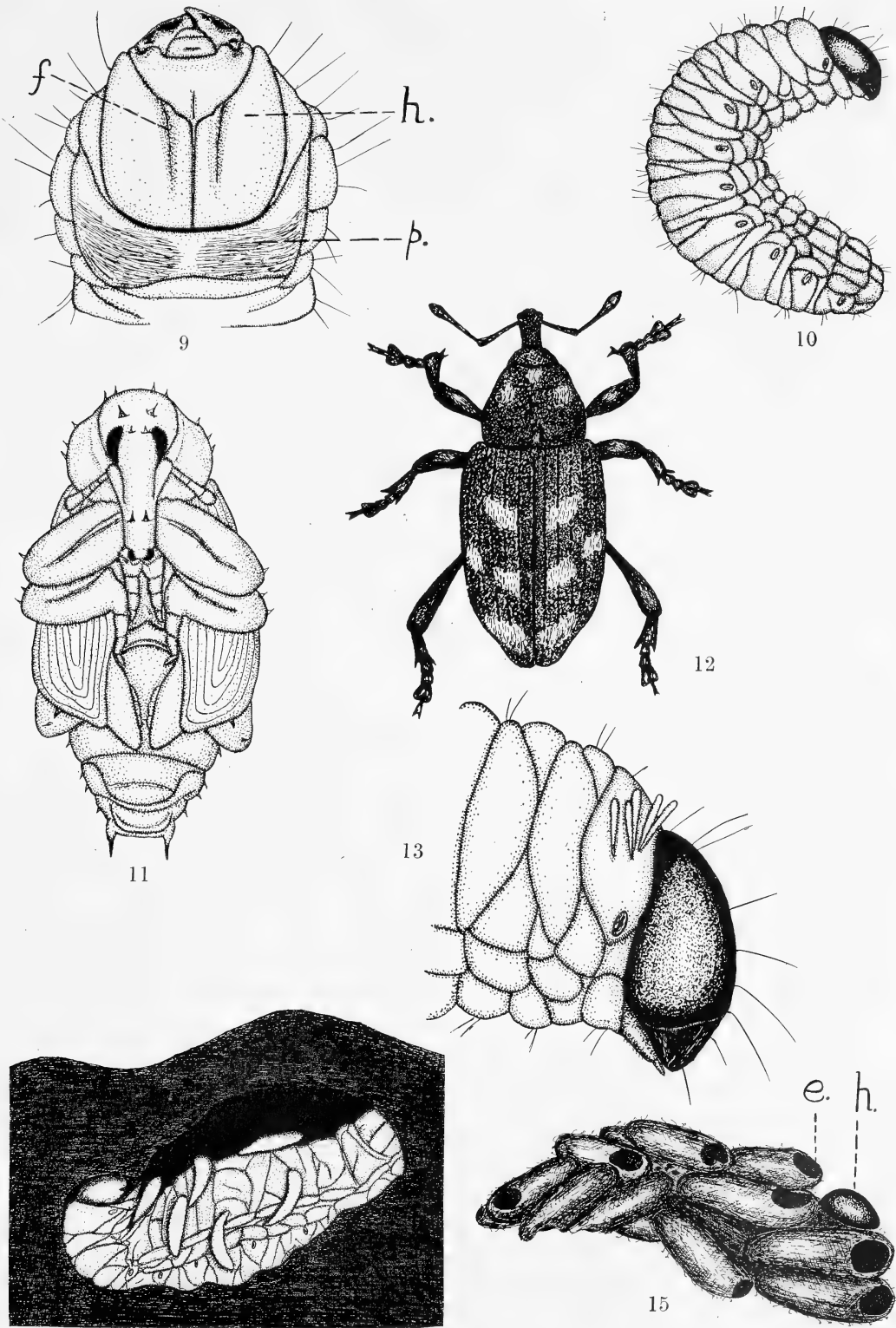
Aberdeenshire.	Nairn.
Elgin and Moray.	Peebles.
Fifeshire.	Renfrew.
Kincardineshire.	Ross-shire.
Midlothian.	

This list is of course incomplete, but it indicates a wide and probably general distribution of *Bracon*. It is probably present wherever *Hylobius abietis* occurs in numbers.

It is certain that the *Bracon* is no inconsiderable factor in helping to keep the numbers of *H. abietis* in check. This useful work might be supplemented by the experimental introduction to an area infested with *Hylobius* of additional numbers of *Bracon*. Laboratory experiments show that the *Bracon* can be reared in large numbers during the summer.











In conclusion I wish to acknowledge my indebtedness to S. J. Gammell, Esq., of Countesswells, for permission to collect *Bracon* cocoons on his estate, and to Dr R. Stewart MacDougall for his kind advice and assistance throughout my work.

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EXPLANATION OF PLATES.

PLATE I.

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| Fig. 1. Male of <i>Bracon</i> sp. $\times 8$ .   | Fig. 5. Third-stage larva. $\times 20$ .       |
| Fig. 2. Female of <i>Bracon</i> sp. $\times 8$ . | Fig. 6. Fourth-stage larva. $\times 12$ .      |
| Fig. 3. Egg. $\times 70$ .                       | Fig. 7. Fifth-stage larva. $\times 15$ .       |
| Fig. 4. First-stage larva. $\times 60$ .         | Fig. 8. Pupa in cut-open cocoon. $\times 10$ . |

PLATE II.

Fig. 9. Head and prothorax of larva of *H. abietis* showing head (*h*); (*f*) epicranial furrows distinguishing it from the larva of *P. notatus*; and (*p*) the prothoracic segmental plate. Magnified.

Fig. 10. Larva of *H. abietis*. Notice the elliptical spiracles.

Fig. 11. Pupa of *H. abietis*. Magnified.

Fig. 12. *H. abietis*, adult. Magnified.

Fig. 13. Fore part of *H. abietis* larva, to show *in situ* eggs of *Bracon*. Highly magnified.

Fig. 14. Larva of *H. abietis* in cavity on bark, with *Bracon* larvæ feeding on it. Magnified.

Fig. 15. Batch of *Bracon* cocoons, showing exit holes (*e*) and head of the dead *Hyllobius* larva (*h*).

(Issued separately March 1, 1917.)

OBITUARY NOTICES.

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Professor Gwynne-Vaughan. By Professor F. O. Bower, F.R.S.

(MS. received December 13, 1916.)

By the death of Professor David Thomas Gwynne-Vaughan, which took place at Reading on September 4, 1915, the Royal Society of Edinburgh has lost a Fellow of high scientific standing, and one whose title to fame was in an essential part based upon work published by the Society itself. He had a very high regard for the Society. On the other hand, the Society had signalled its appreciation of the merits of his work, not only by accepting it for publication in a sumptuous form, but also by the award to him in 1910 of the MakDougall-Brisbane Medal. He was elected a Fellow in the same year. He was only forty-four years of age when he died, so that in the normal course much more scientific work of high quality might reasonably have been expected from him.

He was born on March 12, 1871, at Royston House, Llandovery, being the elder son of Henry Thomas Gwynne-Vaughan of Cynghordy, later of Erwood Hall, Breconshire. His mother was Elizabeth, second daughter of David Thomas, of Royston House, Llandovery. She died in 1874, and Professor Gwynne-Vaughan was her only child. He went to school at Monmouth in 1882, and proceeded with an exhibition from school to Christ's College, Cambridge, where in 1891 he was elected to a scholarship in science. In 1893 he took a First Class in the Natural Sciences Tripos, and left Cambridge to take up a Mastership. This, however, he soon relinquished in order to enter on research. With this object he went to Kew, and was admitted to the Jodrell Laboratory within the Royal Gardens.

This move was the determining point of his career. For the laboratory was then under the direction of Dr D. H. Scott, who soon recognised the qualities which had passed unnoticed among the crowd of undergraduates at Cambridge. Wittingly or unwittingly, Gwynne-Vaughan had come under the influence of the very investigator by whom his patient and acute powers of observation could best be directed into that channel of anatomical inquiry which he subsequently did so much to advance. Stellar problems were in their infancy in 1895. Van Tieghem had broken

fresh ground, and had provided a terminology which was in advance of the observed facts. What was then urgently required was cool and controlled observation: and this was exactly what Gwynne-Vaughan was so well fitted to supply.

He first engaged in the examination of the stelar conditions seen in the Nymphaeaceæ and the Primulaceæ, and the results were published in 1897; the former in a Memoir in the *Transactions of the Linnæan Society* ("On the Morphology and Anatomy of the Nymphaeaceæ"), the latter in the *Annals of Botany* ("On Polystely in the genus *Primula*"). At the British Association Meeting at Liverpool in 1896 he gave a preliminary account of his observations, which showed his hearers that not only a new investigator, but also a new teacher had appeared. It led to his appointment as Assistant in the Botanical Department of Glasgow University. Lang was already a member of the staff there, and for ten years these two, with the Professor, worked together, each in his several way, but with full mutual knowledge, upon problems relating to the Pteridophyta.

It is not always that Scottish students take to a teacher having marked racial characters not their own. But Gwynne-Vaughan, though a Welshman through and through, was a success in Glasgow from the first, with the large medical classes as well as with the smaller classes for women at Queen Margaret College. He must have handled some 1500 students while in Glasgow, and it was clear that they appreciated the energy and single-mindedness of his work with them. On excursions his activity, combined with his sporting instincts and wide natural knowledge, secured for him an enthusiastic personal following. He also took his share in the advanced teaching, laying in certain restricted branches the foundation of those valuable notes which were expanded over the whole area of the science during his later period of teaching in London.

The anatomical experience which Gwynne-Vaughan had gained at Kew, and his grasp of stelar questions as illustrated in flowering plants, fitted him to enter with special insight on his arrival in Glasgow into the investigation of the anatomy of the Filicales. Mr Boodle was already engaged at Kew in similar work. But by mutual consent it was arranged that while he undertook more especially the Schizæaceæ, Gleicheniaceæ, and Hymenophyllaceæ, Gwynne-Vaughan should devote himself to types which showed greater advance in anatomical complexity. Accepting the protostelic state as probably the primitive condition for all, it became necessary by patient observation and comparison to relate to it the more complex states already recognised by Van Tieghem as "polystelic."

Gwynne-Vaughan approached this naturally through those types with a tubular stele in the axis, which he designated "solenostelic," reviving a term already introduced by Van Tieghem. A careful analysis of the anatomy of *Loxsonia*, as a typical example of this structure, was followed by comparisons with numerous other genera, such as *Hypolepis*, *Dennstaedtia* and *Pteris*. The gradual steps to dictyostely were thus traced, and the demonstration given that not only by comparison, but also in the individual life (e.g. *Alsophila*), the transition depends upon the overlapping of the foliar gaps in an abbreviated axis. The facts were embodied in two papers, entitled "Observations on the Anatomy of Solenostelic Ferns," published in the *Annals of Botany*, 1901, 1903. These contain a great body of condensed comparative observation, showing that solenostely and dictyostely are related conditions. The origin of medullary vascular tracts within the solenostele was also traced from their simplest beginnings.

But still there remained the more difficult question how the solenostelic state itself was related to the protostelic. Towards the solution of this problem resort was made to comparison of certain fossils related to the living Osmundaceæ. The work was carried out in happy co-operation with Dr Robert Kidston, F.R.S., of Stirling. This co-operation was real and equivalent. The one partner brought to bear on the problem a wide knowledge of fossils from the stratigraphical point of view: and he had already taken up a cognate inquiry in the Sigillarias. The other supplied critical and expert anatomical experience based upon the study of living plants. The result is a series of beautifully illustrated Memoirs published by the Royal Society of Edinburgh ("On the Fossil Osmundaceæ," *Trans. Roy. Soc. Edin.*, 1907, 1908, 1909, 1910, 1914). The fossil material was of world-wide origin, but chiefly from New Zealand and from Russia. In a sequence of plants, dating from the Permian Period to the present day, and shown to be really related to one another by many structural similarities, an anatomical progression was traced which follows most convincingly the successive stratigraphical horizons. It illustrates steps in the medullation of the stele and in the amplification of the leaf-trace, and it throws light upon the probable origin of the leaf-gap. Some still hesitate in full acceptance of the far-reaching conclusions which were drawn: but in most quarters the Memoirs of this magnificent Series are held as botanical classics; and none can be blind to the validity of the methods, or the acuteness of the internal criticism which they show. Though the details of the progression from the protostele to the confirmed solenostele are not fully demonstrated in them, the foundations are securely laid, and others are already building upon them.

In relation to this work on the Fossil Osmundaceæ there was also published a short but important joint note "On the Origin of the Adaxially-curved Leaf-trace in the Filicales" (*Proc. Roy. Soc. Edin.*, 1908, p. 433). There are also two later papers by Gwynne-Vaughan alone. The first of these, entitled "Some Remarks on the Anatomy of the Osmundaceæ" (*Ann. of Bot.*, July 1911), dealt with the structure of the young plant, and the origin of the medullation, and of the foliar gaps in the individual life. The second was his last completed work, and it described a case of "mixed pith" found by Mrs Gwynne-Vaughan in an anomalous specimen of *Osmunda regalis*. The structure seen in it was held to support the theory that the pith of the Osmundaceæ is phylogenetically stelar, and not cortical.

The two series of Memoirs above mentioned embody the most effective work of Gwynne-Vaughan. But they by no means exhaust it. He originated a most ingenious theory of the stele of *Equisetum* (*Ann. of Bot.*, 1901); he discovered the axillary buds of *Helminthostachys* (*Ann. of Bot.*, 1902); he worked through the anatomy of *Archangiopteris* (*Ann. of Bot.*, 1905). He also wrote on the curious lattice-work structure of certain Fern stems (*Ann. of Bot.*, 1905), and on the minute structure of the tracheæ of Ferns (*Ann. of Bot.*, 1908). In all of these his originality was patent, though restrained. He had also entered upon other work in co-operation with Dr Kidston. A Memoir on *Tempskya* was published in Russia (*Verh. d. russ. kaiserl. mineral. Gesellschaft*, Bd. xlviii, 1911), and a new series "On the Carboniferous Flora of Berwickshire" had been opened with its "Part I, *Stenomyelon*" (*Trans. Roy. Soc. Edin.*, 1912). But here the curtain falls prematurely upon a life of investigation full of promise, as in the past it had been remarkably full of achievement. Only forty-four years of age, we can hardly forecast what Gwynne-Vaughan might yet have done if he had lived out the full span.

But he was not merely a laboratory botanist. He had an acute sense of specific characters, and a good knowledge of the native flora in the field. His systematic analysis of difficult species of Algæ was pertinacious and successful. And equally in the determination of Ferns, his systematic powers were exercised upon the collection brought back by Professor Lang from a journey in the Eastern Tropics. He was also an extensive traveller himself. In 1897 he went up the Amazon and Purus Rivers some 3500 miles, as botanist attached to a rubber-prospecting expedition: but his scientific proclivities were restricted by the jealous demands of the firm that employed him. In 1899 he joined the Skeat Expedition to the Malay Peninsula, and experienced the delights of forest life with

attendant Malays on the borders of Siam. Extracts from his vivid letters home during this period of adventure are given in Scott's obituary notice in the *Annals of Botany*, vol. xxx, pp. v-x. They show not only lively observation, but also a vigorous literary style. Perhaps owing to his own delicate sense of duty to the expedition he was with, he made no private collection in Siam for the purpose of future work. It is, however, an interesting fact that some of the plants which he collected for the Skeat Expedition were among the last of the new species determined by the veteran Sir Joseph Hooker. In 1909 he attended the British Association Meeting at Winnipeg, making the acquaintance of Canadian forests and lakes. Thus as a traveller he had touched three of the great geographical areas of the world.

We have traced Gwynne-Vaughan to Glasgow, where he worked from 1896 to 1907, as assistant, and later as lecturer in Queen Margaret College. In 1902 he co-operated with Professor Bower in producing the second edition of *Practical Botany for Beginners* (Macmillan & Co.), which has gone through several reprints. In 1907 he was appointed Head of the Botanical Department at Birkbeck College, London, and for two years he toiled in formulating elementary and advanced lectures covering the whole area of the science. In 1909 he received the appointment as Professor in Queen's University, Belfast. In 1911 he married Dr H. C. I. Fraser, herself an accomplished botanist, who had succeeded him in the post at Birkbeck College. He was finally transferred from Belfast in 1914 to the Chair of Botany in University College, Reading. But he lived only to complete one full year of duty there. His health had latterly not been good, though he bravely continued his work to the end. He may be said to have run in harness till within two months of his death.

His appreciation among botanists has been quite general. All felt the sincerity, the acuteness, the scrupulous care that characterised his work. An outspoken critic, he was always ready to help with suggestions and with facts, of which he had an uncommon store laid by in very carefully tabulated notes. He became personally known to the general body of British botanists by holding office, first as Secretary (1901, 1909-11), and later as Recorder (1912-13) of Section K of the British Association. Not only was the business of the Section well conducted by him, but he had the power of keeping the body of botanists together in friendly relations. They will feel that by the death of Gwynne-Vaughan they have lost not only a prominent investigator, but also a colleague who had a happy power of promoting unity and co-operation.

Already distinctions had come his way. He was elected to the Linnæan

Society in 1907. In 1910 the Royal Society of Edinburgh awarded to him the MakDougall-Brisbane Medal for his researches published in the Society's *Transactions*, and he was elected a Fellow the same year. In 1912 he was elected M.R.I.A. His friends anticipated for him, at an early date, further and even higher distinctions. But such honours are only the ostensible signs of appreciation currently given by contemporaries. The work of Gwynne-Vaughan is of a nature that will ensure its permanence: and that is the mark of real distinction. His results were always strictly tested and criticised before publication. The consequence is that they will be durable, and take permanent place in the web of Botanical Science.

Personally, Gwynne-Vaughan was of light build. At Cambridge he rowed and played Rugby football. In later years, cycling and fishing were among his amusements. But as the interest of his scientific work gripped him, he sacrificed more and more of the time available for exercise to his laboratory. A prominent characteristic was his dry humour, which was combined with an almost whimsical expression of it. Through this shone constantly the steadfast scientific ideal. As a colleague he was always loyal and helpful. Private and personal interests were wholly effaced by the dominating sense of duty and of camaraderie. Such factors made up a personality as attractive and refreshing as it was original. He was one different from the common run of scientific men: and it is the unique personality that we miss most when it is gone.

**Sir William Turner**, K.C.B., D.L., M.B.Lond., F.R.C.S.S.L. and E., LL.D., D.C.L., D.Sc., F.R.S.S.L. and E., Principal and Vice-Chancellor, University of Edinburgh, Honorary Burgess of the City of Edinburgh. By **Sir James A. Russell**. (With One Plate.)

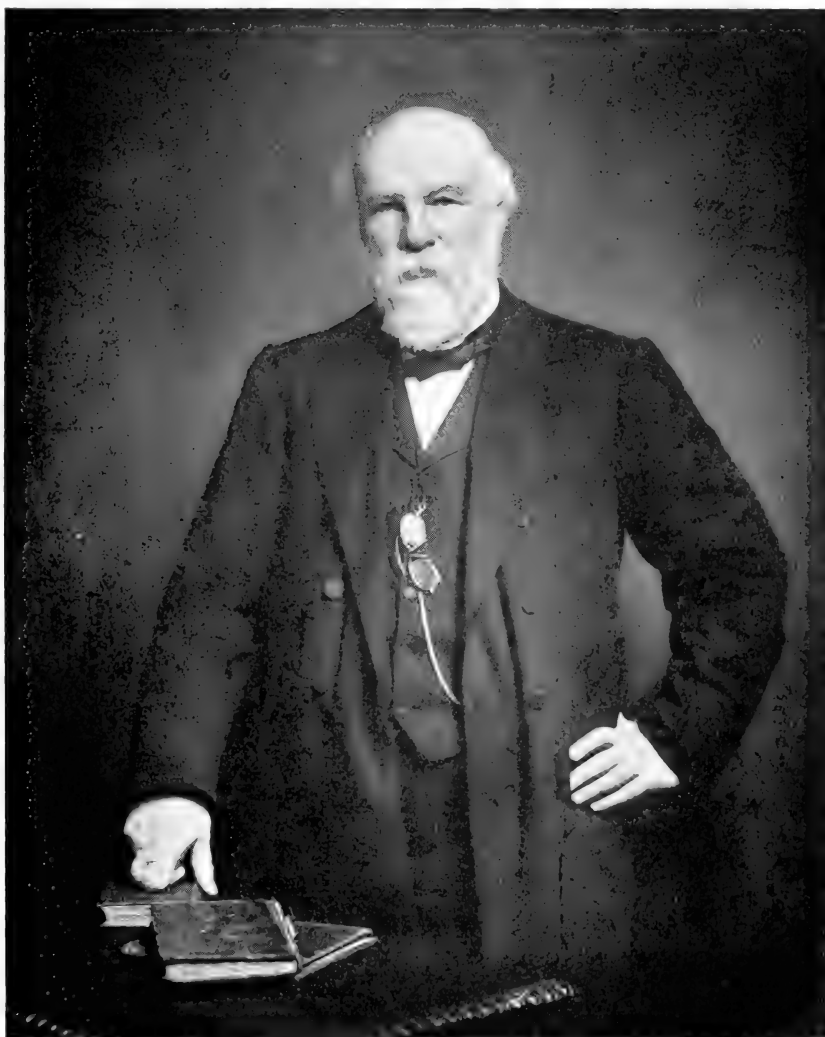
(Read at the Meeting on May 1, 1916.)

AT the eighth Ordinary Meeting the President, Dr Horne, gave utterance to the sorrow with which the Fellows heard the news of the death of Sir William Turner, K.C.B., and expressed the Society's high appreciation of the services which Sir William rendered to it. Indeed, his devotion to its interests was one of the striking features of his distinguished career.

Throughout his long life Sir William had been free from serious illness, and conserved his physical and mental powers to the last. He was cut off by a short illness in the midst of work, at the age of 84, on 15th February 1916. We recall his sturdy frame and rapid walk, his twinkling eyes, strong voice, and dominant personality, his dignity, invariable courtesy, cheerfulness, fund of humour, wonderful memory, and cautious, judicious mind. He was a man of the highest talent, eminently sane and workable, and absolutely free from the want of balance which is not unfrequently associated with genius. Those who had to work with him will always remember his high sense of the importance of work, and his devotion to duty. He possessed extraordinary powers of steady work and an unwearied patience that endeared him to slow or stupid students in his early days, and that later amazed his colleagues on the University Court when he zealously pursued the true inwardness of the dreary details of a University ordinance or hunted the last sixpence in University accounts long after the interest of the others had flagged. When working jointly with an assistant he undertook the heaviest or most disagreeable part of the task himself. His self-control was admirable. He once said to the writer, "If my digestion is in order I defy any man to make me lose my temper." With these qualities Sir William was a tolerant and sincere Christian gentleman. He was a member of St John's Episcopal Church for about sixty years, and served on its vestry for many years. At the same time he attended and took part in University and students' services of the Presbyterian Churches.

In 1863 he married Agnes, eldest daughter of Mr Abraham Logan of Burnhouses, Berwickshire, and this union did much to attach his affec-





*Photo by*

*Elliott & Fry.*

PRINCIPAL SIR WILLIAM TURNER.



tions to Scotland. Lady Turner died in 1908. They had a family of three sons and two daughters, all of whom survive.

He was tenacious of his rights, both public and private, and expressed great indignation when the services of a gamekeeper were offered on behalf of the proprietor to "show him the way" along an old drove road through a deer forest. Turner would not accept the semblance of permission when he claimed a right. Turner had a hearty laugh, and could find interest and amusement in little things. No boy of the party laughed more at the antics of a goose in its vain endeavours to capture a small trout in the River Earn. He was an omnivorous reader, and this, with his sense of humour and retentive memory, made his conversation most interesting. He became a member of the Royal Society Club in 1869, and it is needless to say that his genial presence was much appreciated at the dinners. Turner always showed appreciation of the games and exercises in which students indulged, and yet he did not apply himself to golf or other form of sport. His exercise he got from walking, often taking a roundabout way to the University for the sake of the walk. Boating was also a favourite form of amusement when he lived at the seaside.

He was an enthusiastic Volunteer, being one of the original members of the University Company of Volunteers in 1859. At first the Government did not even furnish rifles, and later, when grants were made, the privates of No. 4 Q.E.R.V.B. had to pay regular subscriptions. Lieutenant Turner's superior officers, the stern Captain Sir Robert Christison, Bart., and the alert Surgeon-Major Sir Douglas Maclagan, gave him the proper training in military discipline. He was promoted to be Major, and finally retired, after thirty-one years' service, as Lieut.-Colonel, Queen's Rifle Volunteer Brigade, Royal Scots, 1889-90, V.D. Only a fortnight before his death he took part in the Officers' Training Corps service in St Giles' Cathedral.

During the period of his activity as demonstrator and Professor he devoted little time to luncheon. It consisted of some sandwiches—to which Lady Turner later made him add a cup of extract of beef in hot water—taken in his private room after lecture at two o'clock while interviewing assistants, students, or other visitors: a very frugal refreshment, which caused the loss of no time and did not lead to relaxation of effort in the afternoon. He did not smoke, but did not object to the use of tobacco by others.

He was noted for shrewdness and insight, and many took advantage of his wise counsel in difficulty or when considering their future course. Former students, and especially former assistants, were supported by him with all the energy and influence that he possessed when they were

applicants for chairs or other posts. No exertions were too great for him in such a case.

Turner always had great enjoyment in fine scenery, and when demonstrator spoke with enthusiasm of Bellagio and the Italian lakes. Later he was carried away by the delights of a driving tour in fine weather, far from the madding crowd, in the north-west of Scotland, and of motor tours in England, when he gathered all that he could learn of the Romans or others who used the open way in past times. Travel on the Continent with members of his family was his chief relaxation in later years, and then he enjoyed cathedrals and other fine or historical buildings to the full. He also visited Canada and the United States.

Turner was thoroughly loyal to the city of his adoption, of which he was a deputy-lieutenant, and no one was more ready to obey the call of the Lord Provost for advice or assistance. Honours that many accepted as mere compliments were to him occasions for the discharge of duty. At the time of his death he was a Vice-President of the Royal Blind Asylum and School, and the chairman of directors acknowledged his services and readiness to make speeches on behalf of the institution when required. He served on the board of Donaldson's Hospital for many years, and was seldom absent from a committee meeting. Sir William Turner was Honorary Professor of Anatomy to the Royal Scottish Academy, an office which he held from 1878. He showed his interest by visits to the life school and in other ways. How he found time for all his many-sided activities can only be explained by his orderly mind, punctuality, and diligence, supported by a very happy home life.

On 7th August 1913 a mural tablet was unveiled, with appropriate speeches, in Sir William Turner's presence, to mark the site of the house in Lancaster in which he was born on 7th January 1832. Afterwards, in the Town Hall, Mr H. L. Storey presented to the Corporation a portrait of his father, the late Sir Thomas Storey four times Mayor of Lancaster, and Sir William Turner delivered an address in appreciation of his old friend. Sir Richard Owen, likewise a native of Lancaster, was born some twenty-eight years before Turner.

Turner's mother, who was Miss Aldren before marriage, lost her husband, William Turner, when he was forty years old and her son William was five. There were another son and daughter who died young. She apprenticed William to Christopher Johnson, surgeon, who gave him a liking for chemistry. Evidently the lad was a diligent apprentice if ignorant of modern views of infection, for he was busy with pestle and mortar pounding drugs for pills while his hands were desquamating after scarlet fever!

When Turner had to choose a medical school he followed Owen's steps to St Bartholomew's Hospital, and there became intimate with his fellow-student George Rolleston, afterwards Linacre Professor of Anatomy and Physiology at Oxford. He also made the acquaintance of Sir James Paget, of whose character and attainments he ever after spoke with admiration. As a student Turner delighted in the theatre, for which he showed little inclination after he came to Edinburgh. He was a distinguished student, holding a scholarship, and worked for about a year in the chemical laboratory in addition to attending lectures and the practical class taken by the ordinary medical student. John Stenhouse, LL.D., F.R.S., his Professor of Chemistry, writing to Turner when candidate for the Chair of Anatomy in 1867, said: "I may state that Dr Rolleston, now Linacre Professor at Oxford, and yourself were by far the ablest and most promising pupils I had during the seven years I held the Professorship at St Bartholomew's." At a lecture, 2nd March 1855, at the Royal Institution, on the economical application of charcoal to sanitary purposes, Dr Stenhouse stated that the charcoal that had been in contact with two dead dogs had been examined by his pupil Mr Turner. Turner's paper, communicated by Sir James Paget, F.R.S. (received 18th May 1854), on examination of the cerebro-spinal fluid, and appearing in the *Proceedings of the Royal Society of London*, was a purely chemical investigation of fluid from a case of spina bifida treated by Sir James Paget. Even so late as 1861 he communicated a chemical paper, "On the Properties of the Secretion of the Human Pancreas," to the *Proceedings of the Royal Society of Edinburgh*, and one, "On the Mode of Elimination of the Metal Manganese when employed Medically," to the *Edinburgh Medical Journal* for April—showing that he had not yet abandoned his test tubes.

Turner told the writer that his intention as a student was to take to chemistry, and that he counted on winning a chemical scholarship by examination, but that a question was set on agricultural chemistry and he was beaten by the son of a farmer. Without the money and the prestige of this scholarship he felt that he could not set up as a chemist. While under this check the illustrious anatomical philosopher Goodsir appeared and took him to become an anatomist, and so changed the direction of his life. Professor Goodsir, the premier anatomist of his day, had gone to the Riviera on leave of absence from the University of Edinburgh for his health, while his class was taken by Dr Struthers, lecturer on anatomy in the Edinburgh Extramural School. When he returned in 1854 he found that he had no demonstrators. In these circumstances he proceeded to London to consult his old friend Professor Sharpey of University College,

who with Sir William Ferguson of King's and Sir James Paget of St Bartholomew's found three young surgeons willing to come to Edinburgh. They were Frederick W. Sayer, who died of fever after a short period of service in Edinburgh; A. M. Edwards, who had already a year's experience as a demonstrator of anatomy; and William Turner, M.R.C.S. 1853. After serving for a few years in the dissecting-room, Edwards entered on a brilliant surgical career in Edinburgh that ended obscurely in Australia. Goodsir sent for Turner to his hotel and asked him how he would describe Scarpa's triangle (a favourite test question). Turner indicated how he would set about it, and Goodsir promptly appointed him to be senior demonstrator. Turner might have been less surprised at Goodsir's quickness had he known of Sir James Paget's letters of recommendation.

Turner came to Edinburgh in the autumn of 1854 to spend what he called the most miserable winter of his life, owing to his lack of experience in lecturing on anatomy. It must be remembered that Turner was at this time himself a student with examinations to pass. He only took his M.B. Lond. in 1857. He had a gold medal and honours in chemistry in 1854.

Turner's duties as senior demonstrator consisted (1) in giving a daily demonstration or lecture on topographical anatomy at 4 p.m., in which he also explained the relation of a knowledge of the parts described to the practice of medicine and surgery; (2) in giving a course of demonstrations on microscopic anatomy; (3) in superintending the work of the dissecting-room; (4) and also, in summer, in giving a course of advanced lectures on some special department of anatomy. Goodsir himself gave the formal scientific lecture at one o'clock, when the facts of human anatomy were illustrated from comparative anatomy or other sciences. The students paid a fee for the four o'clock demonstration, but received no credit from the University for their attendance, as it was not compulsory; and yet the room was always filled.

As Goodsir's health gradually failed he transferred more and more of the duties of the chair to Turner, who had gained his full confidence and highest esteem. After thirteen years' service as demonstrator, Turner was appointed to the Chair of Anatomy in 1867 on Goodsir's death, and held the appointment for thirty-six years until he became Principal in 1903. While demonstrator he had several notable juniors, among them H. S. Wilson, John Cleland, Joseph Bell, Thomas Annandale, Ramsay H. Traquair, John Chiene. Of these, Emeritus-Professor Cleland of Glasgow, upon whom Goodsir's mantle as a philosophical anatomist chiefly fell, and Emeritus-Professor Chiene, C.B., of the Chair of Surgery at Edinburgh, alone survive.

His accession to the chair made little difference to the teaching arrangements, except that the senior demonstrator had to give the four o'clock lectures on topographical anatomy alternately with the professor.<sup>1</sup>

The quality of Turner's teaching of anatomy has been attested not only by many hundreds of pupils, but shown by the number of former assistants who became professors of anatomy. The list includes the late Morrison Watson of Manchester (who married a sister of Lady Turner); Watson's successor at Manchester, the late Alfred Harry Young; the late Daniel John Cunningham, Dublin, and Turner's own successor at Edinburgh; the late John Halliday Scott, of Otago, N.Z.; Johnson Symington, Belfast; Arthur Thomson, Oxford; David Hepburn, Cardiff; Arthur Robinson, King's College, London, and Birmingham, who succeeded Cunningham at Edinburgh; A. M. Paterson, Liverpool; J. T. Wilson, Sydney; Robert Howden, University of Durham; J. C. Lamont, Lahore and University College, Dundee; T. H. Bryce, University of Glasgow; James Musgrove, St Andrews, retired and now succeeded by David Waterston; Alexander Primrose, formerly anatomy, now surgery, Toronto; Richard J. A. Berry, Melbourne.

Turner's incomparable success as a lecturer on anatomy was partly due to the natural gifts of a strong, distinct voice, splendid memory, and earnest emphatic style, so that the students were listening to very clear thoughts put into very clear words. The effect of his style was enhanced by the little-known fact that he usually suffered from stage fright for some minutes before he faced the big one o'clock class. Much, however, was due to preparation. Every statement was carefully arranged to come in the proper order of logical sequence, and side lines of thought that might confuse were rigorously eschewed, however tempting. Then the lectures were in proper perspective and adapted to the audience. Points of scientific or practical interest were always mentioned, but ordinary students were not bored with an excess of detail or with too much advanced anatomy, which was administered to the *élite* in special courses. The precaution was also taken of refreshing the memory of the audience with a *résumé* or summary of the previous lecture, or of facts supposed to be known already. Suppose that Turner had to address a medical audience on new points in the anatomy of the brain, he assumed that their recollection of the more

<sup>1</sup> When Turner became Professor in 1867 he appointed John Chiene to be senior demonstrator. Chiene began teaching surgery in 1870, and was succeeded as senior demonstrator by the late Morrison Watson, who became Professor of Anatomy at Manchester in 1874. The writer, who was appointed Inspector of Anatomy for Scotland in February 1881, and, like Chiene and Watson, was one of Goodsir's pupils, succeeded Watson as senior demonstrator, but resigned this office in 1876, when the direct connection with Goodsir was broken by the appointment of Daniel John Cunningham.

elementary anatomy of the organ had become dim, and would spend the first quarter of an hour or so in running over points supposed to be known already, before embarking upon what was new. With all this there was some attraction in his style that defies analysis; and assistants such as Morrison Watson, D. J. Cunningham, or A. H. Young (who became professors) would sometimes steal behind the screen that cut off the back of the classroom to listen with admiration, at the four o'clock demonstration, to the exposition of the anatomy of femoral hernia or some other favourite topic.

The lectures were always effectively mounted with diagrams, preparations, and fresh dissections. In such large classes it was usually easy to find two prosectors for the one o'clock lecture-class and other two for the four o'clock demonstration. One of the prosectors for the lecture-class, the late Dr James Foulis, deserves special mention, for he was Turner's chief assistant in dissecting the Longniddry whale, and was acknowledged to be the best dissector seen in the Edinburgh School for a generation. In preparing a fresh dissection for the lecture-class, he and his colleague on some special occasions began at seven in the morning and worked continuously until one o'clock.

Turner had the affectionate esteem of the students, and disorder was practically unknown. At the slightest disturbance Turner paused and stared sternly at the spot. The students knew instinctively that he would never appeal to the class, but that if necessary he would note the guilty man and that severe measures would follow. Any bad members of the class did not presume, because they knew him; but at a graduation after he became Principal, two youths who did not know him behaved disgracefully and were expelled. His predecessor had been too lenient, and they had come to think that any conduct would be forgiven.

When Turner arrived in Edinburgh in 1854 to begin his splendid and many-sided career as teacher, man of science, man of affairs, and administrator, he came under the spell of Goodsir as regards teaching and research, and soon made like-minded friends. T. Spencer Cobbold was Conservator of the Anatomical Museum under Goodsir, and published papers on the Anatomy of the Giraffe, Trematode Worms, etc. Lister, born in 1827, came to Edinburgh the year before Turner to have a look at Professor Syme's work, took the place of his infirmary resident, Dr Dewar (called away by his father's illness), married Syme's daughter, and remained in Edinburgh until appointed to the Regius Chair of Surgery in Glasgow. In 1859 Lister and Turner published a joint research. Turner's friend, J. Matthews Duncan, published a paper on the Os Sacrum in 1855.

From the time that he joined the University of Edinburgh until the end



of his life Turner was occupied with research connected with physiology and anatomy in all its branches, descriptive or histological, healthy or pathological, human or comparative, ethnological and teratological—his studies of crania and in anthropology being especially remarkable.

A list (prepared by himself) of 276 writings, with the addition of his last communication to this Society on 5th July 1915, entitled "A Contribution to the Craniology of the People of Scotland: Part II, Prehistoric, Descriptive, and Ethnographical," is appended. The classification is as follows:—

Human Anatomy and Physiology . . . . .	77
Comparative Anatomy and Zoology . . . . .	104
Pathological Anatomy . . . . .	15
Anthropology . . . . .	51
General Addresses, Reviews, etc. . . . .	26
In Memoriam . . . . .	4
	<hr/>
	277

His researches are characterised by fullness and accuracy of observation, precision of statement, and very cautious deduction. Consequently they will continue to be sources of trustworthy information.

When Turner applied for the chair many testimonies of the utility of his researches were given by men engaged in the practice of medicine, as well as by cultivators of science. Robert Barnes found his contributions to the knowledge of abnormal conditions of the uterus and ovaries of remarkable value. Hughlings Jackson said: "By his work some of the driest details of human anatomy have become new points of departure." "He has made surgery safer, pathology clearer, and the method of studying the relations of mind to brain more definite and satisfactory." William Sharpey acknowledged his services to anatomy, and Lionel S. Beale voiced the appreciation of physiologists.

Perhaps the papers most welcomed by medical men were his studies of the brain from 1866 onwards for some twenty years. The paper in the *Journal of Anatomy and Physiology*, 1874, on "The Relation of the Cerebrum to the Outer Surface of the Skull and Head," was a pioneer communication on the subject. Among other papers important to medicine were that on "A System of Anastomosing Arteries connecting the Visceral and Parietal Branches of the Abdominal Aorta," *Brit. and Foreign Med.-Chirurg. Review*, July 1863, and that on "A Supplementary System of Nutrient Arteries for the Lungs," *Reports British Assoc. Advance. Science, Bath*, p. 129, 1863. One of his most outstanding studies, important alike

to science and to practical medicine, which let light into dark places, was that of placentation, begun in 1870, when he published a paper in the *Trans. Roy. Soc. Edin.*, "On the Gravid Uterus and on the Arrangement of the Foetal Membrane in the Cetacea." He lectured on the comparative anatomy of the placenta in the Royal College of Surgeons of England in 1875 and 1876, and returned to the subject from time to time up to 1889.

The anatomy of whales had fascinated the Edinburgh anatomists Knox, John Goodsir, and John Struthers, afterwards Sir John Struthers of Aberdeen. Turner fell a victim to the same fascination in 1860, and continued to write papers on the Cetacea until 1914. He began with a paper in our *Transactions* on the thyroid gland in the Cetacea, with observations on the relations of the thymus to the thyroid in these and some other mammals. Goodsir had a good collection of cetacean specimens in the Anatomical Museum, but Turner increased this very largely, and induced Sir John Struthers to take a benevolent interest in the museum, and especially in the cetacean bones, when he returned to Edinburgh from Aberdeen after he retired from his chair of anatomy.

The great Finner whale stranded at Longniddry in 1869 was undoubtedly the largest subject ever dissected by Turner, and tested the ardour of himself and of his pupil and colleague in the work, Dr James Foulis. This whale, a pregnant female, 78 feet 9 inches in length, was left on the beach for inspection by the public, who were conveyed by special trains; and when the stench of putrefaction became so grievous that ordinary persons could not approach the carcass, it was towed across the Firth to have the blubber removed and other parts turned into money. It was reported that at Longniddry an incautious visitor fell into the tongue when walking on a lower jaw-bone. Turner and Foulis pursued the whale to Kirkcaldy and accomplished feats of observation and dissection, with the result that Foulis had to part with his suit of clothes, and Turner, although more cautious, found that his footprints were an object of great interest to all dogs that crossed his track.

There are papers about the Longniddry whale in the *Proceedings* and *Transactions* of this Society of 1869 and 1870, and Turner was awarded the Neill Prize in 1871 for this investigation. The sternum and ossa innominata are described in the *Journal of Anatomy and Physiology*, 1870.

Turner made his *début* as a craniologist with papers before the British Association Newcastle-on-Tyne meeting in 1863, and these papers, together with papers in our *Proceedings* and in the *Proceedings of the Society of Antiquaries of Scotland* for 1864 and 1865, show that he had accepted the doctrines of Darwin notwithstanding the opposition of Owen and

Goodsir. He began and continued to collect and examine crania from this time. Ancient and modern skulls of every race and from every country, skulls normal and skulls deformed, were all welcome. Former pupils and foreign friends all helped. In one instance a captain and part owner of a steamer got himself into danger in the East when trying to dig up skulls for Turner. At the time of Turner's death he had a number of skulls in the room reserved for him in the Anatomical Department of the University which he had not yet examined. Our *Transactions* are enriched by many papers from his pen on the races of mankind and their craniology. In 1904 he was awarded the Keith Prize for his various memoirs on the craniology of the peoples of Scotland and of India. He edited the second edition in 1863 and the third edition in 1870 of Paget's *Pathology*, revising the pathological while Sir James Paget revised the clinical portion.

Turner prepared an *Atlas of Human Anatomy and Physiology*, with handbook, in 1857, and wrote the article "Anatomy" for the *Encyclopædia Britannica*, ninth edition. This well-balanced article was expanded into *Introduction to Human Anatomy, including the Anatomy of the Tissues*, 1877. An early copy was sent to Oliver Wendell Holmes, who proved that he had read it by sending back a list of errata.

He collected and edited the *Anatomical Memoirs of Professor John Goodsir, F.R.S.*, in two volumes, Edinburgh, 1868. He also arranged and edited the *Scientific Papers and Addresses by Professor George Rolleston, F.R.S.*, two volumes, Oxford, 1874.

In April 1866 Turner became examiner in anatomy for the University of London; and in the autumn of the same year he joined the late Professor G. M. Humphry of Cambridge, also a St Bartholomew's man, in founding the *Journal of Anatomy and Physiology*, an important quarterly magazine, which they edited. Some twenty years later Humphry and Turner helped the late Mr C. B. Lockwood, another St Bartholomew's man, to found the Anatomical Society.

A great many of Turner's papers, especially of those on human anatomy, appeared in the *Journal of Anatomy and Physiology*; but papers appeared in other magazines, and in reports such as those of the *Challenger* and the British Association for the Advancement of Science.

He was elected a Fellow of the Royal Physical Society in 1858, and had the unique distinction of holding the office of President for four successive years, 1863-1867. Besides the presidential address in 1888, vol. x, and a paper by George Logan and W. T., 1867, there are seven papers by him in the *Proceedings* of that Society, of which he became President for a second time, 1885 to 1888.

Next to the University of Edinburgh the Royal Society of Edinburgh held a place near his heart, and of his more important contributions to science thirty-nine are to be found in its *Proceedings* and twenty-four in the *Transactions*. He was elected a Fellow on 4th February 1861, became a Member of Council in 1866, and succeeded Professor Allman as one of the Secretaries to the Ordinary Meetings in 1869. This office he held for twenty-two years, and was associated in it first with Professor Tait and then with Professor Crum Brown. In 1891 he was elected a Vice-President of the Society, and served for ten years, Dr Ramsay H. Traquair taking his place as Secretary.

When the office of President became vacant by the lamented death of Lord Kelvin, the Fellows turned to Sir William Turner, who was elected President in 1908. He occupied the position for five years, and became a permanent Member of Council on retiring. He made an admirable President both at the Council board and at the meetings of the Society.

During the greater part of his fifty-five years as a Fellow he held office in the Society, but one of his most important services was rendered at a critical time before he became President, during the negotiations with the Government when the Society had to remove from the Royal Institution buildings. He loyally supported the late Professor Chrystal in applying for the new rooms in George Street, and it was his speech that convinced the Secretary for Scotland of the justice and weight of the claim made by the Society.

His presidential address on the occasion of the opening of the new building in George Street, 8th November 1909, "On the Rise of Scientific Study in Scotland," contained an historic sketch of the Society, with lists of the officials from the commencement, and an index, under both authors and subjects, of the communications to the *Transactions* from 1889 to 1908, thus supplementing the index published in 1890.

Turner became a Fellow of the Royal Society of London in 1877, and contributed papers to the *Proceedings* and *Transactions*. He served on the Council in 1890.

In 1867 the Medical Professors of the University of Edinburgh were nearly all very distinguished men of marked personality, who did not hesitate to express their views, whether about scientific or medical matters or each other, with a freedom and force that led to serious differences at times. Turner managed to keep friends with every one, and his business capacity was soon recognised in the Senate. His master in University politics was Sir Robert Christison, Bart., who not only imbued him with his own attachment to the interests of the University, but helped to create

in him a certain jealousy of the Extra-mural Medical School, so that in matters involving the interests of the University and the Extra-mural School Turner became a strong University partisan.

He became a Fellow of the Royal College of Surgeons of Edinburgh in 1861, and was elected President of the College in October 1882, but served only one year instead of following the usual course of seeking re-election for a second year. The movement for obtaining the patronage of the Royal Colleges for the Extra-mural School had begun, and with his views of the rights and privileges of the University Turner found the position difficult.

He served as Dean of the Medical Faculty in the University for some years. In November 1889 he was one of the four Assessors elected by the Senatus to the reconstituted University Court, and sat continuously in the Court for twenty-six years as Assessor or as Principal. His power of clear and cogent statement, intense loyalty to the University, cautious wisdom and experience, gained the confidence and respect of the members of the Court, and it is not too much to say that at the end he was regarded by his colleagues with the warmest affection as well as with the reverence due to his age and office.

In 1873 Turner was sent to the General Medical Council as representative of the Universities of Edinburgh and Aberdeen, and held the joint seat until 1883, when he was replaced by Sir John Struthers and was out of the Council for three years. He was then returned as representative of the University of Edinburgh under the Medical Act of 1886, and held office until his resignation in 1905. On the death of Sir Richard Quain in 1898 Turner became President, and retained the position until he resigned it in November 1904. By the special request of Sir Donald MacAlister, his successor in the chair, he remained as an ordinary Member of Council for another year. Many important decisions were taken on his initiative, and many of the reports were drafted by his hand. Undoubtedly he greatly influenced his colleagues by his force of character and clear contributions to the discussions, although he approached educational questions from the Scottish point of view, in which he believed.

When he became President the finances of the Council were unsatisfactory, and criticism of the Council and its officers was severe, and probably some of it was deserved. Reforms had to be carried through in the face of opposition, and Sir Donald MacAlister gives the credit for success to Sir William's manifest fairness and firmness, his grasp of executive detail, and his power of evoking loyal support, and says that his tact, good feeling, and surety of judgment never failed.

In 1893 he became a Fellow of the Royal College of Surgeons of England. He served as an influential member of the Royal Commission, 1881-82, on the Medical Acts. He was President of the Anthropological Section of the British Association, and later of the British Association itself in 1900.

Sir William Turner was associated throughout with the movement which led to the erection of the New Medical School of the University and of the M'Ewan Hall. The first meeting took place in 1869, and the first subscription list was opened 6th April 1874. The amount required to meet the first estimates was obtained or within sight, when it was found that owing to a rapid increase in the number of students the original estimates must be enlarged. Principal Sir Alexander Grant, Bart., the dominating spirit of the movement, felt the difficulty of applying to an exhausted public, but held a meeting in 1883, when it was resolved to launch a Tercentenary appeal. The Tercentenary Festival was celebrated in April 1884, and then Professor Turner and Mr M'Ewan, M.P., took the chief burden on themselves and tramped city and country soliciting subscriptions. Principal Sir Alexander Grant, Bart., died in December 1884, and Turner took his place as Chairman of the Acting Committee. The New Buildings were handed over by the Acting Committee to the Senatus in October 1886.

During their efforts Mr M'Ewan and Sir William Turner felt the hopelessness of raising money for a University Hall, and Mr M'Ewan resolved to present one. The first step consisted in the appointment of Mr M'Ewan, Principal Sir William Muir, Professor Sir William Turner, and Mr John Christison, W.S., as trustees, who obtained the Edinburgh University Extension Act, 1886, and began the erection of the Hall in January 1889. Mr M'Ewan, on behalf of the trustees, handed it over to the Chancellor, the Right Hon. A. J. Balfour, on behalf of the University, 3rd December 1897. The chief burden and responsibility as a trustee naturally fell upon Turner.

The late Mr John Barton, Convener of Trades, whose firm was contractor for the plumbing work of the New Buildings, informed the writer that the contractors greatly preferred to work for Turner. He knew his own mind, there was no dubiety about his instructions, and he could understand and allow for things that went wrong.

Mr Carnegie's trust-deed for the Universities of Scotland is dated 7th June 1901, and Sir William Turner (who had been knighted in 1886 and made K.C.B. in 1901) used his influence so that the new income might be applied to the best advantage.

As he retired from the Anatomical Chair in 1903, after being for some fifty years the outstanding figure in British anatomy, in order to become Principal and Vice-Chancellor of the University of Edinburgh, his influence and leisure for administrative work were increased. He became Joint-Chairman of the University Library Committee, and filled other minor posts. It was in his Principalship that the new Engineering Department was completed at High School Yards in 1905-6, and that part of the Surgical Hospital of the Old Infirmary was transformed into a new Department for Natural Philosophy in 1907. It was sometimes said that his policy was not sufficiently progressive, but it was only limited by the apparent funds available, and Turner's cautious estimate has proved of value in war time. During his Principalship notable improvements were made on the medical and scientific side, new groups for graduation arranged in the Arts curricula, the three-term session introduced, the tutorial system established, and the number of lectureships increased.

Sir William Turner acted along with the late Mr Middleton Rettie, Emeritus-Professor Chiene, C.B., and Emeritus-Professor M'Kendrick as a trustee on the Mary Dick Trust, and this doubtless quickened a desire to affiliate the Royal Dick Veterinary College to the University. He succeeded in carrying out a working arrangement, and had the satisfaction of seeing the institution of the degrees of Bachelor and Doctor of Science in this important branch of applied science.

Undoubtedly Principal Sir William Turner was fond of his own way, and usually got it up to the last; but his wisdom was such that the University must always look back upon his term of office as one of the notable periods of its history and regard his whole long and loyal service with gratitude.

Many honours besides those already mentioned came to Sir William Turner. He received the freedom of the city of Edinburgh in December 1909; the Universities of Dublin and of Cambridge conferred the degree of D.Sc.; the Universities of Glasgow, St Andrews, Aberdeen, and Montreal all enrolled him as LL.D.; Oxford, Toronto, and Durham called him to be D.C.L. He was an Honorary Member of the Royal Irish Academy and of the Royal Medical Society of Edinburgh; Honorary Fellow of the Obstetrical Society of Edinburgh; Foreign Member of the Anthropological Societies of Paris, Rome, and Brussels; Corresponding Member of the Anthropological Society and Royal Prussian Academy, Berlin; Knight of the Royal Prussian Order Pour le Mérite, 1912.

Early in 1895 his portrait was painted by the late Sir George Reid, President of the Royal Scottish Academy, for presentation by subscribers, and is now a family heirloom. Another portrait of Sir William Turner,

which hangs in the University Senate Room, was painted by Sir James Guthrie, President of the Royal Scottish Academy, and was presented by Sir R. Finlay on behalf of the subscribers and accepted by The Right Hon. A. J. Balfour on behalf of the University 13th February 1913.

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## LAWS OF THE SOCIETY.

*Adopted July 3, 1916 ; amended December 18, 1916.*

## I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed fifty-six, of whom not more than twenty may be British subjects, and not more than thirty-six subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

## ELECTION OF FELLOWS.

## II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

## III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

## IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words :

*In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.*

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of two guineas, and a subscription of two guineas for the year of election. He shall continue to pay a subscription of two guineas at the beginning of each session so long as he remains a Fellow. A Fellow may compound for these contributions by a single payment of forty guineas, or on such other terms as the Council may from time to time fix.\*

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows without regard to the limitation of numbers specified in Law I.

\* Law VI does not apply to Fellows elected before 1917, whose terms of Fellowship are determined by the previously existing Laws.

## IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the first Ordinary Meeting in June. The names shall be printed in the circular for the second Ordinary Meeting in June. The election shall be by Ballot, and shall take place at the first Ordinary Meeting in July after the manner prescribed in Laws III and IV for the election of Fellows.

## EXPULSION OF FELLOWS.

## X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

## XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

## MEETINGS OF THE SOCIETY.

## XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

## XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held on the first and third Mondays of each month, from November to March and from May to July, inclusive,

with the exception that in January the Meetings shall be held on the second and fourth Mondays.

The Council shall have power to alter the date of any Ordinary Meeting, if it appears to them conducive to the interests of the Society.

#### XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned; no other business shall be transacted.

### PUBLICATION OF PAPERS.

#### XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

### DISTRIBUTION OF PUBLICATIONS.

#### XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

### CONSTITUTION OF COUNCIL.

#### XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),\* a Curator of the Library and Museum, and twelve ordinary members of Council.

\* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.



## ELECTION OF COUNCIL.

## XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

## XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years.

## XX.

In the event of a vacancy arising in the Council or in any of the offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

## POWERS OF THE COUNCIL.

## XXI.

The Council shall have the following powers :—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

## DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

## XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

## ASSISTANT-SECRETARY AND LIBRARIAN.

## XXVII.

The Council shall appoint an Assistant-Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

## ALTERATION OF LAWS.

## XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

## THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner :—

### I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1917–1918 for the “best communication on a scientific subject, communicated,\* in the first instance, to the Royal Society of Edinburgh during the Sessions 1915–1916 and 1916–1917.” Preference will be given to a paper containing a discovery.

### II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1916–1917, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1916.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society.

\* For the purposes of this award the word “communicated” shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented \* to the Society during the Sessions 1914-15, 1915-16, whether they may have been given in with a view to the prize or not.

### III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1917-1918.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented \* to the Society during the two years preceding the fourth Monday in October 1917,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

### IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

\* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

# RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(See *Minutes of Meeting of January 18, 1915.*)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows :—

1. Papers or essays to be considered shall be arranged in two groups, A and B,  
—Group A to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology and Physics; Group B to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two Prizes shall be awarded to each group in alternate biennial periods, provided papers worthy of recommendation have been communicated to the Society.
3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.
4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider papers coming within their group and to report accordingly.
5. Papers to be considered by the Committees shall fall within the period dating from the last award in groups A and B respectively.

II. With regard to the Neill Prize, the term “Naturalist” shall be understood to include any student in the Sciences composing group B, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

## AWARDS OF THE KEITH, MAKDOUGALL - BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

### I. KEITH PRIZE.

- 1ST BIENNIAL PERIOD, 1827-29.—Dr BREWSTER, for his papers “on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals,” published in the Transactions of the Society.
- 2ND BIENNIAL PERIOD, 1829-31.—Dr BREWSTER, for his paper “on a New Analysis of Solar Light,” published in the Transactions of the Society.
- 3RD BIENNIAL PERIOD, 1831-33.—THOMAS GRAHAM, Esq., for his paper “on the Law of the Diffusion of Gases,” published in the Transactions of the Society.
- 4TH BIENNIAL PERIOD, 1833-35.—Professor J. D. FORBES, for his paper “on the Refraction and Polarization of Heat,” published in the Transactions of the Society.
- 5TH BIENNIAL PERIOD, 1835-37.—JOHN SCOTT RUSSELL, Esq., for his researches “on Hydrodynamics,” published in the Transactions of the Society.
- 6TH BIENNIAL PERIOD, 1837-39.—Mr JOHN SHAW, for his experiments “on the Development and Growth of the Salmon,” published in the Transactions of the Society.
- 7TH BIENNIAL PERIOD, 1839-41.—Not awarded.
- 8TH BIENNIAL PERIOD, 1841-43.—Professor JAMES DAVID FORBES, for his papers “on Glaciers,” published in the Proceedings of the Society.
- 9TH BIENNIAL PERIOD, 1843-45.—Not awarded.
- 10TH BIENNIAL PERIOD, 1845-47.—General Sir THOMAS BRISBANE, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.
- 11TH BIENNIAL PERIOD, 1847-49.—Not awarded.
- 12TH BIENNIAL PERIOD, 1849-51.—Professor KELLAND, for his papers “on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations,” published in the Transactions of the Society.
- 13TH BIENNIAL PERIOD, 1851-53.—W. J. MACQUORN RANKINE, Esq., for his series of papers “on the Mechanical Action of Heat,” published in the Transactions of the Society.
- 14TH BIENNIAL PERIOD, 1853-55.—Dr THOMAS ANDERSON, for his papers “on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances,” published in the Transactions of the Society.
- 15TH BIENNIAL PERIOD, 1855-57.—Professor BOOLE, for his Memoir “on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments,” published in the Transactions of the Society.
- 16TH BIENNIAL PERIOD, 1857-59.—Not awarded.
- 17TH BIENNIAL PERIOD, 1859-61.—JOHN ALLAN BROWN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers “on the Horizontal Force of the Earth’s Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally,” published in the Transactions of the Society.
- 18TH BIENNIAL PERIOD, 1861-63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication “on some Kinematical and Dynamical Theorems.”
- 19TH BIENNIAL PERIOD, 1863-65.—Principal FORBES, St Andrews, for his “Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars,” published in the Transactions of the Society.
- 20TH BIENNIAL PERIOD, 1865-67.—Professor C. PIAZZI SMYTH, for his paper “on Recent Measures at the Great Pyramid,” published in the Transactions of the Society.
- 21ST BIENNIAL PERIOD, 1867-69.—Professor P. G. TAIT, for his paper “on the Rotation of a Rigid Body about a Fixed Point,” published in the Transactions of the Society.
- 22ND BIENNIAL PERIOD, 1869-71.—Professor CLERK MAXWELL, for his paper “on Figures, Frames, and Diagrams of Forces,” published in the Transactions of the Society.

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- 23RD BIENNIAL PERIOD, 1871-73.—Professor P. G. TAIT, for his paper entitled “First Approximation to a Thermo-electric Diagram,” published in the Transactions of the Society.
- 24TH BIENNIAL PERIOD, 1873-1875.—Professor CRUM BROWN, for his Researches “on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear.”
- 25TH BIENNIAL PERIOD, 1875-77.—Professor M. FORSTER HEDDLE, for his papers “on the Rhombohedral Carbonates,” and “on the Felspars of Scotland,” published in the Transactions of the Society.
- 26TH BIENNIAL PERIOD, 1877-79.—Professor H. C. FLEEMING JENKIN, for his paper “on the Application of Graphic Methods to the Determination of the Efficiency of Machinery,” published in the Transactions of the Society; Part II having appeared in the volume for 1877-78.
- 27TH BIENNIAL PERIOD, 1879-81.—Professor GEORGE CHRYSAL, for his paper “on the Differential Telephone,” published in the Transactions of the Society.
- 28TH BIENNIAL PERIOD, 1881-83.—THOMAS MUIR, Esq., LL.D., for his “Researches into the Theory of Determinants and Continued Fractions,” published in the Proceedings of the Society.
- 29TH BIENNIAL PERIOD, 1883-85.—JOHN AITKEN, Esq., for his paper “on the Formation of Small Clear Spaces in Dusty Air,” and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.
- 30TH BIENNIAL PERIOD, 1885-87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., “On Ice and Brines,” and “On the Distribution of Temperature in the Antarctic Ocean,” have been published in the Proceedings of the Society.
- 31ST BIENNIAL PERIOD, 1887-89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.
- 32ND BIENNIAL PERIOD, 1889-91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.
- 33RD BIENNIAL PERIOD, 1891-93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions.
- 34TH BIENNIAL PERIOD, 1893-95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.
- 35TH BIENNIAL PERIOD, 1895-97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.
- 36TH BIENNIAL PERIOD, 1897-99.—Dr JAMES BURGESS, for his paper “on the Definite Integral  $\frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt$ , with extended Tables of Values,” printed in vol. xxxix of the Transactions of the Society.
- 37TH BIENNIAL PERIOD, 1899-1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.
- 38TH BIENNIAL PERIOD, 1901-03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., etc., for his memoirs entitled “A Contribution to the Craniology of the People of Scotland,” published in the Transactions of the Society, and for his “Contributions to the Craniology of the People of the Empire of India,” Parts I, II, likewise published in the Transactions of the Society.
- 39TH BIENNIAL PERIOD, 1903-05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on “The Histology of the Blood of the Larva of *Lepidosiren paradoxa*,” published in the Transactions of the Society within the period.
- 40TH BIENNIAL PERIOD, 1905-07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled “Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord,” published in the Transactions of the Society within the period.
- 41ST BIENNIAL PERIOD, 1907-09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, “On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland.”
- 42ND BIENNIAL PERIOD, 1909-11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon “Sulphur” and upon “Vapour Pressure,” appearing in the Proceedings of the Society.



- 43RD BIENNIAL PERIOD, 1911-1913.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.
- 44TH BIENNIAL PERIOD, 1913-15.—JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of *Lingula* and *Pelagodiscus*," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

## II. MAKDOUGALL-BRISBANE PRIZE.

- 1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.
- 2ND BIENNIAL PERIOD, 1860-62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.
- 3RD BIENNIAL PERIOD, 1862-64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.
- 4TH BIENNIAL PERIOD, 1864-66.—Not awarded.
- 5TH BIENNIAL PERIOD, 1866-68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.
- 6TH BIENNIAL PERIOD, 1868-70.—Not awarded.
- 7TH BIENNIAL PERIOD, 1870-72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblasic or Tubularian Hydroids—since published.
- 8TH BIENNIAL PERIOD, 1872-74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.
- 9TH BIENNIAL PERIOD, 1874-76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.
- 10TH BIENNIAL PERIOD, 1876-78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.
- 11TH BIENNIAL PERIOD, 1878-80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877-78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.
- 12TH BIENNIAL PERIOD, 1880-82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.
- 13TH BIENNIAL PERIOD, 1882-84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.
- 14TH BIENNIAL PERIOD, 1884-86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."
- 15TH BIENNIAL PERIOD, 1886-88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.
- 16TH BIENNIAL PERIOD, 1889-90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part I, of the Society's Transactions.
- 17TH BIENNIAL PERIOD, 1890-92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.
- 18TH BIENNIAL PERIOD, 1892-94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255-263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.
- 19TH BIENNIAL PERIOD, 1894-96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

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- 20TH BIENNIAL PERIOD, 1896-98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.
- 21ST BIENNIAL PERIOD, 1898-1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.
- 22ND BIENNIAL PERIOD, 1900-02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.
- 23RD BIENNIAL PERIOD, 1902-04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.
- 24TH BIENNIAL PERIOD, 1904-06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.
- 25TH BIENNIAL PERIOD, 1906-08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.
- 26TH BIENNIAL PERIOD, 1908-10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."
- 27TH BIENNIAL PERIOD, 1910-12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.
- 28TH BIENNIAL PERIOD, 1912-14.—Professor C. R. MARSHALL, M.D., M.A., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

### III. THE NEILL PRIZE.

- 1ST TRIENNIAL PERIOD, 1856-59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermatogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.
- 2ND TRIENNIAL PERIOD, 1859-61.—ROBERT KAYE GREVILLE, LL.D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.
- 3RD TRIENNIAL PERIOD, 1862-65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.
- 4TH TRIENNIAL PERIOD, 1865-68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemertean, and on some New British Annelids," published in the Transactions of the Society.
- 5TH TRIENNIAL PERIOD, 1868-71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Foetal Membranes in the Cetacea," published in the Transactions of the Society.
- 6TH TRIENNIAL PERIOD, 1871-74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.
- 7TH TRIENNIAL PERIOD, 1874-77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.
- 8TH TRIENNIAL PERIOD, 1877-80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.
- 9TH TRIENNIAL PERIOD, 1880-83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.
- 10TH TRIENNIAL PERIOD, 1883-86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

- 11TH TRIENNIAL PERIOD, 1886-89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.
- 12TH TRIENNIAL PERIOD, 1889-92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.
- 13TH TRIENNIAL PERIOD, 1892-95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.
- 14TH TRIENNIAL PERIOD, 1895-98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.
- 15TH TRIENNIAL PERIOD, 1898-1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix of the Transactions of the Society.
- 16TH TRIENNIAL PERIOD, 1901-04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.
- 17TH TRIENNIAL PERIOD, 1904-07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.
- 1ST BIENNIAL PERIOD, 1907-09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."
- 2ND BIENNIAL PERIOD, 1909-11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).
- 3RD BIENNIAL PERIOD, 1911-13.—Dr W. S. BRUCE, in recognition of the scientific results of his Arctic and Antarctic explorations.
- 4TH BIENNIAL PERIOD, 1913-15.—ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

#### IV. GUNNING VICTORIA JUBILEE PRIZE.

- 1ST TRIENNIAL PERIOD, 1884-87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.
- 2ND TRIENNIAL PERIOD, 1887-90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.
- 3RD TRIENNIAL PERIOD, 1890-93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.
- 4TH TRIENNIAL PERIOD, 1893-96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.
- 1ST QUADRENNIAL PERIOD, 1896-1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.
- 2ND QUADRENNIAL PERIOD, 1900-04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.
- 3RD QUADRENNIAL PERIOD, 1904-08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."
- 4TH QUADRENNIAL PERIOD, 1908-12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

## PROCEEDINGS OF THE STATUTORY GENERAL MEETING

### Beginning the 133rd Session, 1915-1916.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 25, 1915, at 4.30 p.m.

Professor HUDSON BEARE, Vice-President, in the Chair,

the Minutes of the last Statutory Meeting, October 26, 1914, were read, approved, and signed.

The CHAIRMAN nominated as Scrutineers of the Voting Papers, Professor JAMES MACKINNON and Dr J. G. GRAY.

The Ballot for the Election of Office-Bearers and Members of Council was then taken.

The GENERAL SECRETARY announced that Mr G. A. STEWART, Librarian, and Mr W. J. BEATON, Assistant Librarian, were still on service, and that the Council had appointed Miss M. LE HARIVEL as Temporary Assistant Librarian.

In submitting his accounts for the year the TREASURER drew attention to the exceptional conditions under which the Council and the Society had carried on their work and to the manner in which these had influenced the Financial Report. Four hundred pounds of the Society's funds had been invested in the  $4\frac{1}{2}$  per cent. War Loan, distributed as follows:—

Neill Fund . . . . .	£15
Makdougall-Brisbane Fund . . . . .	150
Makerstoun Magnetic Meteorological Observation Fund . . . . .	220
Gunning Victoria Jubilee Prize Fund . . . . .	15

Professor CRICHTON MITCHELL moved the adoption of the Treasurer's Report, and also votes of thanks to the Treasurer and to the auditors, who were reappointed.

Dr KNOTT moved and Dr HORNE seconded the following changes of Rule, which had been announced at the Ordinary Meeting of July 5 and the Special Meeting of June 28:—

#### RULE IX.

Each candidate for admission as an Ordinary Fellow shall be proposed and recommended by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary before the 24th of December of each Session, and, subject to the approval of the Council, shall be exhibited publicly in the Society's Rooms during the succeeding month of January. All recommendations so exhibited shall be considered by the Council at its first meeting in the month of February, and a list of those approved by the Council for election shall be issued to the Fellows not later than the 14th of February.

#### RULE X.

The Recommendation of a Candidate for admission as an Ordinary Fellow shall be in the following terms:—

A.B., a gentleman well versed in Science (or Polite Literature, as the case may be), being to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby recommend him as deserving of that honour, and likely to prove a useful and valuable Fellow.

#### RULE XI.

The Election shall take place at the first Ordinary Meeting of the Society in March of each Session, and only those candidates approved by the Council shall be eligible.

Those candidates, and only those, whose election is supported by a majority of two-thirds of those voting shall be deemed elected.

These alterations will necessitate the rearranging and renumbering of the succeeding Rules. These were agreed to unanimously.

The Scrutineers reported that the following Council had been elected :—

JOHN HORNE, LL.D., F.R.S., F.G.S., President.	
Professor F. O. BOWER, M.A., D.Sc., F.R.S.,	
Professor Sir THOMAS R. FRASER, M.D., LL.D., F.R.C.P.E.,	
F.R.S.,	
BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S.,	} Vice-Presidents.
Professor Sir E. A. SCHÄFER, M.R.C.S., LL.D., F.R.S.,	
The Right Honourable Sir J. H. A. MACDONALD, K.C.B.,	
LL.D., D.L., F.R.S., M.I.E.E.,	
Professor R. A. SAMPSON, M.A., D.Sc., F.R.S.,	
CARGILL G. KNOTT, D.Sc., General Secretary.	
ROBERT KIDSTON, LL.D., F.R.S., F.G.S.,	} Secretaries to Ordinary Meetings.
Professor ARTHUR ROBINSON, M.D., M.R.C.S.,	
JAMES CURRIE, M.A., Treasurer.	
JOHN S. BLACK, M.A., LL.D., Curator of Library and Museum.	

#### ORDINARY MEMBERS OF COUNCIL.

Principal A. P. LAURIE, M.A., D.Sc.	R. STEWART MACDOUGALL, M.A., D.Sc.
Professor J. GRAHAM KERR, M.A., F.R.S.	W. A. TAIT, D.Sc., M.Inst.C.E.
LEONARD DOBBIN, Ph.D.	J. H. ASHWORTH, D.Sc.
ERNEST MACLAGAN WEDDERBURN, M.A.,	Professor C. G. BARKLA, D.Sc., F.R.S.
LL.B., D.Sc.	Professor C. R. MARSHALL, M.A., M.D.
W. B. BLAIKIE, LL.D.	Principal A. CRICHTON MITCHELL, D.Sc., Hon.
Principal O. C. BRADLEY, M.D., D.Sc.	D.Sc. (Geneva).

Society's Representative on } WILLIAM ALLAN CARTER, M.Inst. C.E.  
George Heriot's Trust }

Principal Sir WILLIAM TURNER, K.C.B., D.C.L., F.R.S., former  
President, is a permanent Member of Council.

On the motion of Dr J. S. BLACK, thanks were voted to the Scrutineers.

PROCEEDINGS OF THE ORDINARY MEETINGS,  
Session 1915-1916.

FIRST ORDINARY MEETING.

*Monday, November 1, 1915.*

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The President opened the Session with an Address on the Influence of James Geikie's Researches on the Development of Glacial Geology.

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SECOND ORDINARY MEETING.

*Monday, November 15, 1915.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read :—

1. Preliminary Notice of a Family showing inherited Abnormal Segmentation of the Digits of both Hands. By Dr H. DRINKWATER. (*With Diagram and Lantern Illustrations.*)
  2. The Moulting of the King Penguin. By Professor COSSAR EWART, F.R.S., and Miss DOROTHY MACKENZIE. (*With Lantern Illustrations.*)
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THIRD ORDINARY MEETING.

*Monday, December 6, 1915.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read :—

1. The *Geometria Organica* of Colin MacLaurin. By CHARLES TWEEDIE, M.A., B.Sc. In the absence of the Author, the Paper was read in abstract by Professor G. A. GIBSON.
2. The Anatomy of the Stem of the *Papaveraceæ*. By Professor R. J. HARVEY GIBSON and Miss M. BRADLEY, M.Sc.
3. On a small Collection of Terrestrial Isopoda from Spain, with Descriptions of Four New Species. By W. E. COLLINGE. Communicated by Professor M'INTOSH.

The following Candidate for Fellowship was balloted for, and declared duly elected :—CHARLES STEWART HUNTER, L.R.C.P.E., L.R.C.S.E., D.P.H., Medical Officer of Health, Carnoustie.

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FOURTH ORDINARY MEETING.

*Monday, December 20, 1915.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read :—

1. The Torsional Vibration of Beams of Commercial Section. By E. G. RITCHIE, B.Sc. Communicated by Professor A. H. GIBSON.
2. The Origin of Oil Shale. By E. H. CUNNINGHAM CRAIG, B.A., F.G.S. Communicated by Dr HORNE, F.R.S.
3. The Theory of Circulants from 1880 to 1900. By THOMAS MUIR, C.M.G., LL.D., F.R.S.

## FIFTH ORDINARY MEETING.

*Monday, January 10, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read :—

1. The Optical Rotation and Cryoscopic Behaviour of Sugars dissolved in (a) Formamide, (b) Water. By J. E. MACKENZIE, Ph.D., D.Sc., and SUDHAMOY GHOSH, D.Sc. Communicated by Professor J. WALKER, F.R.S.

2. Note on the Sublimation of Sugars. By SUDHAMOY GHOSH, D.Sc. Communicated by Professor J. WALKER, F.R.S.

3. A Revision of British Idoteidæ, a Family of Marine Isopoda. By WALTER E. COLLINGE, M.Sc., F.L.S. Communicated by Professor M'INTOSH, F.R.S.

## SIXTH ORDINARY MEETING.

*Monday, January 24, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read :—

1. On the Size of Particles in Deep Sea Deposits. By Dr SVEN ODÉN, Upsala. Communicated by the GENERAL SECRETARY.

2. The Dynamics of Cyclones and Anticyclones. Part III. By Dr JOHN AITKEN, F.R.S.

## SEVENTH ORDINARY MEETING.

*Monday, February 7, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read :—

1. The Anatomy and Affinity of *Platyzoma microphyllum*. By JOHN M. THOMPSON, B.Sc. Communicated by Professor BOWER, F.R.S. (*With Lantern Illustrations.*)

2. On the Leaf Trace in some Pinnate Leaves. By R. C. DAVIE, M.A., D.Sc. Communicated by Dr KIDSTON, F.R.S. (*With Lantern Illustrations.*)

## EIGHTH ORDINARY MEETING.

*Monday, February 21, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

By request of the Council, Professor C. G. BARKLA, F.R.S., gave an Address on Recent Work in X-Rays, and its bearing on the Theories of Atomic Structure and of Electro-Magnetic Radiation. (*With Experimental and Lantern Illustrations.*)

## NINTH ORDINARY MEETING.

*Monday, March 6, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The Annual Election of Fellows took place. The following were elected:— ROBERT JOHN TAINSH BELL, M.A., D.Sc., Lecturer in Mathematics in the University of Glasgow, 146 Hyndland Road, Glasgow; FRANCIS ERNEST BRADLEY, M.A., M.Com., LL.D., Barrister-at-Law, Examiner to the Council of Legal Education, Bank of England Chambers, Tib Lane, Manchester; HENRY BRIGGS, M.Sc., A.R.S.M., Lecturer in Mining, Heriot-Watt College, Allermuir, Liberton, Midlothian; C. T. CLOUGH, M.A. (Cambridge), LL.D., District Geologist, H.M. Geological

Survey, Scotland, St Ann's Mount, Polton, Midlothian; JAMES EDWARD CROMBIE, M.A., LL.D., Millowner, Parkhill House, Dyce, Aberdeenshire; E. H. CUNNINGHAM CRAIG, B.A. (Cambridge), Geologist and Mining Engineer, The Dutch House, Beaconsfield; A. W. GIBB, D.Sc., Lecturer in Geology, The University, Aberdeen, 1 Belvidere Street, Aberdeen; The Hon. Lord GUTHRIE, LL.D., Judge of the Court of Session, 13 Royal Circus, Edinburgh; PERCY THEODORE HERRING, M.D., F.R.C.P.Ed., Professor of Physiology, University of St Andrews, Hepburn Gardens, St Andrews; Col. Sir DUNCAN A. JOHNSTON, K.C.M.G., C.B., Colonel, Royal Engineers, 8 Lansdowne Crescent, Edinburgh; HYMAN LEVY, M.A., B.Sc., Research Assistant, Aeronautical Section, National Physical Laboratory, Teddington; JOHN E. MACKENZIE, Ph.D., D.Sc., Lecturer in Chemistry, University of Edinburgh, Major-Adjutant, O.T.C., 2A Ramsay Garden, Edinburgh; W. F. P. M'LINTOCK, D.Sc. (Edinburgh), Royal Scottish Museum, Edinburgh; ROBERT MUIR, M.A., M.D., Sc.D., F.R.S., Professor of Pathology, University of Glasgow, 16 Victoria Crescent, Dowanhill, Glasgow; JAMES RITCHIE, M.A., D.Sc., Royal Scottish Museum, 20 Upper Gray Street, Edinburgh; DAVID RONALD, Civil Engineer, Engineering Inspector, Local Government Board, Burnfield, Falkirk; The Hon. Lord E. T. SALVESEN, Judge of the Court of Session, Dean Park House, Edinburgh; D. R. STEUART, F.I.C., Chemist to the Broxburn Oil Company, Osborne Cottage, Broxburn; J. MARTIN WHITE, Esq., of Balruddery, Balruddery, near Dundee.

The following Communication was read:—

On Leaf Architecture. By Professor F. O. BOWER, F.R.S. (*With Lantern Illustrations.*)

#### TENTH ORDINARY MEETING.

*Monday, March 20, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read:—

1. The Ochil Earthquakes of the Years 1900-1914. By CHARLES DAVISON, Sc.D., F.G.S. Communicated by the GENERAL SECRETARY. (*With Lantern Illustrations.*)
2. Mathematical Note on the Fall of Small Particles through Liquids. By Dr C. G. KNOTT.
3. *Apractoleidus Teretipes*—a new Oxfordian Plesiosaur. By W. R. SMELLIE, M.A., B.Sc. Communicated by Professor J. W. GREGORY, F.R.S.

Dr C. T. CLOUGH, the Hon. Lord GUTHRIE, Professor PERCY T. HERRING, and the Hon. Lord SALVESEN signed the Roll, and were duly admitted Fellows of the Society.

#### ELEVENTH ORDINARY MEETING.

*Monday, May 1, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The following Communications were read:—

1. Obituary Notice of Sir WILLIAM TURNER. By Sir JAMES A. RUSSELL, F.R.C.P.E., LL.D.
2. Clinical Methods of Estimation of Sugar in the Blood. By Dr HARRY RAINY and Miss HAWICK.
3. The Insect Association of a Local Environmental Complex in the District of Holmes Chapel, Cheshire. By Dr ALFRED E. CAMERON. Communicated by Dr STEWART MACDOUGALL. (*With Lantern Illustrations.*)

Dr JAMES RITCHIE signed the Roll, and was duly admitted a Fellow of the Society.

#### TWELFTH ORDINARY MEETING.

*Monday, May 15, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The President read the List of Nominations for Honorary Fellowship.

The following Communications were read:—

1. On the Jurassic Fossil Fungus, *Phycomycites Frodinghamii*. By Dr DAVID ELLIS. (*With Lantern Illustrations.*)
2. On a Possible Explanation of the Satellites of Spectral Lines. By Dr R. A. HOUSTOUN.



## THIRTEENTH ORDINARY MEETING.

*Monday, June 5, 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The Keith Prize and the Neill Prize awards for the Biennial Period 1913-1915.

The Council of the Royal Society of Edinburgh having awarded the Keith Prize to JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus" and on "Sclerocheilus," published in the *Transactions* of the Society, and for other papers on the Morphology and Histology of Polychæta; and the Neill Prize to ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven" and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the *Transactions* of the Society; these Prizes will be presented at the Meeting of July 3, 1916.

The following Communications were read :—

1. The Prothallus of *Tmesipteris Tannensis*. By Professor A. ANSTRUTHER LAWSON, D.Sc.
2. On the Theory of Continued Fractions. By Professor E. T. WHITTAKER, F.R.S.
3. On Captain Weir's Azimuth Diagram and its Anticipation of the Nomogram in Spherical Trigonometry. By HERBERT BELL, M.A., B.Sc. Communicated by the GENERAL SECRETARY.

## LOSS BY FIRE.

By the destructive fire at Messrs Neill & Co.'s Printing Works, the Society lost the finished sheets of many papers which were on the point of being published in the *Transactions* and *Proceedings*. This delayed publication for several months.

## PROPOSED HONORARY FELLOWS.

*As British Honorary Fellows :—*

Sir FRANCIS DARWIN, D.Sc., M.B., F.R.S., Cambridge; J. W. L. GLAISHER, M.A., Sc.D., F.R.S., Trinity College, Cambridge; J. N. LANGLEY, Sc.D., F.R.S., Professor of Physiology, Cambridge; CHARLES LAPWORTH, M.Sc., F.R.S., Emeritus Professor of Geology, University of Birmingham; ALEXANDER MACALISTER, M.D., F.R.S., Professor of Anatomy, Cambridge; ARTHUR SCHUSTER, Emeritus Professor of Physics, University of Manchester.

*As Foreign Honorary Fellows :—*

CHARLES BARROIS, Professor of Geology and Mineralogy, Lille; D. H. CAMPBELL, Professor of Botany, Leland Stanford University, Cal., U.S.A.; M. E. GLEY, Professor of Physiology, Paris; C. GOLGI, Professor of Anatomy, Rome; General W. C. GORGAS, U.S. Army Medical Department; G. B. GRASSI, Professor of Entomology, Rome; E. C. PICKERING, Professor of Astronomy, Cambridge, U.S.A.; E. WARMING, Professor of Botany, University, and Director of the Botanical Gardens, Copenhagen.

## FOURTEENTH ORDINARY MEETING.

*Monday, June 19, 1916.*

Professor Sir Thomas R. Fraser, F.R.S., Vice-President, in the Chair.

The Keith Prize and Neill Prize awards for the Biennial Period 1913-1915.

The Council of the Royal Society of Edinburgh having awarded the Keith Prize to JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus" and on "Sclerocheilus," published in the *Transactions* of the Society, and for other papers on the Morphology and Histology of Polychæta; and the Neill Prize to ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven" and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the *Transactions* of the Society; these Prizes will be presented at the Meeting of July 3, 1916.

The following Communications were read :—

1. The Pharmacological Action of Nitric Esters. By Professor C. R. MARSHALL.
2. The Trachytic and other Allied Rocks of the Clyde Carboniferous Lava Plateaux. By G. W. TYRRELL, A.R.C.S., F.G.S. Communicated by the PRESIDENT.

## THE LAWS OF THE SOCIETY.

At the Ordinary Meeting of June 5, 1916, the President gave notice that the Draft of the Laws of the Society, recently revised by the Council, would be proposed and voted upon at the Ordinary Meeting of July 3, 1916.

Proof copies of these Laws may be seen by Fellows in the Reading Room of the Society.

## PROPOSED HONORARY FELLOWS.

*As British Honorary Fellows:—*

Sir FRANCIS DARWIN, D.Sc., M.B., F.R.S., Cambridge; J. W. L. GLAISHER, M.A., Sc.D., F.R.S., Trinity College, Cambridge; J. N. LANGLEY, Sc.D., F.R.S., Professor of Physiology, Cambridge; CHARLES LAPWORTH, M.Sc., F.R.S., Emeritus Professor of Geology, University of Birmingham; ALEXANDER MACALISTER, M.D., F.R.S., Professor of Anatomy, Cambridge; ARTHUR SCHUSTER, Ph.D., D.Sc., F.R.S., Emeritus Professor of Physics, University of Manchester.

*As Foreign Honorary Fellows:—*

CHARLES BARROIS, Professor of Geology and Mineralogy, Lille; D. H. CAMPBELL, Professor of Botany, Leland Stanford University, Cal., U.S.A.; M. E. GLEY, Professor of Physiology, Paris; C. GOLGI, Professor of Anatomy, Rome; General W. C. GORGAS, U.S. Army Medical Department; G. B. GRASSI, Professor of Comparative Anatomy, Rome; E. C. PICKERING, Professor of Astronomy, Cambridge, U.S.A.; E. WARMING, Emeritus Professor of Botany and Keeper of the Royal Botanic Gardens, Copenhagen.

## FIFTEENTH ORDINARY MEETING.

*Monday, 3rd July 1916.*

Dr John Horne, F.R.S., President, in the Chair.

The following Honorary Fellows were elected:—

*As British Honorary Fellows:—*

Sir FRANCIS DARWIN, D.Sc., M.B., F.R.S., Cambridge; J. W. L. GLAISHER, M.A., Sc.D., F.R.S., Trinity College, Cambridge; J. N. LANGLEY, Sc.D., F.R.S., Professor of Physiology, Cambridge; CHARLES LAPWORTH, M.Sc., F.R.S., Emeritus Professor of Geology, University of Birmingham; ALEXANDER MACALISTER, M.D., F.R.S., Professor of Anatomy, Cambridge; ARTHUR SCHUSTER, Ph.D., D.Sc., F.R.S., Emeritus Professor of Physics, University of Manchester.

*As Foreign Honorary Fellows:—*

CHARLES BARROIS, Professor of Geology and Mineralogy, Lille; D. H. CAMPBELL, Professor of Botany, Leland Stanford University, Cal., U.S.A.; M. E. GLEY, Professor of Physiology, Paris; C. GOLGI, Professor of Anatomy, Rome; General W. C. GORGAS, U.S. Army Medical Department; G. B. GRASSI, Professor of Comparative Anatomy, Rome; E. C. PICKERING, Professor of Astronomy, Cambridge, U.S.A.; E. WARMING, Emeritus Professor of Botany and Keeper of the Royal Botanic Gardens, Copenhagen.

*Change of Laws.*

As announced by the President at the Ordinary Meeting of June 5, 1916, the revised Draft of Laws was proposed, voted upon and adopted.

Professors J. W. GREGORY and JAMES WALKER were appointed the Society's Delegates to Conjoint Board of Scientific Societies.

## The Keith Prize and Neill Prize Awards for the Biennial Period 1913-1915.

The Keith Prize was presented to JAMES HANTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the *Transactions* of the Society, and for other papers on the Morphology and Histology of Polychæta.

The Neill Prize was presented to ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven" and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the *Transactions* of the Society.

The following Communications were read :—

1. On Old Red Sandstone Fossil Plants showing Structure from Rhynie Chert Bed, Aberdeenshire. By Dr R. KIDSTON, F.R.S., and Professor W. H. LANG, F.R.S.

2. Contributions to our Knowledge of British Palæozoic Plants. Part I. Fossil Plants from the Scottish Coal Measures. By Dr R. KIDSTON, F.R.S.

3. Exhibition of a Universal Sun Dial giving Greenwich or any Standard Mean Time ; and of a diagram giving Sunrise and Sunset in mean time for all Longitudes and Latitudes under any Daylight Saving Bill. By Dr W. B. BLAIR, F.R.S.

4. On the heating of Field Coils of Dynamo-Electric Machinery. By Professor MAGNUS MACLEAN.

5. Preliminary Communication on the Effects of Thyroid-feeding upon the Pancreas. By Dr M. KOJIMA, Staff-Surgeon in the Imperial Japanese Navy. Communicated by Professor Sir EDWARD A. SCHÄFER, F.R.S.

6. The Application of Operators to the Solution of the Algebraic Equation. By JAMES LITTLEJOHN, Esq. Communicated by the GENERAL SECRETARY.

7. On Systems of Partial Differential Equations and the Transformation of Spherical Harmonics. By H. BATEMAN, Ph.D. Communicated by Professor WHITTAKER.

Dr JOHN E. MACKENZIE and Dr W. F. P. M'LINTOCK signed the Roll, and were duly admitted Fellows of the Society.

## PROCEEDINGS OF THE STATUTORY GENERAL MEETING

### Ending the 133rd Session, 1915-1916.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 25, 1916, at 4.30 p.m.

Dr John Horne, F.R.S., President, in the Chair.

The Minutes of the last Statutory Meeting, October 25, 1915, were read, approved, and signed.

Dr CLARK TROTTER signed the Roll, and was duly admitted a Fellow of the Society.

The PRESIDENT nominated as Scrutineers of the Voting Paper, Lord SALVESEN and Professor MACKINNON.

The ballot for the election of Office-Bearers and Members of Council was then taken.

The TREASURER submitted his Report for the preceding Session, drawing special attention to the depreciation in value of the Society's investments.

On the motion of Sir E. A. SCHÄFER the Treasurer's Report was adopted.

The SECRETARY moved that Messrs LINDSAY, JAMIESON & HALDANE, C.A., be reappointed auditors for the ensuing session. This was agreed to.

The Scrutineers reported that the Balloting Papers had all been in order, and the following Council had been duly elected:—

JOHN HORNE, LL.D., F.R.S., F.G.S., President.	
BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S.,	
Professor Sir E. A. SCHÄFER, M.R.C.S., LL.D., F.R.S.,	
The Right Hon. Sir J. H. A. MACDONALD, G.C.B.,	
K.C., LL.D., D.L., F.R.S., M.I.E.E.,	}
Professor R. A. SAMPSON, M.A., D.Sc., F.R.S.,	
Professor D'ARCY THOMPSON, C.B., B.A., F.R.S.,	
Professor JAMES WALKER, D.Sc., Ph.D., LL.D., F.R.S.,	
CARGILL G. KNOTT, D.Sc., LL.D., General Secretary.	
Professor ARTHUR ROBINSON, M.D., M.R.C.S.,	}
Professor E. T. WHITTAKER, Sc.D., F.R.S.,	
JAMES CURRIE, M.A., Treasurer.	
A. CRICHTON MITCHELL, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.	

Vice-Presidents.

Secretaries to Ordinary Meetings.

#### ORDINARY MEMBERS OF COUNCIL.

W. B. BLAIKIE, LL.D.	JOHN S. BLACK, M.A., LL.D.
Principal O. C. BRADLEY, M.D., D.Sc.	Sir GEORGE A. BERRY, M.D., C.M., F.R.C.S.
R. STEWART MACDOUGALL, M.A., D.Sc.	JOHN S. FLETT, M.A., D.Sc., LL.D., F.R.S.
W. A. TAIT, D.Sc., M.Inst.C.E.	Professor MAGNUS MACLEAN, M.A., D.Sc.,
J. H. ASHWORTH, D.Sc.	M.Inst.E.E.
Professor C. G. BARKLA, D.Sc., F.R.S.	Professor DAVID WATERSTON, M.A., M.D.,
Professor C. R. MARSHALL, M.A., M.D.	F.R.C.S.E.

Society's Representative on } WILLIAM ALLAN CARTER, M.Inst.C.E.  
George Heriot's Trust, }

The PRESIDENT, in the name of the Society, thanked the Scrutineers for their Report.

The SECRETARY announced that Messrs STEWART & BEATON, the Librarians, were still on active service, and the work of the Library was being efficiently carried on by Miss LE HARIVEL.

**ABSTRACT**  
OF  
**THE ACCOUNTS OF JAMES CURRIE, ESQ.**

*As Treasurer of the Royal Society of Edinburgh.*

SESSION 1915-1916.

**I. ACCOUNT OF THE GENERAL FUND.**

CHARGE.

1. Arrears of Contributions at 1st October 1915 . . . . .	£117 12 0	
2. Contributions for present Session :—		
1. 149 Fellows at £2, 2s. each . . . . .	£312 18 0	
111 Fellows at £3, 3s. each . . . . .	349 13 0	
	<hr/>	
	£662 11 0	
Less—Subscriptions for present Session, included in 1915		
Accounts . . . . .	4 4 0	
	<hr/>	
	£658 7 0	
2. Fees of Admission and Contributions of eighteen new		
Resident Fellows at £5, 5s. each . . . . .	94 10 0	
3. Fees of Admission of two new Non-Resident Fellows at		
£26, 5s. each . . . . .	52 10 0	
	<hr/>	
		805 7 0
3. Contribution for 1916-1917 paid in advance . . . . .		3 3 0
4. Interest received—		
Interest, less Tax £65, 3s. 2 <sup>6</sup> / <sub>10</sub> d. . . . .	£327 3 3	
Annuity from Edinburgh and District Water Trust, less Tax,		
£11, 3s. 1d. . . . .	41 6 11	
	<hr/>	
		368 10 2
5. Transactions and Proceedings sold . . . . .		21 17 5
6. Annual Grant from Government . . . . .		600 0 0
7. Income Tax repaid for year to 5th April 1916 . . . . .		61 3 7
		<hr/>
		£1977 13 2

**Amount of the Charge . . . . .**

DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING :—		
Inhabited House Duty . . . . .	£0 6 3	
Insurance . . . . .	19 3 3	
Coal, etc., to 26th September 1916 . . . . .	37 11 6	
Gas to 8th May 1916 . . . . .	0 5 0	
Electric Light to 1st May 1916 . . . . .	5 16 7	
Water 1915-16 . . . . .	4 4 0	
	<hr/>	
		£67 6 7
2. SALARIES :—		
General Secretary, 1915-16 . . . . .	£100 0 0	
Librarian . . . . .	120 0 0	
Assistant Librarian . . . . .	50 0 0	
Interim Assistant Librarian . . . . .	78 0 0	
Office Keeper . . . . .	94 10 0	
Treasurer's Clerk . . . . .	25 0 0	
	<hr/>	
		467 10 0
		<hr/>
Carry forward . . . . .	£534 16 7	

	Brought forward	£534 16 7	
3. EXPENSES OF TRANSACTIONS :—			
Neill & Co., Ltd., Printers	£484 10 6		
Less—Entered in 1914-15 Accounts	£255 0 0		
Sum received from Prof. T. H. Bryce on account of Dr Young's Paper	56 6 6		
	<hr/>	311 6 6	
		<hr/>	
		£173 4 0	
M'Farlane & Erskine, Lithographers	23 2 9		
Hislop & Day, Engravers	41 9 9		
Orrock & Son, Bookbinders	123 15 6		
Bemrose & Sons, Ltd., Printers	16 2 6		
Alex. Ritchie & Son, Lithographers	70 0 6		
	<hr/>		447 15 0
<i>Note.</i> —Owing to a fire in Messrs Neill's premises a considerable amount of printing has been postponed.			
4. EXPENSES OF PROCEEDINGS :—			
Neill & Co., Ltd., Printers	£224 15 3		
Less—Entered in 1914-15 Accounts	120 0 0		
	<hr/>	£104 15 3	
		<hr/>	
		16 14 6	
Hislop & Day, Engravers	0 12 0		
Orrock & Son, Bookbinders			122 1 9
	<hr/>		
<i>Note.</i> —Owing to a fire in Messrs Neill's premises a considerable amount of printing has been postponed.			
5. BOOKS, PERIODICALS, NEWSPAPERS, ETC. :—			
Otto Schulze & Co., Booksellers	£57 0 2		
James Thin, do.	50 0 10		
R. Grant & Son, do.	13 3 10		
W. Green & Son, Ltd., do.	0 14 0		
Robertson & Scott, News Agents	3 2 4		
Egypt Exploration Funds Subscription	4 4 0		
Ray Society do.	1 1 0		
John Grieg	3 3 0		
	<hr/>		132 9 2
6. OTHER PAYMENTS :—			
Neill & Co., Ltd., Printers	£79 8 0		
E. Sawyers, Purveyor	37 5 4		
S. Duncan, Tailor (uniforms)	5 6 6		
Orrock & Son, Bookbinders	5 1 1		
Andrew H. Baird	5 6 6		
Lindsay, Jamieson & Haldane, C.A., Auditors	6 6 0		
Post Office Telephone Rent	12 0 0		
A. Cowan & Sons, Ltd.	11 3 11		
W. S. Brown & Sons	20 12 0		
Gillies & Wright, Joiners	29 7 10		
R. Graham, Slater	6 13 8		
Charles Jenner & Coy.	12 12 0		
Oliver Typewriter Coy., Ltd.	4 2 3		
Jas. Sinclair (Lantern)	6 10 0		
Petty Expenses, Postages, Carriage, etc.	72 1 6		
	<hr/>		393 16 7
7. NET AMOUNT paid to meet present deficit in connection with the publication of the Napier Tercentenary Memorial Volume			131 10 5
8. SUM transferred on account of Dr F. R. C. Reed's Memoir			5 3 10
9. ARREARS of CONTRIBUTIONS outstanding at 30th September 1916 :—			
Present Session	£74 11 0		
Previous Sessions	69 6 0		
	<hr/>		143 17 0
Amount of the Discharge			<hr/> <hr/> £1911 10 4

# Abstract of Accounts.

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Amount of the Charge . . . . .	£1977 13 2
Amount of the Discharge . . . . .	1911 10 4
Excess of Receipts over Payments for 1915-1916 . . . . .	£66 2 10
FLOATING BALANCE IN FAVOUR OF THE SOCIETY at 1st October 1915 . . . . .	106 16 7
<b>Floating Balance in favour of the Society at 30th September 1916</b> . . . . .	<b>£172 19 5</b>
<i>Being—</i>	
Balance due by Union Bank of Scotland, Ltd., on Account Current . . . . .	£256 14 8
Due by Treasurer . . . . .	16 4 9
	£272 19 5
<i>Deduct—</i> Due to General Secretary . . . . .	100 0 0
	£172 19 5

## II. ACCOUNT OF THE KEITH FUND.

To 30th September 1916.

### CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1915 . . . . .	£62 2 3
2. INTEREST RECEIVED :—	
On £896, 19s. 1d. North British Railway Company 3 per cent. Debenture Stock for year to Whitsunday 1916, less Tax £4, 8s. 2d. . . . .	£22 10 0
On £211, 4s. North British Railway Company 3 per cent. Lien Stock for year to 30th June 1916, less Tax £1, 2s. 8d. . . . .	5 4 0
	27 14 0
3. INCOME TAX repaid for year to 5th April 1916 . . . . .	4 10 4
	£94 6 7

### DISCHARGE.

1. Dr James Hartley Ashworth—Money portion of Prize 1913-15 . . . . .	£49 10 9
2. Alexander Kirkwood & Son, Engravers, Gold Medal . . . . .	16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916 . . . . .	28 15 10
	£94 6 7

## III. ACCOUNT OF THE NEILL FUND

To 30th September 1916.

### CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1915 . . . . .	£33 17 3
2. INTEREST RECEIVED :—	
On £355 London, Chatham and Dover Railway 4½ per cent. Arbitration Debenture Stock for year to 30th June 1916, less Tax £2, 17s. 10d. . . . .	£13 1 8
On £15 four and a half per cent. War Loan, 1925-45, for year to 1st June 1916, less Tax 2s. 8d. . . . .	0 10 10
	13 12 6
3. INCOME TAX repaid for year to 5th April 1916 . . . . .	2 3 7
	£49 13 4

### DISCHARGE.

1. Dr Robert Campbell—Money portion of Prize 1913-15 . . . . .	£15 11 8
2. Alex. Kirkwood & Son, Engravers, for Gold Medal . . . . .	16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916 . . . . .	18 1 8
	£49 13 4

**IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND***To 30th September 1916.***CHARGE.**

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1915 . . . . .	£25 12 6
2. INTEREST RECEIVED :— On £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 for year to 30th June 1916, less Tax £2, 12s. 5d. . . . .	£11 19 7
On £150 four and a half per cent. War Loan, 1925-45, for year to 1st June 1916, less Tax £1, 7s. 1d. . . . .	5 7 11
	<hr/>
	17 7 6
3. INCOME TAX repaid for year to 5th April 1916 . . . . .	2 8 11
	<hr/>
	£45 8 11

**DISCHARGE.**

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916 . . . . .	£45 8 11
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**V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL  
OBSERVATION FUND***To 30th September 1916.***CHARGE.**

1. BALANCE due by General Fund at 1st October 1915 . . . . .	£5 12 4
2. INTEREST RECEIVED :— On £220 four and a half per cent. War Loan, 1925-45, for year to 1st June 1916, less Tax £1, 19s. 7d., Commission 3d. . . . .	7 18 2
3. INCOME TAX repaid for year to 5th April 1916 . . . . .	0 14 10
	<hr/>
	£14 5 4

**DISCHARGE.**

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916 . . . . .	£14 5 4
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**VI. ACCOUNT OF THE GUNNING VICTORIA JUBILEE PRIZE FUND***To 30th September 1916.*

(Instituted by Dr R. H. GUNNING of Edinburgh and Rio de Janeiro.)

**CHARGE.**

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1915 . . . . .	£86 3 7
2. INTEREST RECEIVED :— On £1000 North British Railway Company 3 per cent. Consoli- dated Lien Stock for year to 30th June 1916, less Tax £5, 7s. 9d. . . . .	£24 12 3
On £15 four and a half per cent. War Loan, 1925-45, for year to 1st June 1916, less Tax 2s. 8d. . . . .	0 10 10
	<hr/>
	25 3 1
3. INCOME TAX repaid for year to 5th April 1916 . . . . .	4 0 6
	<hr/>
	£115 7 2



## DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916 . . . . .	£115 7 2
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**VII. DR F. R. C. REED'S MEMOIR ON THE BRACHIOPODA OF THE  
ORDOVICIAN AND SILURIAN ROCKS OF GIRVAN**

*To 30th September 1916.*

## CHARGE.

1. BALANCE due by the Union Bank of Scotland, Ltd., on Deposit Receipt at 1st October 1915 . . . . .	£160 14 10
2. INTEREST RECEIVED :— On Deposit Receipts by the Union Bank of Scotland, Ltd., uplifted . . . . .	5 1 5
3. SUM transferred from General Fund . . . . .	5 3 10
	<hr/> £171 0 1 <hr/>

## DISCHARGE.

1. T. A. Brock, Cambridge, for Drawings . . . . .	£70 17 6
2. London Stereoscopic Coy., Ltd. . . . .	98 8 0
3. East Coast Railway . . . . .	1 14 7
	<hr/> £171 0 1 <hr/>

**STATE OF THE FUNDS BELONGING TO THE ROYAL  
SOCIETY OF EDINBURGH**

*As at 30th September 1916.*

## 1. GENERAL FUND—

1. £2090, 9s. 4d. three per cent. Lien Stock of the North British Railway Company at 59½ per cent. . . . .	£1249 1 0
2. £8519, 14s. 3d. three per cent. Debenture Stock of do. at 61½ per cent. . . . .	5239 12 5
3. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 120 per cent. . . . .	1050 0 0
4. £1811 four per cent. Debenture Stock of the Caledonian Railway Company at 84 per cent. . . . .	1521 4 10
5. £35 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 86½ per cent. . . . .	30 5 6
6. Arrears of Contributions, as per preceding Abstract of Accounts . . . . .	143 17 0
	<hr/> £9234 0 9 <hr/>

<i>Add</i> Floating Balance in favour of the Society, as per preceding Abstract of Accounts . . . . .	172 19 5
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AMOUNT . . . . .	<hr/> £9407 0 2 <hr/>
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Exclusive of Library, Museum, Pictures, etc., Furniture of the Society's Rooms at George Street, Edinburgh.

## 2. KEITH FUND—

1. £896, 19s. 1d. three per cent. Debenture Stock of the North British Railway Company at 61½ per cent. . . . .	£551 12 6
2. £211, 4s. three per cent. Lien Stock of do. at 59½ per cent. . . . .	126 3 10
3. Balance due by Union Bank of Scotland, Ltd., on Account Current . . . . .	28 15 10
	<hr/> AMOUNT . . . . . £706 12 2 <hr/>

**3. NEILL FUND—**

1. £355 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at $86\frac{1}{2}$ per cent.	£307	1	6
2. £15 four and a half per cent. War Loan, 1925-45, at $94\frac{5}{8}$ per cent.	14	3	11
3. Balance due by Union Bank of Scotland, Ltd., on Account Current	18	1	8
AMOUNT	£339	7	1

**4. MAKDOUGALL-BRISBANE FUND—**

1. £365 four per cent. Consolidated Preference Stock No. 2 of the Caledonian Railway Company at $75\frac{3}{4}$ per cent.	£276	9	9
2. £150 four and a half per cent. War Loan, 1925-45, at $94\frac{5}{8}$ per cent.	141	18	9
3. Balance due by Union Bank of Scotland, Ltd., on Account Current	45	8	11
AMOUNT	£463	17	5

**5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—**

1. £220 four and a half per cent. War Loan, 1925-45, at $94\frac{5}{8}$ per cent.	£208	3	6
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	14	5	4
AMOUNT	£222	8	10

**6. GUNNING VICTORIA JUBILEE PRIZE FUND—**Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £1000 three per cent. Consolidated Lien Stock of the North British Railway Company at $59\frac{3}{4}$ per cent.	£597	10	0
2. £15 four and a half per cent. War Loan, 1925-45, at $94\frac{5}{8}$ per cent.	14	3	11
3. Balance due by Union Bank of Scotland, Ltd., on Account Current	115	7	2
AMOUNT	£727	1	1

EDINBURGH, *14th October* 1916.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1915-1916, and have found them to be correct. The securities of the various Investments at 30th September 1916, as noted in the above Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE, C.A.,  
*Auditors.*

# THE COUNCIL OF THE SOCIETY.

October 1916.

## PRESIDENT.

JOHN HORNE, LL.D., F.R.S., F.G.S.

## VICE-PRESIDENTS.

BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S., formerly District Superintendent and Acting Paleontologist of the Geological Survey of Scotland.  
 SIR EDWARD ALBERT SCHÄFER, M.R.C.S., LL.D., F.R.S., Professor of Physiology in the University of Edinburgh.  
 THE RIGHT HON. SIR J. H. A. MACDONALD, P.C., G.C.B., K.C., F.R.S., M.Inst.E.E.  
 PROFESSOR R. A. SAMPSON, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland.  
 PROFESSOR D'ARCY THOMPSON, C.B., B.A., F.R.S., Professor of Natural History in the University College, Dundee.  
 PROFESSOR JAMES WALKER, D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh.

## GENERAL SECRETARY.

CARGILL G. KNOTT, D.Sc., LL.D., Lecturer on Applied Mathematics in the University of Edinburgh.

## SECRETARIES TO ORDINARY MEETINGS.

ARTHUR ROBINSON, M.D., M.R.C.S., Professor of Anatomy in the University of Edinburgh.  
 E. T. WHITTAKER, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh.

## TREASURER.

JAMES CURRIE, M.A.

## CURATOR OF LIBRARY AND MUSEUM.

A. CRICHTON MITCHELL, D.Sc., Hon. D.Sc. (Geneva).

## COUNCILLORS.

W. B. BLAIKIE, LL.D.	C. R. MARSHALL, M.A., M.D., Professor of Materia Medica and Therapeutics in the Medical School, Dundee.
O. CHARNOCK BRADLEY, M.D., D.Sc., M.R.C.V.S., Principal of the Royal (Dick) Veterinary College, Edinburgh.	J. SUTHERLAND BLACK, M.A., LL.D.
R. STEWART MACDOUGALL, M.A., D.Sc., Lecturer on Agricultural Entomology in the University of Edinburgh.	SIR GEORGE A. BERRY, M.D., C.M., F.R.C.S.
W. A. TAIT, D.Sc., M.Inst.C.E.	JOHN S. FLETT, M.A., D.Sc., LL.D., F.R.S., Director of the Geological Survey of Scotland.
J. H. ASHWORTH, D.Sc., Lecturer on Invertebrate Zoology in the University of Edinburgh.	MAGNUS MACLEAN, M.A., D.Sc., M.Inst.C.E., Professor of Electrical Engineering in the Royal Technical College, Glasgow.
C. G. BARKLA, D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh.	DAVID WATERSTON, M.A., M.D., F.R.C.S.E., Professor of Anatomy in the University of St Andrews.

*Society's Representative on George Heriot's Trust.*

WILLIAM ALLAN CARTER, M.Inst.C.E.

OFFICE, LIBRARY, ETC., 22, 24 George Street, Edinburgh. TEL. No., 2881.

# ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

*Corrected to January 1917.*

N.B.—*Those marked \* are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.

K.           "           "           "           Keith Medal.  
N.           "           "           "           Neill Medal.  
V. J.       "           "           "           the Gunning Victoria Jubilee Prize.  
C.           "           "           "           contributed one or more Communications to the  
Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.				Service on Council, etc.
1898	C.	* Abetromby, the Hon. John, LL.D., 62 Palmerston Place, Edinburgh		
1898		Adami, Prof. J. G., M.A., M.D. Cantab., F.R.S., Professor of Pathology in M'Gill University, Montreal		
1911		* Adams, Archibald Campbell, A.M.Inst.Mech.E., A.M.Inst.E.E., Consulting Engineer, 1 Old Smithhills, Paisley		
1896		* Affleck, Sir Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row, Edinburgh		
1875	C. K.	Aitken, John, LL.D., F.R.S., Ardenlea, Falkirk	5	
	V. J.			
1895		* Alford, Robert Gervase, M.Inst.C.E., Three Gables, Woodburn Park Road, Tunbridge Wells, Kent		
1889		Alison, John, M.A., Head Master, George Watson's College, Edinburgh		
1894		Allan, Francis John, M.D., C.M. Edin., M.O.H. City of Westminster, Westminster City Hall, Charing Cross Road, London		
1888	C.	Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California		
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.	10	
1883		Anderson, Sir Robert Rowand, LL.D., 16 Rutland Square, Edinburgh		
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh, 29 Lutton Place, Edinburgh		
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.		
1905		* Andrew, George, M.A., B.A., H.M.I.S., Balgillo Cottage, Seafield Road, Broughty Ferry		
1881	C.	Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork	15	
1915		Anthony, Charles, M.Inst.C.E., M. Am. Soc. C.E., F.R.San.I., F.R.A.S., F.R.Met.S., F.R.M.S., F.C.S., General Manager Water Works Company, Casilla de Correo 149, Bahia Blanca, Argentina		
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., Nylstroom, Smoke Lane, Reigate		
1899		Appleyard, James R., Royal Technical Institute, Salford, Manchester		
1893		Archer, Walter E., Union Club, Trafalgar Square, London, S.W.		
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	20	
1907		* Archibald, James, M.A., Head Master, St Bernard's School, 1 Leamington Terrace, Edinburgh		
1911	C. K.	* Ashworth, James Hartley, D.Sc., Lecturer on Invertebrate Zoology, University of Edinburgh, 69 Braid Avenue, Edinburgh		1912-14, 1915-
1907		* Badre, Muhammad, Ph.D., Almuneerah, Cairo, Egypt		

# Alphabetical List of the Ordinary Fellows of the Society, 407

Date of Election.			Service on Council, etc
1896	C.	* Baily, Francis Gibson, M.A., M.Inst.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh, Newbury, Colinton, Midlothian	1909-12
1877	C.	Balfour, I. Bayley, M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Garden, Inverleith House, Edinburgh 25	1888-91.
1905	C.	Balfour-Browne, William Alexander Francis, M.A., Barrister-at-Law, 26 Barton Road, Cambridge	
1892	C.	Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace, Edinburgh	
1902	C.	Bannerman, W. B., C.S.I., I.M.S., M.D., D.Sc., Surgeon General, Indian Medical Service, Madras, India	
1889		Barbour, A. H. F., M.A., M.D., LL.D., F.R.C.P.E., 4 Charlotte Square, Edinburgh	
1886		Barclay, A. J. Gunion, M.A., 729 Great Western Road, Glasgow 30	
1883	C.	Barclay, G. W. W., M.A., Raeden House, Aberdeen	
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII Sanatorium, Midhurst	
1914	C.	* Barkla, Charles Glover, D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, Littledene, 34 Priestfield Road, Edinburgh	1915-
1882	C.	Barnes, Henry, M.D., LL.D., 6 Portland Square, Carlisle	
1904		Barr, Sir James, M.D., LL.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool 35	
1874		Barrett, Sir William F., F.R.S., M.R.I.A., formerly Professor of Physics, Royal College of Science, Dublin, 31 Devonshire Place, London, W.	
1887		Bartholomew, J. G., LL.D., F.R.G.S., The Geographical Institute, Duncan Street, Edinburgh	1909-12.
1895	C.	Barton, Edwin H., D.Sc., F.R.S., A.M.Inst.E.E., F.P.S.L., Professor of Experimental Physics, University College, Nottingham	
1904		* Baxter, William Muirhead, Glenalmond, Sciennes Gardens, Edinburgh	
1913		Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S. (Eng.), L.R.C.P. (Lond.), D.P.H. (Camb.), Medical Officer of Health and School Medical Officer, City of Carlisle, 15 Brunswick Street, Carlisle 40	
1888		Beare, Thomas Hudson, B.Sc., M.Inst.C.E., Professor of Engineering in the University of Edinburgh	1907-09. V-P 1909-15.
1897	C.	* Beattie, John Carruthers, D.Sc., Professor of Physics, South African College, Cape Town	
1892		Beck, Sir J. H. Meiting, Kt., M.D., M.R.C.P.E., Drostdy, Tulbagh, Cape Province, South Africa	
1893	C. B.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London 45	
1887		Begg, Ferdinand Faithfull, 5 Whittington Avenue, London, E.C.	
1906		Bell, John Patrick Fair, F.Z.S., Fulforth, Wotton Gilbert, Durham	
1916		* Bell, Robert John Tainsh, M.A., D.Sc., Lecturer in Mathematics in the University of Glasgow, 146 Hyndland Road, Glasgow	
1915		Bell, Walter Leonard, M.D. Edin., F.S.A.Scot., 123 London Road, North Lowestoft, Suffolk	
1900		* Bennett, James Bower, C.E., 12 Castle Street, Edinburgh 50	
1887		Bernard, J. Mackay, of Dunsinnan, B.Sc., Dunsinnan, Perth	
1893	C.	* Berry, Sir George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens, Edinburgh	1916-
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria, Australia	
1904		* Beveridge, Erskine, LL.D., St Leonards Hill, Dunfermline	
1880	C.	Birch, De Burgh, C.B., M.D., Professor of Physiology in the University of Leeds, 8 Osborne Terrace, Leeds 55	
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884	C.	Black, John S., M.A., LL.D., 6 Oxford Terrace, Edinburgh	1891-94. 1916- Cur. 1906-16. 1914-
1897	C.	* Blaikie, Walter Biggar, LL.D., The Loan, Colinton	
1904	C.	* Bles, Edward J., M.A., D.Sc., Elterholm, Cambridge	
1898	C.	* Blyth, Benjamin Hall, M.A., V.P.Inst.C.E., 17 Palmerston Place, Edinburgh 60	
1894		* Bolton, Herbert, M.Sc., F.G.S., F.Z.S., Director of the Bristol Museum and Art Gallery, Bristol	
1915		* Boon, Alfred Archibald, D.Sc., Assistant Professor of Chemistry, Heriot-Watt College, Edinburgh	

Date of Election.			Service on Council, etc.
1872	C.	Bottomley, J. Thompson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	
1886	C.	Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow	1887-90, 1893-96, 1907-09. V-P 1910-16.
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., A.Inst.C.E., A.Inst.M.E., M.Inst.E.E., etc., 4 Albert Square, Manchester 65	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	
1916		Bradley, Francis Ernest, M.A., M.Com., LL.D., Barrister-at-Law, Examiner to the Council of Legal Education, Bank of England Chambers, Tib Lane, Manchester	
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Principal, Royal Dick Veterinary College, f Edinburgh	1907-10, 1915-
1886	-	Bramwell, Byrom, M.D., F.R.C.P.E., LL.D., 23 Drumsheugh Gardens, Edinburgh	1890-93.
1907	*	Bramwell, Edwin, M.B., F.R.C.P.E., F.R.C.P. Lond., 24 Walker Street, Edinburgh 70	
1912		Bridger, Adolphus Edward, M.D. (Edin.), F.R.C.P. (Edin.), B.Sc. (Paris), B.L. (Paris), Foley Lodge, Langham Street, London, W.	
1916	C.	* Briggs, Henry, M.Sc., A.R.S.M., Lecturer in Mining, Heriot-Watt College, Allermuir, Liberton, Midlothian	
1895		Bright, Charles, M.Inst.C.E., M.Inst.E.E., F.R.A.S., F.G.S., Consulting Engineer to the Commonwealth of Australia, The Grange, Leigh, Kent, and Members' Mansions, Victoria Street, London, S.W.	
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Thaxted, Dunmow, Essex. 75	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, South African College, Cape Town	
1864	C. K. B.	Brown, Alex. Crum, M.A., M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent, Edinburgh	1865-68, 1869-72, 1873-75, 1876-78, 1911-13. Sec. 1879-1905. V-P 1905-11.
1898		* Brown, David, F.C.S., F.I.C., J.P., Willowbrae House, Willowbrae Road, Edinburgh	
1911		* Brown, David Rainy, Chemical Manufacturer (J. F. Macfarlan & Co.), 93 Abbeyhill, Edinburgh	
1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street, Edinburgh 80	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 64 Upper Berkeley Street, Portman Square, London, W.	
1909	B. C.	* Brownlee, John, M.A., M.D., D.Sc., Medical Research Committee, Statistical Department, 34 Guildford Street, Russell Square, London, W.C.	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh	
1906	C. N.	* Bruce, William Speirs, LL.D., Director of the Scottish Oceanographical Laboratory, Edinburgh, Antarctica, Joppa, Midlothian	1909-12.
1898	C. K.	* Bryce, T. H., M.A., M.D. (Edin.), Professor of Anatomy in the University of Glasgow, 2 The University, Glasgow 85	1911-14.
1870	C. K.	Buchanan, John Young, M.A., F.R.S., 26 Norfolk Street, Park Lane, f London, W.	1878-81, 1884-86.
1905		Bunting, Thomas Lowe, M.D., 27 Denton Road, Scotswood, Newcastle-on-Tyne	
1902	*	Burgess, A. G., M.A., Mathematical Master, Edinburgh Ladies' College, 64 Strathearn Road, Edinburgh	
1887		Burnet, Sir John James, Architect, 18 University Avenue, Hillhead, Glasgow	
1888		Burns, Rev. T., D.D., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh 90	
1915	*	Butchart, Raymond Keiler, B.Sc., University College, Dundee, 8 Martin Street, Maryfield, Dundee	

# Alphabetical List of the Ordinary Fellows of the Society. 409

Date of Election.			Service on Council, etc.
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., Linlithgow	
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh	95
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Dalhousie University, Halifax, Nova Scotia	
1904		* Campbell, Charles Duff, Scottish Liberal Club, Princes Street, Edinburgh	
1915	C. N.	* Campbell, Robert, D.Sc., Lecturer in Petrology, University of Edinburgh, 7 Muirend Avenue, Juniper Green, Midlothian	
1899	C.	* Carlier, Edmund W. W., M.D., M.Sc., F.E.S., Professor of Physiology, University, Birmingham	100
1910		Carnegie, David, M.Inst.C.E., M.Inst.Mech.E., M.I.S.Inst., 17 Eglinton Road, Plumstead, London, S.E.	
1905	C.	* Carse, George Alexander, M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street, Edinburgh	
1901		Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905		Carter, Joseph Henry, F.R.C.V.S., Stone House, Church Street, Burnley, Lancashire	
1898		* Carter, Wm. Allan, M.Inst.C.E., 32 Great King Street, Edinburgh (Society's Representative on George Heriot's Trust)	1911-14. 105
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., Waldegrave Park, Strawberry Hill, Middlesex, and Sandacres Lodge, Parkstone-on-Sea, Dorset	
1908		Cavanagh, Thomas Francis, M.D., The Hospital, Bella Coola, B.C., Canada	
1882		Cay, W. Dyce, M.Inst.C.E., 39 Victoria Street, Westminster, London	
1899		Chatham, James, Actuary, 7 Belgrave Crescent, Edinburgh	
1912		Chaudhuri, Banawari Lal, B.A. (Cal.), B.Sc. (Edin.), Assistant Superintendent, Natural History Section, Indian Museum, 120 Lower Circular Road, Calcutta, India	110
1874		Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains	1884-86, 1904-06.
1891		Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, Craiglea Drive, Edinburgh	
1911		* Clark, William Inglis, D.Sc., 29 Lauder Road, Edinburgh	
1903		* Clarke, William Eagle, LL.D., F.L.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, 35 Braid Road, Edinburgh	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne	115
1913		* Clegghorn, Alexander, M.Inst.C.E., Marine Engineer, 14 Hatfield Drive, Kelvinside, Glasgow	
1916		Clough, C. T., M.A. (Cambridge), LL.D., District Geologist H.M. Geological Survey, Scotland, St Ann's Mount, Polton, Midlothian. (Deceased Aug. 27, 1916.)	
1904	C.	Coker, Ernest George, M.A., D.Sc., F.R.S., M.Inst.C.E., Professor of Civil and Mechanical Engineering, University of London, University College, Gower Street, London, W.C.	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bourne-mouth, W.	
1888	V. J. C.	Collie, John Norman, Ph.D. D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S., Professor of Organic Chemistry in the University College, Gower Street, London	120
1904		* Colquhoun, Walter, M.A., M.B., 18 Walmer Crescent, Ibrox, Glasgow	
1909	C.	* Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace, Edinburgh	
1886		Connan, Daniel M., M.A.	
1905		* Corrie, David, F.C.S., Nobel's Explosives Company, Polmont, Stirlingshire	
1914		* Coutts, William Barron, M.A., B.Sc., 33 Dalhousie Terrace, Edinburgh	125
1911		* Cowan, Alexander C., Papermaker, Valleyfield House, Penicuik, Midlothian	
1916	C.	Craig, E. H. Cunningham, B.A. (Cambridge), Geologist and Mining Engineer, The Dutch House, Beaconsfield	
1908		Craig, James Ireland, M.A., B.A., Controller of the Department of General Statistics, 14 Abdin Street, Cairo: The Koubbeh Gardens, near Cairo, Egypt	

Date of Election.			Service on Council, etc.
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place, Edinburgh.	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Mathematics in the South African College, Cape Town 130	
1887		Crawford, William Caldwell, 1 Lockharton Gardens, Colinton Road, Edinburgh	
1870		Crichton-Browne, Sir Jas., M.D., LL.D., D.Sc., F.R.S., Lord Chancellor's Visitor and Vice-President and Treasurer of the Royal Institution of Great Britain, 45 Hans Place, S.W., and Royal Courts of Justice, Strand, London	
1916		* Crombie, James Edward, M.A., LL.D., Millowner, Parkhill House, Dyce, Aberdeenshire	
1886		Croom, Sir John Halliday, M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, late President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square, Edinburgh	
1914		* Cumming, Alexander Charles, D.Sc., Lecturer in Chemistry, University, Edinburgh, 3 Gardiner Road, Blackhall, Midlothian 135	
1898		* Currie, James, M.A. Cantab. (TREASURER), Larkfield, Goldenacre, Edinburgh	Treas. 1906-
1904		* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock	
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	
1884		Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone Manor, Bow, North Devon	
1894		* Denny, Sir Archibald, Bart., LL.D., Cardross Park, Cardross, Dumbartonshire 140	
1869	C. V. J.	Dewar, Sir James, Kt., M.A., LL.D., D.C.L. D.Sc., F.R.S., V.P.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London	1872-74.
1905		* Dewar, James Campbell, C.A., 27 Douglas Crescent, Edinburgh	
1906		* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane	
1904		Dickinson, Walter George Burnet, F.R.C.V.S., Boston, Lincolnshire	
1884		Dickson, the Right Hon. Charles Scott, Lord Justice-Clerk, K.C., LL.D., 22 Moray Place, Edinburgh 145	
1888	C.	Dickson, Henry Newton, M.A., D.Sc., 160 Castle Hill, Reading	
1876	C.	Dickson, J. D. Hamilton, M.A., Senior Fellow and formerly Tutor, St Peter's College, Cambridge	
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, University Avenue, Los Angeles, California, U.S.A.	
1897		* Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh.	
1904		* Dobbie, Sir James Johnston, Kt., M.A., D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London, 4 Vicarage Gate, Kensington, London, W. 150	1905-08.
1881	C.	Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 6 Wilton Road, Edinburgh	1904-07, 1913-16.
1905		* Donaldson, Rev. Wm. Galloway, F.R.G.S., F.E.I.S., The Manse, Forfar	
1882	C.	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1910		* Douglas, Loudon MacQueen, Author and Lecturer, 3 Lauder Road, Edinburgh	
1908	C.	Drinkwater, Harry, M.D., M.R.C.S. (Eng.), F.L.S., Lister House, Wrexham, North Wales 155	
1901		* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1904		* Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh	
1903		Dunstan, John, M.R.C.V.S., Inversnaid, Liskeard, Cornwall	
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent	
1906	C.	Dyson, Sir Frank Watson, Kt., M.A., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich 160	1907-10.
1893		Edington, Alexander, M.D., Howick, Natal	
1904		* Edwards, John, 4 Great Western Terrace, Kelvinside, Glasgow	
1904		* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith	
1875		Elliot, Daniel G., American Museum of Natural History, Central Park West, New York, N.Y., U.S.A.	
1906	C.	* Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College, Glasgow 165	



# Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.			Service on Council, etc.
1897	C.	*Erskine-Murray, James Robert, D.Sc., 4 Great Winchester Street, London, E.C.	
1884		Evans, William, F.F.A., 38 Morningside Park, Edinburgh	
1879	C. N.	Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.Z.S., Regius Professor of Natural History, University of Edinburgh, Craigybield, Penicuik, Midlothian	1882-85, 1904-07. V-P 1907-12.
1902		*Ewen, John Taylor, B.Sc., M.I.Mech.E., H.M. Inspector of Schools, 104 King's Gate, Aberdeen	
1878	C.	Ewing, Sir James Alfred, K.C.B., M.A., B.Sc., LL.D., M.Inst.C.E., F.R.S., Principal of the University of Edinburgh, formerly Director of Naval Education, Admiralty	1888-91.
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Guy's Hospital (Bacteriological Department), London	
1910		*Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 37 Queen's Crescent, Edinburgh	
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Lecturer on Geography, The University, Glasgow	
1888	C.	Fawsitt, Charles A., Coney Park, Bridge of Allan	175
1883	C.	Felkin, Robert W., M.D., F.R.G.S., Whare Ra, Havelock North, Hawk's Bay, New Zealand	
1899		*Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow	
1907		*Fergus, Edward Oswald, 12 Clairmont Gardens, Glasgow	
1904		*Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh	
1898		*Findlay, John R., M.A. Oxon., 27 Drumsheugh Gardens, Edinburgh	180
1899		*Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Emeritus Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, 23 Dundonald Road, Glasgow, W.	
1911		Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, 136 Glebe Street, St Rollox, Glasgow	
1906		*Fleming, Robert Alexander, M.A., M.D., F.R.C.P.E., Assistant Physician, Royal Infirmary, 10 Chester Street, Edinburgh	
1900	C. N.	*Flett, John S., M.A., D.Sc., LL.D., F.R.S., Director of the Geological Survey of Scotland, 33 George Square, Westminster, S.W.	1916-
1872		Forbes, Professor George, M.A., M.Inst.C.E., M.Inst.E.E., F.R.S., F.R.A.S., 11 Little College Street, Westminster, S.W.	185
1892		*Ford, John Simpson, F.C.S., 7 Corrennie Drive, Edinburgh	
1910		*Fraser, Alexander, Actuary, 17 Eildon Street, Edinburgh	
1896		*Fraser, John, M.B., F.R.C.P.E., formerly one of H.M. Commissioners in Lunacy for Scotland, 54 Great King Street, Edinburgh	
1915		*Fraser, Rev. Joseph Robert, U.F. Manse, Kinneff, Scotland	
1867	C. K. B.	Fraser, Sir Thomas R., Kt., M.D., LL.D., Sc.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, Honorary Physician to the King in Scotland, 13 Drumsheugh Gardens, Edinburgh	1870-73, 1877-79, 1883-86, 1894-97. V-P 1911-16.
1914		*Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 17 Eildon Street, Edinburgh	
1891		Fullarton, J. H., M.A., D.Sc., 23 Porchester Gardens, London, W.	
1891		Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 41 Queen's Road, Aberdeen	
1907		*Galbraith, Alexander, Superintendent Engineer, Cunard Line, Liverpool, 93 Trinity Road, Bootle, Liverpool.	
1888	C.	Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh, St Margaret's, Craiglockhart, Midlothian	195
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1899		Gatehouse, T. E., A.M.Inst.C.E., M.Inst.M.E., M.Inst.E.E., Fairfield, 128 Tulse Hill, London, S.W.	
1867		Gayner, Charles, M.D., F.L.S.	
1909	C.	*Geddes, Auckland C., M.D., Professor of Anatomy, M'Gill University, Montreal, Canada	
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	200

Date of Election.			Service on Council, etc.
1861	C. B.	Geikie, Sir Archibald, O.M., K.C.B., D.C.L. Oxf., D.Sc., LL.D., Ph.D., Late Pres. R.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey	1869-72, 1874-76, 1879-82.
1914		Gemmell, John Edward, M.B., C.M., Hon. Surgeon Hospital for Women and Maternity Hospital; Hon. Gynaecologist, Victoria Central Hospital, Liscard, 28 Rodney Street, Liverpool	
1909		* Gentle, William, B.Sc., 12 Mayfield Road, Edinburgh	
1914		* Gibb, Alexander, A.M.Inst.C.E., Rosyth, Fife	
1916		* Gibb, A. W., D.Sc., Lecturer in Geology, The University, Aberdeen, 1 Belvidere Street, Aberdeen 205	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, 15 South Lauder Road, Edinburgh	
1912	C.	* Gibson, Arnold Hartley, D.Sc., Professor of Engineering, University College, Dundee	
1910		* Gibson, Charles Robert, Lynton, Mansewood, by Pollokshaws	
1890		Gibson, George A., M.A., LL.D., Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow	1905-08, 1912-13.
1911		Gidney, Henry A. J., L.M. and S. Socts. Ap. (Lond.), F.R.C.S. (Edin.), D.P.H. (Camb.), D.O. (Oxford), Army Specialist Public Health, c/o Thomas Cook & Sons, Ludgate Circus, London 210	
1900		Gilchrist Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne	
1880		Gilruth, George Ritchie, Surgeon, 53 Northumberland Street, Edinburgh	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Administrator, Government House, Darwin Northern Territory, Australia	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., 40 Lennox Gardens, London, S.W.	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer and Senior Demonstrator of Anatomy, King's College, University of London, 22 Regent's Park Terrace, London, N.W. 215	
1898		* Glaister, John, M.D., F.R.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow	
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E.	
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh	
1913	C.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), B.A. (Cantab.), Lecturer in Geology, University of London, King's College, Strand, W.C.	
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich 220	
1891		Graham, Richard D., 11 Strathearn Road, Edinburgh	
1898	C.	* Gray, Albert, A., M.D., 4 Clairmont Gardens, Glasgow	
1883	C.	Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow	1903-06. V-P 1906-09.
1910		Gray, Bruce M'Gregor, C.E., A.M.Inst.C.E., Westbourne Grove, Selby, Yorkshire	
1909	C.	* Gray, James Gordon, D.Sc., Lecturer in Physics in the University of Glasgow, 11 The University, Glasgow 225	1913-15.
1910		* Green, Charles Edward, Publisher, Gracemount House, Liberton	
1886		Greenfield, W. S., M.D., F.R.C.P.E., LL.D., Emeritus Professor of General Pathology in the University of Edinburgh, Kirkbrae, Elie, Fife	
1897		Greenlees, Thomas Duncan, M.D. Edin., Rostrevor, Kirtleton Avenue, Weymouth, Dorset	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S., Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow	1908-11.
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Major, H.M. Indian Medical Service, United Service Club, Calcutta, India 230	
1905		Greig, Robert Blyth, LL.D., F.Z.S., Board of Agriculture for Scotland, 29 St Andrew Square, Edinburgh	
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace, Edinburgh	
1907	C.	* Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E.	

# Alphabetical List of the Ordinary Fellows of the Society. 413

Date of Election.			Service on Council, etc.
1911	C.	* Gunn, James Andrew, M.A., M.D., D.Sc., Department of Pharmacology, University Museum, Oxford 235	
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon	
1916		* Guthrie, The Hon. Lord, LL.D., Judge of the Court of Session, 13 Royal Circus, Edinburgh	
1911		* Guy, William, F.R.C.S., L.R.C.P., L.D.S.Ed., Consulting Dental Surgeon, Edinburgh Royal Infirmary; Dean, Edinburgh Dental Hospital and School; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1911		Hall-Edwards, John Francis, L.R.C.P. (Edin.), Hon. F.R.P.S., Senior Medical Officer in charge of X-ray Department, General Hospital, Birmingham, 141A and 141B Great Charles Street (Newhall Street), Birmingham	
1905	B. C.	* Halm, Jacob E., Ph.D., Chief Assistant Astronomer, Royal Observatory, Cape Town, Cape of Good Hope 240	
1899		Hamilton, Allan M'Lane, M.D., LL.D., Great Barrington, Mass., University Club, New York, U.S.A.	
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., F.S.A. Scot., Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia	
1914		Harrison, Edward Philip, Ph.D., Professor of Physics, Presidency College, University of Calcutta, The Observatory, Alipore, Calcutta	
1888	C.	Hart, D. Berry, M.D., F.R.C.P.E., 13 Northumberland Street, Edinburgh	
1914	C.	Harvey-Gibson, Robert John, M.A., F.L.S., D.L. for the County Palatine of Lancaster, M.R.S.G.S., Professor of Botany, University of Liverpool, 18 Gambier Terrace, Liverpool 245	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff	
1892	C.	* Heath, Thomas, B.A., formerly Assistant Astronomer, Royal Observatory, Edinburgh, 11 Cluny Drive, Edinburgh	
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India	
1890	C.	Helme, T. Arthur, M.D., M.R.C.P., M.R.C.S., 3 St Peter's Square, Manchester	
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul, Warwick's Bench Road, Guildford, Surrey 250	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer Zoological Laboratories, University, Bristol	
1890	C.	Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	
1881	C. N.	Herdman, W. A., D.Sc., LL.D., F.R.S., Past Pres. L.S., Professor of Natural History in the University of Liverpool Croxteth Lodge, Ullet Road, Liverpool	
1916		* Herring, Percy Theodore, M.D., F.R.C.P.Ed., Professor of Physiology, University of St Andrews, Hepburn Gardens, St Andrews	
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight 255	
1902		* Hinxman, Lionel W., B.A., Geological Survey Office, 33 George Sq., Edinburgh	
1904		Hobday, Frederick T. G., F.R.C.V.S., 6 Berkely Gardens, Kensington, London, W.	
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Ordnance College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent	
1911		Holland, William Jacob, LL.D. St Andrews, etc., Director Carnegie Institute, Pittsburg, Pa., 5545 Forbes Street, Pittsburg, Pa., U.S.A.	
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland (PRESIDENT), 20 Merchiston Gardens Edinburgh 260	1902-05, 1906-07, 1914-15. V-P 1907-1913. P 1915-
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904	C.	* Horsburgh, Ellice Martin, M.A., B.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 11 Granville Terrace, Edinburgh	
1897		Houston, Alex. Cruikshanks, M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	
1912	C.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow, 11 Cambridge Drive, Glasgow	
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-on-Tyne 265	
1899		Howie, W. Lamond, F.C.S., 26 Neville Court, Abbey Road, Regent's Park, London, N.W.	

Date of Election.			Service on Council, etc.
1883	C.	Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Director of the Welsh National Museum : Crowland, Llandaff, Wales	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt	
1886		Hunt, Rev. H. G. Bonavia, Mus.D. Dub., Mus.B. Oxon., The Vicarage, Burgess Hill, Sussex	
1916	*	Hunter, Charles Stewart, L.R.C.P.E., L.R.C.S.E., D.P.H., Medical Officer of Health, Carnoustie, Dalhousie Villa, Carnoustie	270
1911		Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., Resident Engineer Nitrate Railways, Iquique, Chile, and Maybole, Ayrshire	
1887	C.	Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian	
1887	C.	Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 54 Harley Street, London	
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., 5 Portland Place, London, W.	
1912	*	Inglis, Robert John Mathieson, A.M.Inst.C.E., Engineer, Northern Division, North British Railway, Tantah, Peebles	275
1904	C.	Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal	
1904	*	Ireland, Alexander Scott, S.S.C., 2 Buckingham Terrace, Edinburgh	
1914		Jack, John Noble,	
1875		Jack, William, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow	1888-91.
1894		Jackson, Sir John, C.V.O., LL.D., 48 Belgrave Square, London	280
1889		James, Alexander, M.D., F.R.C.P.E., 14 Randolph Crescent, Edinburgh	
1901	*	Jardine, Robert, M.D., M.R.C.S., F.R.F.P.S. Glas., 20 Royal Crescent, Glasgow	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), etc., Bootham Park Private Mental Hospital, York	
1906	C.	* Jehu, Thomas James, M.A., M.D., F.G.S., Professor of Geology in the University of Edinburgh : 23 Great King Street, Edinburgh	
1900	*	Jerdan, David Smiles, M.A., D.Sc., Ph.D., Temora, Colinton, Midlothian	285
1916	*	Johnston, Col. Sir Duncan A., K.C.M.G., C.B., Colonel Royal Engineers, 8 Lansdowne Crescent, Edinburgh	
1895		Johnston, Col. Henry Halero, C.B., Late A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney	
1903	C.	* Johnston, Thomas Nicol, M.B., C.M., Poggie, Humbie, East Lothian	
1874		Jones, Francis, M.Sc., Lecturer on Chemistry, 17 Whalley Road, Whalley Range, Manchester	
1905		Jones, George William, M.A., B.Sc., LL.B., Scottish Tutorial Institute, Edinburgh and Glasgow, 25 North Bridge : Coraldene, Kirk Brae, Liberton, Edinburgh	290
1888		Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London	
1915		Kennal, James Hermann Rosenthal, Managing Director and Engineer-in-Chief of Babcock & Wilcox, Ltd., Kennal Manor, Chislehurst, Kent	
1907	*	Kemp, John, M.A., Sea Bank School, North Berwick	
1912		Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Assistant Professor of Neurology, Cornell University, New York, 20 West 50th Street, New York, U.S.A.	
1909		Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 126 Queen's Road, Finsbury Park, London, N.	295
1908	*	Kerr, Andrew William, F.S.A. Scot., Royal Bank House, St Andrew Square, Edinburgh	
1903	C. N.	* Kerr, John Graham, M.A., F.R.S., Professor of Zoology in the University of Glasgow	1904-07, 1913-16.
1891		Kerr, Joshua Law, M.D., The Chequers, Mittagong, Sydney, Australia	
1913	*	Kerr, Walter Hume, M.A., B.Sc., Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh	
1908		Kidd, Walter Aubrey, M.D., Heatherdown, Alum Bay, Freshwater, I. of W.	300
1886	C. N.	Kidston, Robert, LL.D., F.R.S., F.G.S., 12 Clarendon Place, Stirling	1891-94, 1903-06. Sec. 1909-16.
1907	*	King, Archibald, M.A., B.Sc., formerly Rector of the Academy, Castle Douglas ; Junior Inspector of Schools, La Maisonnette, Clarkston, Glasgow	
1880		King, W. F., Lonend, Russell Place, Trinity, Leith	

# Alphabetical List of the Ordinary Fellows of the Society. 415

Date of Election.			Service on Council, etc.
1883		Kinnear, The Right Hon. Lord, P.C., one of the Senators of the College of Justice, 2 Moray Place, Edinburgh	
1878		Kintore, The Right Hon. the Earl of, P.C., G.C.M.G., M.A. Cantab., LL.D. Cambridge, Aberdeen, and Adelaide, Keith Hall, Inverurie, Aberdeenshire 305	
1901		* Knight, Rev. G. A. Frank, M.A., 52 Sardinia Terrace, Hillhead, Glasgow	
1907		* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, St James' School, Glasgow, The Shielling, Uddingston, by Glasgow	
1880	C. K.	Knott, C. G., D.Sc., LL.D., Lecturer on Applied Mathematics in the University of Edinburgh, formerly Professor of Physics, Imperial University, Japan (GEN. SECRETARY), 42 Upper Gray Street, Edinburgh	1894-97, 1898-1901, 1902-05. Sec. 1905-12. Gen. Sec. 1912-
1878	C.	Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews	
1910	C.	* Lauder, Alexander, D.Sc., F.I.C., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 310	
1885	C.	Laurie, A. P., M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh }	1908-11, 1913-16.
1894	C.	* Laurie, Malcolm, B.A. D.Sc., F.L.S., 19 Merchiston Park, Edinburgh	
1910	C.	* Lawson, A. Anstruther, B.Sc., Ph.D., D.Sc., F.L.S., Professor of Botany, University of Sydney, New South Wales, Australia	
1905		* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druin-darroch, Banchory, Kincardineshire	
1910	C.	* Lee, Gabriel W., D.Sc., Palæontologist, Geological Survey of Scotland, 33 George Square, Edinburgh 315	
1903		* Leighton, Gerald Rowley, M.D., Local Government Board, 125 George Street, Edinburgh	
1874	C. K.	Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Veterinary Surgeon, Lecturer on Veterinary Science, Veterinary Infirmary, 12 Derwent Street, Derby	
1916	C.	* Levy, Hymen, M.A., B.Sc., Research Assistant, Aeronautical Section, National Physical Laboratory, Teddington	
1914	C. N.	Lewis, Francis John, D.Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada 320	
1905		* Lightbody, Forrest Hay, 53 Queen Street, Edinburgh	
1889		Lindsay, Rev. James, M.A., D.D., B.Sc., F.R.S.L., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Science Master, Royal High School, 33 Lauriston Gardens, Edinburgh	
1912		* Linlithgow, The Most Honourable the Marquis of, Hopetoun House, South Queensferry	
1903		Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay, Groom & Co., Bombay, India 325	
1903		* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine, Dean of the Faculty of Medicine in the University of Edinburgh, 11 Rutland Street, Edinburgh	
1898		* Lothian, Alexander Veitch, M.A., B.Sc., Training College, Cowcaddens, Glasgow	
1884		Low, George M., Actuary, 11 Moray Place, Edinburgh	
1888		Lowe, D. F., M.A., LL.D., formerly Headmaster of Heriot's Hospital School, Lauriston, 19 George Square, Edinburgh	
1900		Lusk, Graham, Ph.D., M.A., Professor of Physiology, Cornell University Medical College, New York, N.Y., U.S.A. 330	
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., 8 Holland Road, Cheltenham	
1907		MacAlister, Donald Alexander, A.R.S.M., F.G.S., 26 Thurloe Square, South Kensington, London, S.W.	
1883		M'Bride, P., M.D., F.R.C.P.E., 10 Park Avenue, Harrogate, and Hill House, Withypool, Dunster, Somerset	
1903		* M'Cormick, Sir W. S., M.A., LL.D., Secretary to the Carnegie Trust for the Universities of Scotland, 13 Douglas Crescent, Edinburgh 335	1910-13.
1905		* Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen	1908-11.

## Proceedings of the Royal Society of Edinburgh.

Date of Election.			Service on Council, etc.
1897	C.	* Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Stewarton, Kilmacolm	
1904		* Macdonald, John A., M.A., B.Sc., King Edward VII School, Johannesburg, Transvaal	
1886		Macdonald, The Right Hon. Sir J. H. A., P.C., G.C.B., K.C., LL.D., F.R.S., { M.Inst.E.E. (VICE-PRESIDENT), 15 Abercromby Place, Edinburgh	1889-92. V-P
1904		Macdonald, William, B.Sc., M.Sc., Agriculturist, Editor <i>Transvaal Agricultural Journal</i> , Department of Agriculture, Pretoria Club, Pretoria, Transvaal 340	1914-
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh	
1901	C.	* MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh, 9 Dryden Place, Edinburgh	1914-
1910		Macewen, Hugh Allen, M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London	
1885	C.	Macfarlane, J. M., D.Sc., Professor of Botany and Director of the Botanic Garden, University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A. 345	
1897		* MacGillivray, Angus, C.M., M.D., D.Sc., 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, Middlesex	
1903		* M'Intosh, Donald C., M.A., D.Sc., 3 Glenisla Gardens, Edinburgh	
1911		M'Intosh, John William, A.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W.	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, Pres. Ray Society, 2 Abbotsford Crescent, St Andrews 350	1885-88.
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., 11 Rothesay Place, Edinburgh	1875-78, 1885-88, 1893-94, 1900-02, V-P 1894-1900.
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	
1912	C.	M'Kendrick, Anderson Gray, M.B., Major, Indian Medical Service, Officiating Statistical Officer to the Government of India, The Pasteur Institute, Kasauli, India	
1900	C.	* M'Kendrick, John Souttar, M.D., F.R.F.P.S.G., 2 Buckingham Terrace, Hillhead, Glasgow 355	
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal, College of Hygiene and Physical Training, Dunfermline	
1916	C.	* Mackenzie, John E., Ph.D., D.Sc., Lecturer in Chemistry, University of Edinburgh, Major-Adjutant, O.T.C., 2A Ramsay Garden, Edinburgh	
1911		* M'Kenzie, Kenneth John, M.A., Master of Method to Leith School Board, 24 Dudley Gardens, Leith	
1894		* Mackenzie, Robert, M.D., Napier, Nairn	
1904	C.	* Mackenzie, W. Leslie, M.A., M.D., D.P.H., LL.D., Medical Member of the Local Government Board for Scotland, 4 Clarendon Crescent, Edinburgh 360	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	
1904		* Mackintosh, Donald James, M.V.O., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow	
1869	C.	Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent, Edinburgh	
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., M.Inst.C.E., Professor of Electrical Engineering in the Royal Technical College, 51 Kerrsland Terrace, Hillhead, Glasgow 365	1916-
1913		* M'Lellan, Dugald, M.Inst.C.E., District Engineer, Caledonian Railway, 20 Kingsburgh Road, Murrayfield, Edinburgh	
1916	C.	* M'Lintock, W. F. P., D.Sc. (Edin.), Royal Scottish Museum, Edinburgh	
1907	C.	* Macnair, Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow	
1898	C.	Mahālanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV, 37 Lower Chitpore Road, Calcutta, India 370	

# Alphabetical List of the Ordinary Fellows of the Society. 417

Date of Election.			Service on Council, etc.
1908		Mallik, Devendranath, Sc.D., B.A., Professor of Mathematics, Astronomical Observatory, Presidential College, Calcutta, India	
1912		Malonev, William Joseph, M.D.(Edin.), Professor of Neurology at Fordham University, New York City, N.Y., U.S.A.	
1913		Marchant, Rev. James, F.R.A.S., Director, National Council for Promotion of Race-Regeneration; Literary Adviser to House of Cassell; 42 Great Russell Street, London, W.C.	
1880	C.	Marsden, R. Sydney, M.D., C.M., D.Sc., D.P.H., Hon. L.A.H. Dub., M.R.I.A., F.I.C., M.O.H., Rowallan House, Carnes Road, and Town Hall, Birkenhead	
1909	C. B.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica and Therapeutics, Medical School, Dundee, Arnshean, Westfield Terrace, West Newport, Fife 375	1915-
1882	C.	Marshall, D. H., M.A., Professor, Union and Alwington Avenue, Kingston, Ontario, Canada	
1901	C.	* Marshall, F. H. A., Sc.D., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	
1912		* Martin, Sir Thomas Carlaw, LL.D., J.P., Director, Royal Scottish Museum, 18 Blackford Road, Edinburgh	
1913		Masson, George Henry, M.D., D.Sc., M.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1885	C.	Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne 380	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London	1902-04.
1911		Mathews, Gregory Macalister, F.L.S., F.Z.S., Foulis Court, Fair Oaks, Hants	
1906		* Mathieson, Robert, F.C.S., St Serf's, Innerleithen	
1902		Matthews, Ernest Romney, A.M.Inst.C.E., F.G.S., Chadwick Professor of Municipal Engineering in the University of London, University College, Gower Street, London, W.C.	
1901	C.	* Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A. 385	
1888		Methven, Cathcart W., M.Inst.C.E., F.R.I.B.A., Durham, Natal, S. Africa	
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y., U.S.A.	
1885	C. B.	Mill, Hugh Robert, D.Sc., LL.D., 62 Camden Square, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow 390	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S., D.P.H., Lt.-Col. R.A.M.C., formerly Army Specialist in Hygiene, c/o National Provincial Bank, Fargate, Sheffield	
1905		* Milne, Archibald, M.A., B.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive, Edinburgh	
1905		* Milne, C. H., M.A., Head Master, Daniel Stewart's College, 4 Campbell Road, Murrayfield, Edinburgh	
1904	C.	* Milne, James Robert, D.Sc., Lecturer on Natural Philosophy, 5 North Charlotte Street, Edinburgh	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen 395	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, Meloyne, Malone Park, Belfast	
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India (CURATOR OF LIBRARY AND MUSEUM), The Observatory, Eskdalemuir, Langholm, Dumfriesshire	1915-16. Cur. 1916-
1897		Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., Cruach, Lochgilphead	
1899		* Mitchell-Thomson, Sir Mitchell, Bart., 6 Charlotte Square, Edinburgh 400	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906	C.	Moffat, Rev. Alexander, M.A. B.Sc., Professor of Physical Science, Christian College, Madras, India	
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., Combe Bank, near Sevenoaks, Kent	
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay, India	
1896		* Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens, Edinburgh 405	

Date of Election.			Service on Council, etc.
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony	
1914		Mort, Spencer, M.D., Ch.B., F.R.C.S.E., Lieut.-Col. R.A.M.C., Medical Officer in Charge, Edmonton Military Hospital, Silver Street, Upper Edmonton, London, N.	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Captain, Professor of Medical Jurisprudence, 26 Park Street, Wellesley, Calcutta, India	
1892	C.	Mossmann, Robert C., 62 Camden Square, London, N.W.	
1916		* Muir, Robert, M.A., M.D., Sc.D., F.R.S., Professor of Pathology, University of Glasgow, 16 Victoria Crescent, Dowanhill, Glasgow 410	
1874	C. K.	Muir, Sir Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Elmcote, Sandown Road, Rondebosch, South Africa	1885-88. V-P 1888-91.
1888	C.	Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers	
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., c/o Dunlop Rubber Co., Ltd., Aston Cross, Birmingham	
1887		Mukhopādhyay, Asūtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta, India	
1891	C.	Munro, Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Society of Antiquaries of Ireland, Elmbank, Largs, Ayrshire 415	1894-97, 1900-03. V-P 1903-08.
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent, Edinburgh	
1907		Musgrove, James, M.D., F.R.C.S. Edin. and Eng., LL.D., Emeritus-Professor of Anatomy, University of St Andrews, The Swallowgate, St Andrews	
1902		Myne, Rev. R. S., M.A., B.C.L. Oxford, F.S.A. Lond., Great Amwell, Herts	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P., 28 Angus Street, Adelaide, S. Australia	
1897		Nash, Alfred George, B.Sc., F.R.G.S., C.E., Belretiro, Mandeville, Jamaica, W.I. 420	
1906		* Newington, Frank A., M.Inst.C.E., M.Inst.E.E., 7 Wester Coates Road, Edinburgh	
1898		Newman, Sir George, M.D., D.P.H., Cambridge, Lecturer on Preventive Medicine, St Bartholomew's Hospital, University of London: Grim's Wood, Harrow Weald, Middlesex	
1884		Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park, Edinburgh	1885-87, 1892-95, 1897-1900.
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place, Edinburgh	
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham 425	
1888		Ogilvie, F. Grant, C.B., M.A., B.Sc., LL.D., Secretary of the Board of Education for the Science Museum and the Geological Survey, and Director of the Science Museum, 15 Evelyn Gardens, London, S.W.	1901-03.
1888		Oliphant, James, M.A., 11 Heathfield Park, Willesden Green, London	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W.	
1895	C.	Oliver, Sir Thomas, M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1915		* Orr, Lewis P., F.F.A., Secretary of Scottish Life Assurance Co., 14 Learmonth Gardens, Edinburgh 430	
1914		* Oswald, Alfred, Lecturer in German, Glasgow Provincial Training College, Nordheim, Bearsden, Glasgow	
1908		Page, William Davidge, F.C.S., F.G.S., M.Inst.M.E., 10 Clifton Dale, York	
1905		Pallin, William Alfred, F.R.C.V.S., Veterinary-Major, Royal Horse Guards, London	
1914		Pare, John William, M.B., C.M., M.D., L.D.S., Lecturer in Dental Anatomy, National Dental Hospital, 9A Cavendish Square, London, W.	
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian 435	
1886	C.	Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., F.R.S., Professor of Physiology in the University of Glasgow, University, Glasgow	1894-97, 1904-06, 1909-12.
1892		* Paulin, Sir David, Actuary, 6 Forres Street, Edinburgh	



# Alphabetical List of the Ordinary Fellows of the Society. 419

Date of Election.			Service on Council, etc.
1881	C. N.	Peach, Benjamin N., LL.D., F.R.S., F.G.S. (VICE-PRESIDENT), formerly District Superintendent and Acting Palæontologist of the Geological Survey of Scotland, 72 Grange Loan, Edinburgh	1905-08, 1911-12. V-P 1912-
1907		* Pearce, John Thomson, B.A., B.Sc., School House, Tranent	
1914		Pearson, Joseph, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon 440	
1904		* Peck, James Wallace, M.A., Chief Inspector, National Health Insurance, Scotland, 83 Princes Street, Edinburgh	
1889		Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	
1887	C. B.	Peddle, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee, The Weisha, Ninewells, Dundee	1904-07, 1908-11.
1893		Perkin, Arthur George, F.R.S., 8 Montpelier Terrace, Hyde Park, Leeds	
1913	C.	* Philip, Alexander, M.A., LL.B., Writer, The Mary Acre, Brechin 445	
1889		Philip, Sir R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square, Edinburgh	
1907	C.	Phillips, Charles E. S., Castle House, Shooter's Hill, Kent	
1914		* Pilkington, Basil Alexander, 20 Queen's Avenue, Blackhall, Midlothian	
1905		* Pinkerton, Peter, M.A., D.Sc., Rector, High School, Glasgow, 44 Hamilton Park Terrace, Hillhead, Glasgow	
1908	C.	* Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Bacteriological Laboratory, Nairobi, British East Africa 450	
1911		* Pirie, James Simpson, Civil Engineer, 28 Scotland Street, Edinburgh	
1906		Pitchford, Herbert Watkins, F.R.C.V.S., Bacteriologist and Analyst, Natal Government, The Laboratory, Pietermaritzburg, Natal	
1886		Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1888		Prain, Sir David, Lt.-Col., Indian Medical Service (Retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.L.S., F.R.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. K. Bayer Akad. Wiss., etc.; Director, Royal Botanic Gardens, Kew, Surrey	
1902		* Preller, Charles du Riche, M.A., Ph.D., A.M.Inst.C.E., 61 Melville Street, Edinburgh 455	
1892		* Pressland, Arthur J., M.A. Camb., Edinburgh Academy	
1875	C.	Prevost, E. W., Ph.D., Weston, Ross, Herefordshire	
1915		Price, Frederick William, M.D., M.R.C.P. Edin., Physician to the Great Northern Hospital, London, 133 Harley Street, London, W.	
1908		* Pringle, George Cossar, M.A., Rector of Peebles Burgh and County High School, Bloomfield, Peebles	
1903		* Pullar, Laurence, Dunbarney, Bridge of Earn, Perthshire 460	
1911		Purdy, John Smith, M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Chief Health Officer for Tasmania, Islington, Hobart, Tasmania	
1898		* Purves, John Archibald, D.Sc., 52 Queen Street, Exeter	
1897	C.	* Rainy, Harry, M.A., M.B., C.M., F.R.C.P.Ed., 16 Great Stuart Street, Edinburgh	
1899		* Ramage, Alexander G., Marchfield, Davidson's Mains, Midlothian	
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, formerly Curator of Australian Museum, Sydney, N.S.W.: "Truro," Queensborough Road, Croydon, N.S.W. 465	
1914		* Ramsay, Peter, M.A., B.Sc., Head Mathematical Master, George Watson's College, 63 Comiston Drive, Edinburgh	
1911		* Rankin, Adam A., British Astronomical Association, West of Scotland Branch, 324 Crow Road, Broomhill, Glasgow, W.	
1891		Rankine, John, K.C., M.A., LL.D., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place, Edinburgh	
1904		Ratliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham	
1900		Raw, Nathan, M.D., M.R.C.P. (London), B.S., F.R.C.S., D.P.H., 66 Rodney Street, Liverpool 470	
1883	C.	Readman, J. B., D.Sc., F.C.S., Belmont, Hereford	
1889		Redwood, Sir Boverton, Bt., D.Sc. (Hon.), F.I.C., F.C.S., A.Inst.C.E., The Cloisters, 18 Avenue Road, Regent's Park, London, N.W.	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., Barrister-at-Law, National Liberal Club, Whitehall Place, London	
1902		Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants	

Date of Election.			Service on Council, etc.
1913		Reid, Harry Avery, F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand 475	
1908	C.	* Rennie, John, D.Sc., Lecturer on Parasitology, and Assistant to the Professor of Natural History, University of Aberdeen, 60 Desswood Place, Aberdeen	
1914		Renshaw, Graham, M.B., M.R.C.S., L.R.C.P., L.S.A., Surgeon, Bridge House, Sale, Manchester	
1913		* Richardson, Harry, M.Inst.E.E., M.Inst.M.E., General Manager and Chief Engineer, Electricity Supply, Dundee and District, The Cottage, Craigie, Broughty Ferry	
1908		Richardson, Linsdall, F.L.S., F.G.S., Organising Inspector of Technical Education for the Gloucestershire Education Committee, 10 Oxford Parade, Cheltenham, Glos.	
1875		Richardson, Ralph, W.S., 29 Eglinton Crescent, Edinburgh 480	
1916	C.	* Ritchie, James, M.A., D.Sc., Royal Scottish Museum, 20 Upper Gray Street, Edinburgh	
1914	C.	* Ritchie, James Bonnyman, D.Sc., Science Master, Kelvinside Academy, Glasgow	
1906	C.	* Ritchie, William Thomas, M.D., F.R.C.P.E., 9 Atholl Place, Edinburgh	
1898	C.	Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John's Street, Manchester 485	
1900		* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	
1896		* Robertson, Robert, M.A., 25 Mansionhouse Road, Edinburgh	
1902	C.	* Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews	
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 2 Mayfield Gardens, Edinburgh	
1910	C.	* Robinson, Arthur, M.D., M.R.C.S., Professor of Anatomy, University of Edinburgh (SECRETARY), 35 Coates Gardens, Edinburgh 490	1910-12. Sec. 1912-
1916		* Ronald, David, Civil Engineer, Engineering Inspector, Local Government Board, Burnfield, Falkirk	
1881		Rosebery, The Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh	
1909	C.	* Ross, Alex. David M.A., D.Sc., F.R.A.S., Professor of Mathematics and Physics, University of Western Australia, Perth, Western Australia	
1906		* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, Dunaura, Hough Street, Falkirk	
1902	C. K.	* Russell, James, 22 Glenorchy Terrace, Edinburgh 495	
1880	C.	Russell, Sir James A., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane, Edinburgh	
1904		Sachs, Edwin O., Architect, Chairman of the British Fire Prevention Committee, Vice-President of the International Fire Service Council, 8 Waterloo Place, Pall Mall, London, S.W.	
1906		Saleeby, Caleb William, M.D., 13 Greville Place, London	
1916		* Salvesen, The Hon. Lord E. T., Judge of the Court of Session, Dean Park House, Edinburgh	
1914		* Salvesen, Theodore Emile, 37 Inverleith Place, Edinburgh 500	
1912	C.	* Sampson, Ralph Allen, M.A., D.Sc., F.R.S. (VICE-PRESIDENT), Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh, Royal Observatory, Edinburgh	1912-15 V-P 1915-
1903		* Samuel, John S., 8 Park Avenue, Glasgow	
1903		* Sarolea, Charles, Ph.D., D.Litt., Lecturer on French Language, Literature, and Romance Philology, University of Edinburgh, 21 Royal Terrace, Edinburgh	
1891		Sawyer, Sir James, K.T., M.D., F.R.C.P., F.S.A., J.P., Consulting Physician to the Queen's Hospital, 31 Temple Row, Birmingham	
1900	C.	* Schäfer, Sir Edward Albert, M.R.C.S., LL.D., F.R.S. (VICE-PRESIDENT), Professor of Physiology in the University of Edinburgh 505	1900-93, 1906-09. V-P 1913-
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.	
1902		Senn, Nicholas, M.D., LL.D., Professor of Surgery, Rush Medical College, Chicago, U.S.A.	
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place, Edinburgh	

# Alphabetical List of the Ordinary Fellows of the Society. 421

Date of Election.			Service on Council, etc.
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street, Edinburgh	
1911	C.	Simpson, Sutherland, M.D., D.Sc. (Edin.), Professor of Physiology, Medical College, Cornell University, Ithaca, N.Y., U.S.A., 118 Eddy Street, Ithaca, N.Y., U.S.A.	
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	
1903		* Skinner, Robert Taylor, M.A., Head Master, Donaldson's Hospital, Edinburgh	
1901		* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1891	C. K.	Smith, Alexander, B.Sc., Ph.D., Department of Chemistry, Columbia University, New York, N.Y., U.S.A.	
1882	C.	Smith, C. Michie, C.I.E., B.Sc., F.R.A.S., formerly Director of the Kodaikānal and Madras Observatories, Winsford, Kodaikānal, South India	515
1915		* Smith, James Lorrain, Professor of Pathology, University of Edinburgh, 11 Bruntsfield Crescent, Edinburgh	
1911		* Smith, Stephen, B.Sc., Goldsmith, 31 Grange Loan, Edinburgh	
1907	C.	Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1880		Smith, William Robert, M.D., D.Sc., LL.D., Professor of Forensic Medicine and Toxicology in King's College, University of London, and Principal of the Royal Institute of Public Health, 36 Russell Square, London, W.C.	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Medical Officer of Health, Coventry	520
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 31 Cameron Street, Dunfermline	
1889		Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpean Professor of Rural Economy and Fellow of St John's College in the University of Oxford, 121 Banbury Road, Oxford	
1911	C.	* Sommerville, Duncan McLaren Young, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1882		Sorley, James, 82 Onslow Gardens, London	525
1896		* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive, Edinburgh	
1874	C.	Sprague, T. B., M.A., LL.D., Actuary, West Holme, Woldingham, Surrey	1885-87.
1906		Squance, Thomas Coke, M.D., F.R.M.S., F.S.A.Scot., Physician and Pathologist in the Sunderland Infirmary, President Sunderland Antiquarian Society, Sunderland Naturalists' Association, 15 Grange Crescent, Sunderland	
1891		Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh	
1915		* Steggall, John Edward Aloisius, M.A., Professor of Mathematics at University College, Dundee, in St Andrews University, Woodend, Perth Road, Dundee	530
1912	C.	Stephenson, John, M.B., D.Sc. (Lond.), Indian Medical Service, Professor of Biology, Government College, Lahore, India	
1910		* Stephenson, Thomas, F.C.S., Editor of the <i>Prescriber</i> , Examiner to the Pharmaceutical Society, 6 South Charlotte Street, Edinburgh	
1916		* Steuart, D. R., F.I.C., Chemist to the Broxburn Oil Company, Osborne Cottage, Broxburn	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1884		Stevenson, David Alan, B.Sc., M.Inst.C.E., 84 George Street, Edinburgh	535
1888	C.	Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road, Edinburgh	
1902		* Stockdale, Herbert Fitton, Director of the Royal Technical College, Glasgow, Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05.
1906		Story, Fraser, Professor of Forestry, University College, Bangor, North Wales	
1907		* Strong, John, M.A., Rector, Royal High School, Edinburgh, 27 Grange Road, Edinburgh	540
1903		Sutherland, David W., M.D., M.R.C.P., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India	
1905		Swithinbank, Harold William, Denham Court, Denham, Bucks	
1912		* Syme, William Smith, M.D. (Edin.), 10 India Street, Glasgow	
1885	C.	Symington, Johnson, M.D., F.R.C.S.E., F.R.S., Professor of Anatomy in Queen's College, Belfast	1892-93.

Date of Election.			Service on Council, etc.
1904	C.	* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	545
1898		Tait, William Archer, D.Sc., M.Inst.C.E., 38 George Square, Edinburgh	
1895		Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, University of Utah, Salt Lake City, Utah, U.S.A.	1914-
1890	C.	Tanakadate, Aikitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy	550
1892		Thackwell, J. B., M.B., C.M., 423A Battersea Park Road, London, S.W.	
1885	C.	Thompson, D'Arey W., C.B., B.A., F.L.S., F.R.S. (VICE-PRESIDENT), Professor of Natural History in University College, Dundee	1892-95, 1896-99, 1907-10, 1912-15. V-P 1916-
1905		* Thoms, Alexander, 7 Playfair Terrace, St Andrews	
1887		Thomson, Andrew, M.A., D.Sc., F.I.C., Rector, Perth Academy, Ardenlea, Pitcullen, Perth	
1911		* Thomson, Frank Wyville, M.A., M.B., C.M., D.P.H., D.T.M., Lt.-Col. I.M.S. (Retired), Bonsyde, Linlithgow	555
1896		* Thomson, George Ritchie, M.B., C.M., General Hospital, Johannesburg, Transvaal	
1903		Thomson, George S., F.C.S., Ferma Albion, Marculesci, Roumania	
1906		* Thomson, Gilbert, M.Inst.C.E., 164 Bath Street, Glasgow	
1887	C.	Thomson, J. Arthur, M.A., LL.D.; Regius Professor of Natural History in the University of Aberdeen	1906-08.
1906	C.	Thomson, James Stuart, M.Sc., Ph.D., Zoological Department, University, Manchester	560
1880		Thomson, John Millar, LL.D., F.R.S., Professor of Chemistry in King's College, London, 5 Chepstow Crescent, London, W. (till March 31, 1917)	
1899		* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	
1912	C.	Thomson, Robert Black, M.B. Edin., Professor of Anatomy, South African College, Cape Town	
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent, Edinburgh	
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of the Cape of Good Hope, University Buildings, Cape Town	565
1876	C.	Thomson, William, Royal Institution, Manchester	
1911		* Tosh, James Ramsay, M.A., D.Sc. (St Ands.), Railway Temperance Hotel, Helensburgh	
1914		Tredgold, Alfred Frank, L.R.C.P., M.R.C.S., Hon. Consulting Physician to National Association for the Feeble-minded, 6 Dapdune Crescent, Guildford, Surrey	
1915		* Trotter, George Clark, M.D., Ch.B. Edin., D.P.H. (Aberdeen), Medical Officer of Health, Paisley, Remuera, Paisley	
1888		Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan, Edinburgh	570
1905		* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street, Edinburgh	
1906	C.	* Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P., Lecturer on Medical Physics, Surgeons' Hall, Physician in charge of Radium Treatment, Royal Infirmary, Edinburgh, 37 George Square, Edinburgh	
1895		Turton, Albert H., M.I.M.M., 233 George Road, Erdington, Birmingham	
1898	C.	* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, Duns, Berwickshire	
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcester-shire	575
1910		Vincent, Swale, M.D. Lond., D.Sc. Edin., etc., Professor of Physiology, University of Manitoba, Winnipeg, Canada	
1891	C. B.	Walker, James, D.Sc., Ph.D., LL.D., F.R.S. (VICE-PRESIDENT), Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road, Edinburgh	1903-05, 1910-13. V-P 1916-
1873	C.	Walker, Robert, M.A., LL.D., University, Aberdeen	
1902		* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C.	Wallace, R., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh	580
1898		Wallace, Wm., M.A., Belvedere, Alberta, Canada	
1891		Walsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London	

# Alphabetical List of the Ordinary Fellows of the Society. 423

Date of Election.			Service on Council, etc.
1907		Waters, E. Wynston, Medical Officer, H.B.M. Administration, E. Africa, Malindi, British East Africa Protectorate, <i>via</i> Mombasa	
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University, St Andrews	1916-
1911		* Watson, James A. S., B.Sc., etc., Assistant in Agriculture, University of Edinburgh, 15 Dick Place, Edinburgh 585	
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley	
1907		* Watt, Andrew, M.A., Secretary to the Scottish Meteorological Society, 6 Woodburn Terrace, Edinburgh	1912-14.
1911		Watt, James, W.S., F.F.A., 24 Rothesay Terrace, Edinburgh	
1911		* Watt, Rev. Lauchlan Maclean, B.D., Minister of St Stephen's Parish, 7 Royal Circus, Edinburgh	
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynaecology, Rush Medical College, 1748 Harrison Street, Chicago, Ill., U.S.A. 590	
1907	B. C.	* Wedderburn, Ernest Maclagan, M.A., LL.B., W.S., D.Sc., 7 Dean Park Crescent, Edinburgh	1913-16.
1903	C.	* Wedderburn, J. H. Maclagan, M.A., D.Sc., 95 Mercer Street, Princeton, N.J., U.S.A.	
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma	
1896		Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., D.C.L., Professor of Philosophy in the University of Michigan, Ann Arbor, U.S.A.	
1909	C.	* Westergaard, Reginald Ludovic Andreas Emil, Ph.D., Professor of Technical Mycology, Heriot-Watt College, Hafnia, Liberton, Edinburgh 595	
1916		* White, J. Martin, Esq., of Balruddery, Balruddery, near Dundee	
1896	C.	White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales	
1911		* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A. (Scot.), Lynwood, Hatton Place, Edinburgh	
1912	C.	* Whittaker, Edmund Taylor, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh (SECRETARY), 35 George Square, Edinburgh	1912-15 Sec.
1879		Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen 600	1916-
1908		* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen	
1910	C.	* Williamson, William, F.L.S., 79 Morningside Drive, Edinburgh	
1900		Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees	
1911		* Wilson, Andrew, M.Inst.C.E., 51 Queen Street, Edinburgh	
1902		* Wilson, Charles T. R., M.A., F.R.S., 14 Cranmer Road, Cambridge, Sidney Sussex College, Cambridge 605	
1895		Wilson-Barker, David, R.N.R., F.R.G.S., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," off Greenhithe, Kent	
1882		Wilson, George, M.A., M.B., LL.D.	
1891		Wilson, John Hardie, D.Sc., University of St Andrews, 39 South Street, St Andrews	
1902		Wilson, William Wright, F.R.C.S.E., M.R.C.S., Cottesbrook House, Acock's Green, Birmingham	
1908		* Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith 610	
1886	C.	Woodhead, German Sims, M.D., F.R.C.P.E., Professor of Pathology in the University of Cambridge	1887-90.
1884		Woods, G. A., M.R.C.S., 1 Hammelton Road, Bromley, Kent	
1911		* Wrigley, Ruric Whitehead, B.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1890		Wright, Johnstone Christie, Conservative Club, Edinburgh	
1896		* Wright, Sir Robert Patrick, Chairman of the Board of Agriculture for Scotland, Kingarth, Colinton, Midlothian 615	
1882		Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow	
1892		Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.	
1896	C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton	
1904		Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal	

## LIST OF HONORARY FELLOWS OF THE SOCIETY

*At January 1, 1917.*

HIS MOST GRACIOUS MAJESTY THE KING.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW I).

Elected

- 1900 Adolf Ritter von Baeyer, Professor of Chemistry, Universität, München, Germany.
- 1916 Charles Barrois, Professor of Geology and Mineralogy, Université, Lille, France.
- 1905 Waldemar Christofer Brögger, Professor of Mineralogy and Geology, K. Frederiks Universitet, Christiania, Norway.
- 1916 Douglas Houghton Campbell, Professor of Botany, Leland Stanford Junior University, California, U.S.A.
- 1905 Moritz Cantor, Professor of Mathematics, Universität, Heidelberg: Gaisbergstrasse 15, Heidelberg, Germany.
- 1902 Jean Gaston Darboux, Secrétariat de l'Institut, Paris, Professor of Higher Geometry, Sorbonne, Paris.
- 1910 Hugo de Vries, Professor of Plant Anatomy and Physiology, Lunteren, Holland.
- 1908 Emil Fischer, Professor of Chemistry, Universität, Berlin, Germany.
- 1916 Marcel Eugène Emile Gley, Professor of Physiology, Collège de France, Paris, Membre de l'Académie de Médecine, Paris: 14, rue Monsieur le Prince, Paris.
- 1910 Karl F. von Goebel, Professor of Botany, Universität, München, Germany.
- 1916 Camillo Golgi, Professor of Pathology, Università, Pavia, Italy.
- 1916 William Crawford Gorgas, General, U.S. Army Medical Corps.
- 1916 Gio. Battista Grassi, Professor of Comparative Anatomy, Regia Università, Roma, Italy: Via Agostino Depretis N. 91, Rome.
- 1905 Paul Heinrich von Groth, Professor of Mineralogy, Universität, München, Germany.
- 1888 Ernst Haeckel, Professor of Zoology, Universität, München, Germany.
- 1913 George Ellery Hale, Director of Mount Wilson Solar Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.
- 1883 Julius Hann, Emeritus Professor of Cosmical Physics, Universität, Wien, Austria.
- 1910 Jacobus Cornelius Kapteyn, Professor of Astronomy, Universiteit, Groningen, Holland.
- 1897 Gabriel Lippmann, Professor of Physics, Université, Paris, France.
- 1895 Carl Menger, Professor of Political Economy, Universität, Wien, Austria: Wien ix, Fuchstallerg 2, Austria.
- 1910 Albert Abraham Michelson, Professor of Physics, University, Chicago, U.S.A.
- 1897 Fridtjof Nansen, Professor of Oceanography, K. Frederiks Universitet, Christiania, Norway.
- 1908 Henry Fairfield Osborn, Professor of Zoology, Columbia University and American Museum of Natural History, New York, N.Y., U.S.A.
- 1910 Wilhelm Ostwald, Emeritus Professor of Phys. Chemistry, Universität, Leipzig: Gross-Bothen, bei Leipzig, Germany.
- 1908 Ivan Petrovitch Pawlov, Emeritus Professor of Physiology, Kais. Inst. Exper. Med., Petrograd: Wedenskaja Strasse 4, Petrograd, Russia.
- 1916 Edward Charles Pickering, Professor of Practical Astronomy and Director of the Astronomical Observatory, Harvard College, Cambridge, Mass.
- 1889 Georg Hermann Quincke, Emeritus Professor of Physics, Bergstrasse 41, Heidelberg, Germany.
- 1913 Santiago Ramón y Cajal, Professor of Histology and Pathological Anatomy, Universidad, Madrid, Spain.
- 1908 Magnus Gustaf Retzius, Emeritus Professor of Anatomy, Högskolan, Stockholm, Sweden.
- 1908 Augusto Righi, Professor of Experimental Physics, Regia Università, Bologna, Italy.
- 1913 Vito Volterra, Professor of Mathematical Physics, Regia Università, Rome, Italy.
- 1905 Wilhelm Waldeyer, Professor of Anatomy, Universität, Berlin, Germany.
- 1916 Eugenius Warming, Emeritus Professor of Botany at the University of Copenhagen and Director of the Botanical Garden: Bjerregaardsvej 5, Copenhagen, Valby.
- 1905 Wilhelm Wundt, Professor of Philosophy, Universität, Leipzig, Germany.

Total, 34.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW I).

- 1916 Sir Francis Darwin, Kt., D.Sc., M.B., F.R.S., Hon. Fellow, Christ's College, Cambridge.
- 1900 Sir David Ferrier, Kt., M.A., M.D., LL.D., F.R.S., Emer. Professor of Neuro-Pathology, King's College, London, 34 Cavendish Square, London, W.

## Elected

- 1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Math.D., F.R.S., Chief Professor of Mathematics in the Imperial College of Science and Technology, London, formerly Sadlerian Professor of Pure Mathematics in the University of Cambridge, Imperial College of Science, London, S.W.
- 1910 Sir James George Frazer, D.C.L., LL.D., Litt.D., F.B.A., Fellow of Trinity College, Cambridge, Professor of Social Anthropology in the University of Liverpool, Trinity College, Cambridge.
- 1916 James Whitbread Lee Glaisher, M.A., Sc.D., F.R.S., Fellow of Trinity College, Cambridge.
- 1908 Sir Alexander B. W. Kennedy, Kt., LL.D., F.R.S., Past Pres. Inst. C.E., A7, Albany, Piccadilly, London, W.
- 1913 Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., Professor of Mathematics in the University of Manchester.
- 1916 John Newport Langley, Sc.D., LL.D., F.R.S., Fellow of Trinity College, Professor of Physiology in the University of Cambridge.
- 1908 Sir Edwin Ray Lankester, K.C.B., LL.D., F.R.S., 29 Thurloe Place, S. Kensington, London, S.W.
- 1916 Charles Lapworth, F.R.S., LL.D., M.Sc., F.G.S., Professor of Geology and Physiography in the University of Birmingham.
- 1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., M.P. University of Cambridge since 1911, Lucasian Professor of Mathematics in the University of Cambridge, St John's College, Cambridge.
- 1900 Archibald Liversidge, M.A., LL.D., F.R.S., Emer.-Professor of Chemistry in the University of Sydney, Fieldhead, Combe Warren, Kingston, Surrey.
- 1916 Alexander Macalister, M.A., M.D., LL.D., F.R.S., F.S.A., Fellow of St John's College, Cambridge, Professor of Anatomy in the University of Cambridge.
- 1886 The Rt. Hon. Lord Rayleigh, O.M., P.C., J.P., D.C.L., LL.D., D.Sc. Dub., F.R.S., Corresp. Mem. Inst. of France, Terling Place, Witham, Essex.
- 1916 Arthur Schuster, Ph.D., D.Sc., LL.D., D. ès Sc. Geneva, Secretary of the Royal Society, London, Emer.-Professor of Physics in the University of Manchester.
- 1908 Charles Scott Sherrington, M.A., M.D., LL.D., F.R.S., Waynflete Professor of Physiology in the University of Oxford, Physiological Laboratory, Oxford.
1913. Sir William Turner Thiselton-Dyer, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew: The Ferns, Witcombe, Gloucester.
- 1905 Sir Joseph John Thomson, D.Sc., LL.D., F.R.S., Cavendish Professor of Experimental Physics, University of Cambridge, Trinity College, Cambridge.
- 1900 Sir Thomas Edward Thorpe, Kt., C.B., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, Imperial College of Science and Technology, South Kensington, London, S.W., Whinfield Salcombe, South Devon.

Total, 19.

## CHANGES IN FELLOWSHIP DURING SESSION 1915-16

### ORDINARY FELLOWS OF THE SOCIETY ELECTED

ROBERT JOHN TAINSH BELL, M.A., D.Sc.	COL. SIR DUNCAN A. JOHNSTON, K.C.M.G.,
FRANCIS ERNEST BRADLEY, M.A.,	C.B.
M.Com., LL.D.	HYMAN LEVY, M.A., B.Sc.
HENRY BRIGGS, M.Sc., A.R.S.M.	JOHN E. MACKENZIE, Ph.D., D.Sc.
C. T. CLOUGH, M.A., LL.D.	W. F. P. M'LINTOCK, D.Sc.
E. H. CUNNINGHAM CRAIG, B.A.	ROBERT MUIR, M.A., M.D., Sc.D., F.R.S.
JAMES EDWARD CROMBIE, M.A., LL.D.	JAMES RITCHIE, M.A., D.Sc.
A. W. GIBB, D.Sc.	DAVID RONALD, C.E.
THE HON. LORD GUTHRIE, LL.D.	THE HON. LORD E. T. SALVESEN.
PERCY THEODORE HERRING, M.D.,	D. R. STEUART, F.I.C.
F.R.C.P.E.	J. MARTIN WHITE.

### ORDINARY FELLOWS DECEASED

SIR STAIR AGNEW, K.C.B., M.A.	NORMAN HAY FORBES, F.R.C.S.E.,
ALEX. RUSSELL BROWN, M.A., B.Sc.,	L.R.C.P. Lond., M.R.C.S. Eng.
CAPTAIN 8TH K.O.S.B.	J. A. HARVIE-BROWN, LL.D, F.Z.S.
JAMES BURGESS, C.I.E., LL.D., Hon.	REV. GEORGE P. LAING.
A.R.I.B.A., F.R.G.S.	NICHOLAS HENRY MARTIN, F.L.S., F.C.S.
ROBERT CAIRD, LL.D.	HENRY O'CONNOR, A.M.Inst.C.E.
C. T. CLOUGH, M.A., LL.D.	THOMAS PARKER, M.Inst.C.E.
JOHN COOK, M.A., LL.D.	SIR A. R. SIMPSON, M.D.
CHARLES A. COOPER, LL.D.	GEORGE SMITH, F.C.S.
ARTHUR DUKINFELD DARBISHIRE,	PRINCIPAL SIR WM. TURNER, K.C.B., M.B.,
M.A.	F.R.C.S.L. and E., LL.D., D.C.L., D.Sc.,
DAVID DOUGLAS.	F.R.S.

### ORDINARY FELLOWS RESIGNED

GEORGE DUTHIE, M.A.	HENRY WALKER, M.A., D.Sc.
JOHN HANNAY THOMPSON, M.Sc., M.Inst.C.E., M.Inst.Mech.E.	

### FOREIGN ORDINARY FELLOWS DECEASED

EMIL CLÉMENT JUNGFLEISCH.	ELIÉ METCHNIKOFF.
CHARLES RENÉ ZEILLER.	

### BRITISH HONORARY FELLOW DECEASED

SIR WILLIAM RAMSAY, K.C.B., LL.D., F.R.S.



List of Library Exchanges, Presentations, etc.

1. TRANSACTIONS AND PROCEEDINGS OF LEARNED SOCIETIES, ACADEMIES, ETC., RECEIVED BY EXCHANGE OF PUBLICATIONS, AND LIST OF PUBLIC INSTITUTIONS ENTITLED TO RECEIVE COPIES OF THE TRANSACTIONS AND PROCEEDINGS OF THE ROYAL SOCIETY OF EDINBURGH. (*For convenience certain Presentations are included in this List.*)

T.P. prefixed to a name indicates that the Institution is entitled to receive *Transactions* and *Proceedings*. P. indicates *Proceedings*.  
(T.) signifies that only the Papers bearing on a particular class of subject are supplied.

AFRICA (BRITISH CENTRAL).

ZOMBA.—*Scientific Department*. Meteorological Observations, Fol. (*Presented by H.M. Acting Commissioner and Consul-General.*)

AMERICA (NORTH). (*See CANADA, UNITED STATES, AND MEXICO.*)

AMERICA (SOUTH).

- T.P. BUENOS AYRES (ARGENTINE REPUBLIC).—*Museo Nacional*. Anales.  
P. *Sociedad Physis*. Boletin.  
*Oficina Meteorologica Argentina*. Anales. (*Presented.*)  
CORDOBA—  
T.P. *Academia Nacional de Ciencias de la Republica Argentina*. Boletin.  
T.P. *National Observatory*. Annals.—Maps.  
T.P. LA PLATA (ARGENTINE REPUBLIC).—*Museo de La Plata*.  
LIMA (PERU). *Cuerpo de Ingenieros de Minas del Peru*. Boletin. (*Presented.*)  
P. MONTEVIDEO (URUGUAY).—*Museo Nacional*. Anales (Flora Uruguay).  
T.P. PARÀ (BRAZIL).—*Museu Paraense de Historia Natural e Ethnographia*. Boletin.  
P. QUITO (ECUADOR).—*Observatorio Astronómico y Meteorológico*.  
RIO DE JANEIRO (BRAZIL).—  
T.P. *Observatorio*. Anuario.—Boletin Mensal.  
P. *Museu Nacional*. Revista (Archivos).  
SANTIAGO (CHILI).—  
T.P. *Société Scientifique du Chili*. Actes.  
P. *Deutscher Wissenschaftlicher Verein*.  
P. SAN SALVADOR.—*Observatorio Astronómico y Meteorológico*.  
VALPARAISO (CHILI).—*Servicio Meteorologico*. Anuario. (*Presented.*)

AUSTRALIA.

*Australasian Association for the Advancement of Science*.—Reports. (*Presented.*)

ADELAIDE—

- P.      *University Library.*
- T.P.    *Royal Society of South Australia.* Transactions and Proceedings.—Memoirs.
- P.      *Royal Geographical Society of Australasia (South Australian Branch).*  
Proceedings.  
*Observatory.* Meteorological Observations. 4to. (*Presented.*)

BRISBANE—

- T.P.    *University of Queensland.*
- P.      *Royal Society of Queensland.* Transactions.
- P.      *Royal Geographical Society (Queensland Branch).* Queensland Geo-  
graphical Journal.
- P.      *Government Meteorological Office.*
- P.      *Water Supply Department.*
- P.      GEELONG (VICTORIA).—*Gordon Technical College.*
- T.P.    HOBART.—*Royal Society of Tasmania.* Proceedings.

MELBOURNE—

- Commonwealth Bureau of Census and Statistics.* Official Year Book.  
By G. H. Knibbs. (*Presented.*)
  - National Museum.* Memoirs. (*Presented.*)
  - T.P.    *University Library.*
  - P.      *Royal Society of Victoria.* Proceedings.—Transactions.
- PERTH, W.A.—
- P.      *Geological Survey.* Annual Progress Reports.—Bulletins.  
*Government Statistician's Office.* Monthly Statistical Abstract. (*Pre-  
sented.*)

SYDNEY—

- T.P.    *University Library.* Calendar.—Reprints of Papers from Science Labora-  
tories.
- T.P.    *Department of Mines and Agriculture (Geological Survey), N.S.W.*  
Records.—Annual Reports.—Palæontology. Mineral Resources.
- T.P.    *Linnean Society of South Wales.* Proceedings.
- T.P.    *Royal Society of New South Wales.* Journal and Proceedings.
- T.P.    *Australian Museum.* Records.—Reports.—Memoirs.—Catalogues.  
*N.S.W. Government.* Fisheries Report. (*Presented.*)

AUSTRIA.

CRACOW—

- T.P.    *Académie des Sciences.* Rozprawy Wydziału matematyczno-przyrod-  
niezego (Proceedings, Math. and Nat. Sciences Cl.).—Rozprawy  
Wydziału filologicznego (Proc., Philological Section).—Rozprawy  
Wydziału historyczno - filozoficznego (Proc., Hist.-Phil. Section).—  
Sprawozdanie Komisji do badania historyi sztuki w Polsce (Proc.,  
Commission on History of Art in Poland).—Sprawozdanie Komisji  
fizyjograficznej (Proc., Commission on Physiography).—Geological  
Atlas of Galicia ; Text, Maps.—Bulletin International, etc.

GRATZ—

- T.P.    *Naturwissenschaftlicher Verein für Steiermark.* Mittheilungen.
- P.      *Chemisches Institut der K. K. Universität.*
- P.      LEMBERG.—*Société Scientifique de Chevtchenko.*

## PRAGUE—

- P. *Deutscher Nat.-Med. Verein für Böhmen* “*Lotos*.”—“*Lotos*.”
- T.P. *K. K. Sternwarte*. Magnetische und Meteorologische Beobachtungen. Astronomische Beobachtungen.
- T.P. *K. Böhmisches Gesellschaft*. Sitzungsberichte: Math.-Naturw. Classe; Phil.-Hist.-Philol. Classe.—Jahresbericht,—and other publications.
- T.P. *Ceská Akademie Císarë Františka Josefa pro Vědy Slovesnost a Umění*. Almanach.—Vestník (Proceedings).—Rozpravy (Transactions): Phil.-Hist. Class; Math.-Phys. Cl.; Philol. Cl.—Historický Archiv.—Bulletin International, Résumé des Travaux présentés,—and other publications of the Academy.
- P. SARAJEVO (BOSNIA).—*The Governor-General of Bosnia-Herzegovina*. Ergebnisse der Meteorologischen Beobachtungen.

## TRIESTE—

- P. *Società Adriatica di Scienze Naturali*.
- P. *Museo Civico di Storia Naturale*.
- P. *Osservatorio Marittimo*. Rapporto Annuale.

## VIENNA—

- T.P. *Kais. Akademie der Wissenschaften*. Denkschriften: Math.-Naturwissenschaftliche Classe; Philosophisch-Historische Classe—Sitzungsberichte der Math.-Naturwissenschaftlichen Classe; Abtheil. I, IIA, IIB, III; Philosoph.-Historische Classe.—Almanach.—Mittheilungen der Erdbeben Commission.
- T.P. *K. K. Geologische Reichsanstalt*. Abhandlungen.—Jahrbücher.—Verhandlungen.
- T.P. *Oesterreichische Gesellschaft für Meteorologie*. Meteorologische Zeitschrift.
- T.P. *K. K. Zoologisch-Botanische Gesellschaft*. Verhandlungen.—Abhandlungen.
- P. *K. K. Naturhistorisches Hofmuseum*. Annalen.
- K. K. Central-Anstalt für Meteorologie und Erdmagnetismus*. Jahrbücher. 4to.—Allgemeiner Bericht und Chronik. 8vo. (*Presented*.)
- K. K. Militär Geographisches Institut*. Astronomisch-Geodätischen Arbeiten.—Astronomische Arbeiten. 4to.—Längenbestimmungen. 4to.—Die Ergebnisse der Triangulierungen. 4to. (*Presented*.)
- Zoologisches Institut der Universität und der Zoologischen Station in Triest*. Arbeiten. (*Purchased*.)

## BELGIUM.

## BRUSSELS—

- T.P. *Académie Royale des Sciences, des Lettres et des Beaux Arts de Belgique*. Mémoires.—Bulletins.—Annuaire.—Biographie Nationale.
- T.P. *Musée Royal d'Histoire Naturelle*. Mémoires.
- T.P. *Musée du Congo*. Annales.—Botanique. Zoologie. Ethnographie et Anthropologie. Linguistique, etc.
- T.P. *L'Observatoire Royal de Belgique, Uccle*. Annuaire.—Annales Astronomiques.—Annales Météorologiques.—Annales.—Physique du Globe.—Bulletin Climatologique.—Observations Météorologiques.

BRUSSELS—continued—

- T.P.    *Société Scientifique. Annales.*  
P.       *Société Belge d'Astronomie. Ciel et Terre. (Purchased.)*  
T.P.    GHENT.—*University Library.*  
T.P.    LOUVAIN.—*University Library.*

BOSNIA-HERZEGOVINA. (See AUSTRIA.)

BULGARIA.

- P.       SOFIA.—*Station Central Météorologique de Bulgarie. Bulletin Mensuel.—*  
            *Bulletins Annuaire.*

CANADA.

- EDMONTON (ALBERTA).—*Department of Agriculture. Annual Report.—*  
            *(Presented.)*  
P.       HALIFAX (NOVA SCOTIA).—*Nova Scotian Institute of Science. Proceedings*  
            *and Transactions.*  
T.P.    KINGSTON.—*Queen's University.*  
MONTREAL—  
P.       *Natural History Society. Proceedings.*  
P.       *Canadian Society of Civil Engineers. Transactions.—Annual Reports.*  
OTTAWA—  
T.P.    *Royal Society of Canada. Proceedings and Transactions.*  
T.P.    *Geological Survey of Canada. Annual Reports.—Palæozoic Fossils.—*  
            *Maps, Memoirs, and other Publications.*  
P.       *Literary and Scientific Society. Transactions.*  
T.P.    QUEBEC.—*Literary and Philosophical Society. Transactions.*  
TORONTO—  
T.P.    *University. University Studies. (History. Psychological Series. Geo-*  
            *logical Series. Economic Series. Physiological Series. Biological*  
            *Series. Physical Science Series. Papers from the Chemical Labora-*  
            *tory.) etc.*  
T.P.    *Canadian Institute. Transactions.*  
P.       *Royal Astronomical Society of Canada. Journal.—Astronomical Handbook.*

CAPE COLONY. (See UNION OF SOUTH AFRICA.)

CEYLON.

COLOMBO—

- T.P.    *Museum. Spolia Zeylanica. Annual Report.*

CHINA.

HONG KONG—

- P.       *Royal Observatory. Monthly Meteorological Bulletin. Report.*

# DENMARK.

## COPENHAGEN—

- T.P. *Académie Royale de Copenhague.* Mémoires : Classe des Sciences.—Oversigt.
- P. *Naturhistorisk Forening.* Videnskabelige Meddelelser.
- P. *Danish Biological Station.* Report.
- Conseil Permanent International pour l'Exploration de la Mer.* Publications de circonstance.—Rapports et Procès-Verbaux de Réunions.—Bulletin des Résultats acquis pendant les croisières périodiques.—Bulletin Statistique. (*Presented.*)
- Kommissionen for Havundersogelser.* Meddelelser : Série Fiskeri. Série Plankton. Série Hydrografi.—Skrifter. (*Presented.*)
- University (Zoological Museum).* Reports of the Danish Ingolf-Expedition. (*Presented.*)

# EGYPT.

- T.P. CAIRO.—*School of Medicine.* Records.
- Ministry of Finance (Survey Dept. : Archæological Survey of Nubia).* Bulletin, Reports, Papers. (*Presented.*)

# ENGLAND AND WALES.

- T.P. ABERYSTWYTH.—*National Library of Wales.*
- BIRMINGHAM—
- P. *Philosophical Society.* Proceedings.
- University.* Calendar. (*Presented.*)
- CAMBRIDGE—
- T.P. *Philosophical Society.* Transactions and Proceedings.
- T.P. *University Library.—Observatory.* Report.—Observations.
- T.P. CARDIFF.—*University College of South Wales.*
- COVENTRY.—Annual Report of the Health of the City. (*Presented by Dr Snell.*)
- P. ESSEX.—*Essex Field Club.* The Essex Naturalist.
- T.P. GREENWICH.—*Royal Observatory.* Astronomical, Magnetical, and Meteorological Observations.—Photo-heliographic Results and other Publications.
- T.P. HARPENDEN (HERTS.).—*Rothamstead Exp. Station.* (*Lawes Agricultural Trust.*)
- LEEDS—
- T.P. *Philosophical and Literary Society.* Reports.
- P. *Yorkshire Geological and Polytechnic Society.* Proceedings.
- LIVERPOOL—
- T.P. *University College Library.*
- P. *Biological Society.* Proceedings and Transactions.
- P. *Geological Society.* Proceedings.
- LONDON—
- P. *Admiralty.* Nautical Almanac and Astronomical Ephemeris.—Health of the Navy (Annual Report).

## LONDON—continued—

- P.      *Aeronautical Society of Great Britain.* Aeronautical Journal. Aeronautical Classics. Reports of the Bird Construction Committee.
- T.P.    *Anthropological Institute.* Journal.
- T.P.    *Athenæum Club.*  
*British Antarctic Expedition, 1907-09.* Reports on Scientific Investigations. (Presented.)
- T.P.    *British Association for the Advancement of Science.* Reports.
- T.P.    *British Museum (Copyright Office).* Reproductions from Illuminated Manuscripts.
- T.P.    *British Museum. Natural History Department.* Catalogues, Monographs, Lists, etc. *National Antarctic Expedition, 1901-04.* Publications.
- T.P.    *Chemical Society.* Journal. Abstract of Proceedings.
- P.      *Faraday Society.* Transactions.
- T.P.    *Geological Society.* Quarterly Journal.—Geological Literature.—Abstract of Proceedings.
- T.P.    *Geological Survey of the United Kingdom.* Summary of Progress. Memoirs.
- P.      *Geologists' Association.* Proceedings.
- T.P.    *Hydrographic Office.*
- T.P.    *Imperial Institute.*
- T.P.    *Institution of Civil Engineers.* Minutes of Proceedings, etc.
- T.P.    *Institution of Electrical Engineers.* Journal.
- P.      *Institution of Mechanical Engineers.* Proceedings.
- T.P.    *International Catalogue of Scientific Literature.* (Purchased.)
- T.P.    *Linnean Society.* Journal: Zoology; Botany.—Transactions: Zoology; Botany.—Proceedings.
- P.      *Mathematical Society.* Proceedings.
- (T.)P. *Meteorological Office.* Report of the Meteorological Committee to the Lords Commissioners of H.M. Treasury.—Reports of the International Meteorological Committee.—Hourly Readings.—Weekly Weather Reports.—Monthly and Quarterly Summaries.—Meteorological Observations at Stations of First and Second Order, and other Publications. Geophysical Journal.—Geophysical Memoirs.
- Mineralogical Society of Great Britain and Ireland.* Mineralogical Magazine and Journal. (Presented.)
- National Antarctic Expedition, 1901-04.* (Presented.)
- Optical Society.* Transactions. (Purchased.)
- P.      *Pharmaceutical Society.* Journal.—Calendar.
- P.      *Physical Society.* Proceedings.
- T.P.    *Royal Astronomical Society.* Monthly Notices.—Memoirs.
- T.P.    *Royal College of Surgeons.*
- T.P.    *Royal Geographical Society.* Geographical Journal.
- T.P.    *Royal Horticultural Society.* Journal.
- T.P.    *Royal Institution.* Proceedings.
- P.      *Royal Meteorological Society.* Quarterly Journal.
- T.P.    *Royal Microscopical Society.* Journal.
- P.      *Royal Photographic Society.* Photographic Journal.

LONDON—*continued*—

- T.P. *Royal Society*. Philosophical Transactions.—Proceedings.—Year-Book.  
—*National Antarctic Expedition, 1901-04*, Publications; and other  
Publications.
- T.P. *Royal Society of Arts*. Journal.
- T.P. *Royal Society of Literature*. Transactions.—Reports.
- T.P. *Royal Society of Medicine*. Proceedings.
- T.P. *Royal Statistical Society*. Journal.
- T.P. *Society of Antiquaries*. Proceedings.—*Archæologia*; or Miscellaneous  
Tracts relating to Antiquity.
- Society of Chemical Industry*. Journal. (*Presented*.)
- Society of Psychical Research*. Journal.—Proceedings. (*Presented by*  
*W. C. Crawford, Esq.*)
- Solar Physics Committee*. Annual Report, and other Publications.  
(*Presented*.)
- T.P. *United Service Institution*.
- T.P. *University College*. Calendar.
- T.P. *University*.
- T.P. *Zoological Society*. Transactions.—Proceedings.
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- T.P. BERGEN.—*Museum.* Aarsberetning.—Aarbog.—An Account of the Crustacea of Norway. By G. O. Sars.

CHRISTIANIA—

- T.P. *K. Norske Frederiks Universitet.* Nyt Magazin for Naturvidenskaberne.  
 —Archiv for Mathematik og Naturvidenskab.  
 T.P. *Meteorological Institute.* Jahrbuch.  
*Videnskabs-Selskab.* Forhandlingar.—Skrifter (Math. Nat. Kl.); (*Presented.*)  
 P. STAVANGER.—*Museum.* Aarshefte.  
 T.P. THRONDHJEM.—*Kgl. Norske Videnskabers Selskab.* Skrifter.  
 P. TROMSÖ.—*Museum.* Aarshefter.—Aarsberetning.

PHILIPPINE ISLANDS.

- P. MANILA.—*Bureau of Science.* Ethnological Survey Publications. Bureau of Forestry. Annual Report.

PORTUGAL.

- T.P. COIMBRA.—*University.* Anuario. Archivo Bibliographico.—Revista.  
 LISBON—  
 T.P. *Academia das Ciências de Lisboa.* Boletim.—Actas.  
 T.P. *Sociedade de Geographia.*  
*Observatorio do Infante D. Luiz.* Annaes. (*Presented.*)  
 PORTO. *Academia Polytechnica.* Annaes Scientificos.

QUEENSLAND. (*See AUSTRALIA.*)

ROUMANIA.

BUCHAREST—

- T.P. *Academia Romana.* Analele. Bulletin de la Section Scientifique.—Also Publications relating to the History, etc., of Roumania. Bibliografia Romanesca.—Catalogues, etc.  
 P. *Institut Météorologique.* Analele.

# RUSSIA.

T.P. DORPAT (JURJEW).—*University*. Inaugural Dissertations. Acta. Sitzungsberichte der Naturforscher Gesellschaft bei der Universität.—Schriften.

T.P. EKATHERINEBOURG.—*Société Ouralienne d'Amateurs des Sciences Naturelles*. Bulletin.

## KAZAN—

T.P. *Imperial University*. Ucheniuiya Zapiski.

P. *Société Physico-Mathématique de Kazan*. Bulletin.

T.P. KIEV.—*University*. Universitetskiya Isvyaistiya.

## MOSCOW—

T.P. *Société Impériale des Naturalistes*. Bulletin.—Nouveaux Mémoires.

T.P. *L'Observatoire Impériale*. Annales.

T.P. *Société Impériale des Amis d'Histoire Naturelle, d'Anthropologie et d'Ethnographie*.

T.P. *Imperial University*.

T.P. *Musée Polytechnique*.

P. *Observatoire Magnétique et Météorologique de l'Université Impériale*.

P. ODESSA.—*Société des Naturalistes de la Nouvelle Russie*. Zapiski.

T.P. POULKOVA.—*Nicolai Hauptsternwarte*. Publications.—Annales.

## PETROGRAD—

T.P. *Académie Impériale des Sciences*. Mémoires : Classe Phys.-Math ; Classe Hist.-Phil.—Bulletins.

T.P. *Commission Sismique Permanente*. Comptes Rendus.—Bulletin.

T.P. *Comité Géologique*. Mémoires.—Bulletins.—Carte Géologique : Région Aurifère d'Ienissei : de l'Amour : de Lena.

*Commission Royale Russe pour la Mesure d'un Arc de Méridien au Spitzberg.*

Missions Scientifiques pour la Mesure d'un Arc de Méridien au Spitzberg entreprises en 1899–1902, sous les auspices des Gouvernements Suédois et Russe. Mission Russe. 4to. (Presented.)

T.P. *Imperial University*. Scripta Botanica.

T.P. *Institut Impérial de Médecine Expérimentale*. Archives des Sciences Biologiques.

T.P. *Physikalische Central-Observatorium*. Annalen.

T.P. *Physico-Chemical Society of the University of Petrograd*. Journal.

T.P. *Russian Ministry of Marine*.

P. *Imperial Russian Geographical Society*.

P. *K. Mineralogische Gesellschaft*. Verhandlungen (Zapiski).—Materialien zur Geologie Russlands.

P. *Société des Naturalistes (Institut Zoologique de l'Université ; Section de Géologie et de Minéralogie)*. Travaux et Suppléments.

P. *Société Astronomique Russe*.

P. TIFLIS.—*Physikalisches Observatorium*. Beobachtungen.

# SCOTLAND.

T.P. ABERDEEN.—*University Library*. Calendar.—*University Studies*.—*Library Bulletin*.

P. BERWICKSHIRE.—*Naturalists' Club*. Proceedings.

T.P. DUNDEE.—*University College Library*.

## EDINBURGH—

- T.P. *Advocates' Library.*  
P. *Botanical Society.* Transactions and Proceedings.  
*Carnegie Trust for the Universities of Scotland.* Report. (*Presented.*)  
P. *Faculty of Actuaries in Scotland.* Transactions.  
*Field Naturalists' and Microscopical Society.* Transactions. (*Presented.*)  
P. *Fishery Board for Scotland.* Annual Reports. Scientific Investigations.  
—Salmon Fisheries. Fifth Report of the Fishery and Hydrographical  
Investigations in the N. Sea and Adjacent Waters (1908-1911). Fol.  
Lond. 1913.  
P. *Geological Society.* Transactions.  
*Geological Survey of Scotland.* Memoirs, Maps, etc. (*Presented by H.M.*  
*Government.*)  
T.P. *Highland and Agricultural Society of Scotland.* Transactions.  
P. *Mathematical Society.* Proceedings.—Mathematical Notes.  
P. *Pharmaceutical Society.* (*North British Branch.*)  
*Registrar-General's Returns of Births, Deaths, and Marriages.* (*Presented.*)  
T.P. *Royal Botanic Garden.* Notes.  
T.P. *Royal College of Physicians.*  
P. *Royal College of Physicians' Laboratory.* Laboratory Reports.  
T.P. *Royal Medical Society.*  
T.P. *Royal Observatory.* Annals.—Annual Report.  
T.P. *Royal Physical Society.* Proceedings.  
*Royal Scottish Academy.* Annual Reports. (*Presented.*)  
P. *Royal Scottish Geographical Society.* Scottish Geographical Magazine.  
T.P. *Royal Scottish Society of Arts.* Transactions.  
P. *Scottish Meteorological Society.* Journal.  
*Scottish National Antarctic Expedition.* Publications. (*Presented.*)  
T.P. *University Library.* Calendar.

## GLASGOW—

- P. *Geological Society.* Transactions.  
*Royal Technical College.* Calendar. (*Presented.*)  
T.P. *Inst. of Engineers and Shipbuilders in Scotland.* Transactions.  
P. *Marine Biological Association of the West of Scotland.* Annual Report.  
(*See MILLPORT.*)  
P. *Natural History Society.*—Glasgow Naturalist.  
T.P. *Royal Philosophical Society.* Proceedings.  
T.P. *University.* Calendar.  
P. *University Observatory.*  
P. MILLPORT.—*Marine Biological Association of the West of Scotland.* Annual  
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T.P. PERTH.—*Perthshire Society of Natural Science.* Proceedings.  
T.P. ST ANDREWS.—*University Library.* Calendar.

## MADRID—

## SPAIN.

- T.P. *Real Academia de Ciencias Exactas, Fisicas y Naturales.* Memorias.—  
Revista.—Anuario.  
T.P. *Instituto Geológico de España.* Boletín.—Memorias.  
P. VILAFRANCA DEL PANADES (CATALUÑA).—*Observatorio Meteorológico.*

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- P. GOTHENBURG.—*Kongl. Vetenskaps och Vitterhets Samhälle. Handlingar.*
- T.P. LUND.—*University. Acta Universitatis Lundensis* (Fysiografiska Sällskapets Handlingar.—Theologi.—Medicina).
- T.P. STOCKHOLM.—*Kong. Svenska Vetenskaps-Akademie. Handlingar.*—Arkiv för Zoologi.—Arkiv för Matematik, Astronomi och Fysik.—Arkiv för Botanik.—Arkiv för Kemi. Mineralogi och Geologi.—Meteorologiska Iakttagelser i Sverige.—Astronomiska Iakttagelser.—Lefnadsteckningar.—Arsbok.—Accessionskatalog.—Meddelanden från K. Vetenskaps Akademiens Nobelinstitut.—Les Prix Nobel.
- P. *Svenska Sällskapet för Antropologi och Geografi. Ymer.*  
*Commission Royale Suédoise pour la Mesure d'un Arc de Méridien au Spitzberg. Missions Scientifiques pour la Mesure d'un Arc de Méridien au Spitzberg entreprises en 1899–1902, sous les auspices des Gouvernements Suédois et Russe. Mission Suédoise. 4to. (Presented.)*
- UPSALA—
- T.P. *Kongliga Vetenskaps Societeten (Regia Societas Scientiarum). Nova Acta.*
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- T.P. BASLE.—*Naturforschende Gesellschaft. Verhandlungen.*
- BERN—  
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- T.P. *Société Helvétique des Sciences Naturelles. (Allgemeine Schweizerische Gesellschaft für die gesammten Naturwissenschaften.) Comptes Rendus.*  
 —Actes (Verhandlungen).—Nouveaux Mémoires.
- P. *Naturforschende Gesellschaft. Mittheilungen.*
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- NEUCHÂTEL—
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- T.P. *Naturforschende Gesellschaft. Vierteljahrsschrift.*
- P. *Schweizerische Botanische Gesellschaft. Berichte (Bulletin).*  
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TASMANIA. (See AUSTRALIA.)

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- P. CONSTANTINOPLE.—*Société Impériale de Médecine*. Gazette Médicale d'Orient.

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CAPE TOWN—

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 T.P. *Royal Astronomical Observatory*. Reports.—Annals.—Meridian Observations.—Independent Day Numbers.  
 P. *Geological Commission* (now *Survey*). Annual Reports.  
 T.P. *South African Museum*. Annals.  
 P. *South African Association for the Advancement of Science*. Journal.  
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- T.P. *Geological Society of South Africa*. Transactions and Proceedings.  
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- P. *Geological Survey of Natal*. Annual Reports.—Reports on the Mining Industry of Natal.  
 T.P. *Government Museum*. Annals.—Catalogues.

PRETORIA—

- Dept. of Mines—Geological Survey*. Reports.—Memoirs.—Maps. (*Presented.*)  
 T.P. *Transvaal Museum*. Annals.

UNITED STATES OF AMERICA.

ALBANY—

- T.P. *New York State Library*. Annual Reports.—Bulletins.  
*State Museum*. Annual Reports.—Bulletin. *New York State Education Department*. Annual Reports.  
 P. ALLEGHENY.—*Observatory*. Publications, etc.  
 P. ANN ARBOR.—*Michigan Academy of Sciences*. Reports. (*University.*)  
 P. ANNAPOLIS (MARYLAND).—*St John's College*.  
 P. AUSTIN.—*Texas Academy of Sciences*. Transactions.  
 T.P. BALTIMORE.—*Johns Hopkins University*. American Journal of Mathematics.—American Chemical Journal.—American Journal of Philology.—University Studies in Historical and Political Science.—Memoirs from the Biological Laboratory.—University Circulars.  
*Johns Hopkins Hospital*. Bulletins.—Reports. (*Presented.*)  
 T.P. *Maryland Geological Survey*. Publications.  
*Maryland Weather Service*. Reports. (*Presented.*)  
*Peabody Institute*. Annual Reports. (*Presented.*)

## BERKELEY (CALIFORNIA)—

- T.P. *University of California*.—University Chronicle.—Reports of Agricultural College.—Publications (Zoology, Botany, Geology, Physiology, Pathology, and American Archæology and Ethnology).—Memoirs.  
*Academy of Pacific Coast History*. Publications.

## BOSTON—

- T.P. *Bowditch Library*.  
T.P. *Boston Society of Natural History*. Memoirs.—Proceedings.—Occasional Papers.  
T.P. *American Academy of Arts and Sciences*. Memoirs.—Proceedings.  
P. BROOKLYN.—*Institute of Arts and Sciences*. Museum Reports.—Bulletins.  
P. BUFFALO.—*Society of Natural Sciences*. Bulletin.

CALIFORNIA. (See SAN FRANCISCO, SACRAMENTO, BERKELEY, MOUNT HAMILTON, MOUNT WILSON, and STANFORD.)

## CAMBRIDGE—

- T.P. *Harvard University*. — *Museum of Comparative Zoology*. Memoirs. — Bulletins.—Annual Reports.  
T.P. *Astronomical Observatory, Harvard College*. Annals.—Annual Reports.—Observatory Circulars.  
P. CHAPEL HILL (NORTH CAROLINA).—*E. Mitchell Scientific Society*. Journal.  
P. CHARLOTTESVILLE.—*Philosophical Society, University of Virginia*. Bulletin; Scientific Series and Humanistic Series.—Proceedings.

## CHICAGO—

- T.P. *Field Museum of Natural History*. Publications: Geological Series; Botanical Series; Zoological Series; Ornithological Series; Anthropological Series.—Annual Reports.  
P. *University of Chicago*.  
T.P. *Yerkes Observatory (University of Chicago)*. Publications.  
P. *Academy of Sciences*. Bulletins.—Special Publications.—Bulletins of the Natural History Survey.

## CINCINNATI—

- P. *Observatory (University)*. Publications.—University Record.  
P. *Society of Natural History*. Journal.  
T.P. CLEVELAND (OHIO).—*Geological Society of America*. Bulletins.  
T.P. CLINTON (IOWA).—*Litchfield Observatory, Hamilton College*.  
COLORADO SPRINGS.—*Colorado College*. Colorado College Studies. (*Presented*.)  
P. CONNECTICUT.—*Connecticut Academy of Arts and Sciences*. Transactions.—Memoirs.  
P. DAVENPORT.—*Academy of Natural Sciences*. Proceedings.  
P. DENVER (COLORADO).—*Scientific Society of Colorado*. Proceedings.  
T.P. DES MOINES (IOWA).—*Iowa Academy of Sciences*. Proceedings.  
P. GARRISON, N.Y.—Editor, *American Naturalist*.  
T.P. GRANVILLE (OHIO).—*Denison University and Scientific Association*. Bulletin of the Scientific Laboratories.  
P. INDIANAPOLIS.—*Indiana Academy of Sciences*. Proceedings.  
IOWA CITY—  
P. *Geological Survey*. Annual Reports.

## IOWA CITY—continued—

- T.P. *State University. Laboratories of Natural History.* Bulletins.—Contributions from the Physical Laboratories.

IOWA. (See DES MOINES.)

## ITHACA (N.Y.)—

- P. The Editor, *Physical Review.* (Cornell University.)  
P. The Editors, *Journal of Physical Chemistry.* (Cornell University.)  
T.P. LAWRENCE (KANSAS).—*University of Kansas.* Science Bulletin (University Quarterly).  
P. LINCOLN (NEBRASKA).—*University of Nebraska. Agricultural Experiment Station.* Bulletins.

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- T.P. *Wisconsin University. Washburn Observatory.* Observations.  
P. *Wisconsin Academy of Sciences, Arts, and Letters.* Transactions.  
P. *Geological and Natural History Survey.* Bulletins.  
P. MASSACHUSETTS.—*Tufts College Library.* Tufts College Studies.  
P. MERIDEN (CONN.).—*Meriden Scientific Association.*

MICHIGAN. (See ANN ARBOR.)

## MINNEAPOLIS (MINN.)—

- T.P. { *University of Minnesota.* Studies.—Bulletin of the School of Mines.  
{ *Geological and Natural History Survey of Minnesota.* Reports.  
P. *Botanical Survey.*

MISSOURI. (See ST LOUIS and ROLLA.)

- P. MOUNT HAMILTON (CALIFORNIA).—*Lick Observatory.* Bulletins.—Publications.  
T.P. MOUNT WILSON (CALIFORNIA).—*Solar Observatory.* Contributions.—Reports.  
T.P. NEWHAVEN (CONN.).—*Yale College. Astronomical Observatory of Yale Observatory.* Transactions.—Reports.  
T.P. NEW ORLEANS.—*Academy of Sciences.*

## NEW YORK—

- T.P. *American Mathematical Society.* Bulletins.—Transactions.  
T.P. *American Museum of Natural History.* Bulletins.—Memoirs.—American Museum Journal.—Annual Reports.—Anthropological Papers.—Guide Leaflets.—Handbook Series.—Monograph Series.  
P. *American Geographical Society.* Bulletin.—Geographical Review.  
P. *American Institute of Electrical Engineers.* Proceedings.

NEW YORK. (See also ALBANY.)

## PHILADELPHIA—

- T.P. *American Philosophical Society for Promoting Useful Knowledge.* Proceedings.—Transactions.  
T.P. *Academy of Natural Science.* Proceedings.—Journal.  
T.P. *University of Pennsylvania.* Publications:—Philology, Literature, and Archæology, Mathematics, etc. Contributions from the Zoological and Botanical Laboratories. University Bulletins.—Theses.—Calendar.  
P. *Franklin Institute.* Journal.  
T.P. *Geological Survey of Pennsylvania.*  
P. *Wagner Free Institute of Science.* Transactions.  
P. *Geographical Society.* Bulletin.

PHILADELPHIA—*continued*—

- P.      *Commercial Museum.*
- P.      PORTLAND (MAINE).—*Society of Natural History.* Proceedings.
- P.      PRINCETON, N.J.—*University.* Annals of Mathematics.—*University Observatory.* Contributions.
- P.      ROCHESTER.—*Academy of Science.* Proceedings.
- T.P.   ROLLA (MISS.).—*Bureau of Geology and Mines.* Biennial Reports, etc.
- T.P.   SALEM.—*Essex Institute.*

SAINT LOUIS—

- T.P.   *Academy of Sciences.* Transactions.
- P.      *Missouri Botanical Garden.* Annual Reports. Annals.
- P.      *Washington University.* University Studies.
- T.P.   SAN FRANCISCO (CALIFORNIA).—*Academy of Sciences.* Proceedings.—*Memoirs.*—Occasional Papers.
- P.      STANFORD (CALIFORNIA).—*University.* Publications. (*Presented.*)
- P.      TOPEKA.—*Kansas Academy of Science.* Transactions.
- T.P.   URBANA.—*University of Illinois.* Bulletins of *State Geological Survey, State Laboratory of Natural History, and Engineering Experiment Station.*

WASHINGTON—

- T.P.   *U.S. National Academy of Sciences.* Memoirs.
- T.P.   *Bureau of Ethnology.* Annual Reports.—Bulletins.
- T.P.   *U.S. Coast and Geodetic Survey.* Annual Reports, etc.
- T.P.   *U.S. Commission of Fish and Fisheries.* Reports.—Bulletins.
- T.P.   *U.S. Naval Observatory.* Reports.—Observations.
- T.P.   *U.S. Geological Survey.* Bulletins.—Annual Reports.—Monographs.—*Geologic Atlas of the United States.*—Mineral Resources.—Professional Papers.—Water Supply and Irrigation Papers.
- Geological Society of America.* (See CLEVELAND.)
- T.P.   *Weather Bureau.* (*Department of Agriculture.*) Monthly Weather Review.—Bulletins.—Reports.—Bulletin of the Mount Weather Observatory (now embodied in Monthly Weather Review).
- T.P.   *Smithsonian Institution.* Miscellaneous Collections.—The same (Quarterly Issue).—Contributions to Knowledge.—Reports.—Annals of the Astrophysical Observatory.—Harriman Alaska Expedition, Vol. XIV. 4to.
- T.P.   *Surgeon-General's Office.* Index Catalogue of the Library. 4to.
- T.P.   *Carnegie Institution of Washington.* Year-Books.—Publications. Classics of International Law.—*Carnegie Foundation for the Advancement of Teaching.* Annual Report.—Bulletin.
- T.P.   *American Association for the Advancement of Science.* Proceedings.
- P.      *U.S. National Museum.* Bulletins.—Reports.—Proceedings.—Contributions from the U.S. National Herbarium.
- P.      *Department of Agriculture.* (*Division of Economic Ornithology, and Mammalogy.*) Bulletin.
- P.      *U.S. Patent Office.*
- Washington Academy of Sciences, Journal of the.* (*Purchase.*)
- Bureau of Standards.* *Department of Commerce and Labour.* Bulletins. (*Presented.*)—Technologic Papers.

WISCONSIN. (See MADISON.)

VICTORIA. (See AUSTRALIA.)



List of Periodicals and Annual Publications added to the  
Library by Purchase, etc.

*Periodicals not found in this List will be found in Exchange List.*

*Annals (Works of Reference), see end of List.*

Acta Mathematica.

American Journal of Science and Arts.

\* ——— Naturalist.

\* ——— Journal of Mathematics.

\* ——— Chemical Journal.

\* ——— Journal of Philology.

Anatomischer Anzeiger.

——— ———— Ergänzungshefte.

Annalen der Chemie (Liebig's).

\* ——— der Physik.

\* ——— der Physik. (Beiblätter.)

Annales de Chimie.

——— d'Hygiène Publique et de Médecine Légale.

——— de Physique.

——— des Sciences Naturelles. Zoologie et Paléontologie.

——— des Sciences Naturelles. Botanique.

Annali dell' Islam. (*Presented.*)

Annals and Magazine of Natural History (Zoology, Botany, and Geology).

——— of Botany.

\* ——— of Mathematics. (Princeton, N.J.)

Anthropologie (L').

Arbeiten-Zoologisches Institut der Universität und der Zoologischen Station in Triest.

\* Archiv för Matematik og Naturvidenskab.

\* Archiv für Biontologie.

Archives de Biologie.

——— de Zoologie Expérimentale et Générale.

\* ——— des Sciences Biologiques.

——— des Sciences Physiques et Naturelles.

——— Italiennes de Biologie.

\* Arkiv för Matematik, Astronomi och Fysik. (Stockholm.)

\* ——— för Kemi, Mineralogi och Geologi. „

\* ——— för Botanik. „

\* ——— för Zoologi. „

Astronomie (L').

Astronomische Nachrichten.

Astrophysical Journal.

Athenæum.

Bericht über die Wissenschaftlichen Leistungen in der Naturgeschichte der  
niederen Thiere. Begründet von R. Leuckart.

Bibliotheca Mathematica.

Bibliothèque Universelle et Revue Suisse.

——— ——— See Archives des Sciences Physiques et Naturelles.

Biologisches Centralblatt.

\* Received by exchange.

Blackwood's Magazine.

Bollettino delle Pubblicazioni Italiane. (*Presented.*)

Bookman.

Botanische Zeitung.

Botanisches Centralblatt.

———— Beiheft.

British Rainfall.

Bulletin Astronomique.

———— des Sciences Mathématiques.

———— Mensuel de la Société Astronomique de Paris. *See* L'Astronomie.

Cambridge British Flora. By C. E. Moss.

Catalogue of Scientific Papers, 1800–1900. Name Index.

Catalogue of Scientific Papers, 1800–1900. Subject Index (Mathematics, Mechanics, Physics, Part I.)

Centralblatt für Bakteriologie und Parasitenkunde.

———— für Mineralogie, Geologie und Palæontologie.

Ciel et Terre.

Contemporary Review.

Crelle's Journal. *See* Journal für Reine und Angewandte Mathematik.

Dictionary, New English. Ed. by Sir J. A. H. Murray.

Dingler's Polytechnisches Journal.

Edinburgh Medical Journal.

———— Review.

Egypt Exploration Fund. Publications.

\* Electrician.

Encyklopädie der Mathematischen Wissenschaften.

Engineering.

English Mechanic and World of Science.

\* Essex Naturalist.

Fauna und Flora des Golfes von Neapel.

Flora.

Fortnightly Review.

\* Gazette Médicale d'Orient.

\* Geographical Journal.

\* Geographical Magazine (Scottish).

\* Geographie (La).

Geological Magazine.

Göttingische Gelehrte Anzeigen.

Indian Antiquary.

———— Engineering. (*Presented.*)

Indian Journal of Medical Research. (*Presented.*)

Intermédiaire (L') des Mathématiciens.

International Catalogue of Scientific Literature.

Internationale Revue der Gesamten Hydrobiologie und Hydrographie.

Jahrbücher für Wissenschaftliche Botanik (Pringsheim).

Jahresbericht über die Fortschritte der Chemie und verwandter Theile anderer Wissenschaft.

Journal de Conchyliologie.

\* Received by exchange.

Journal des Débats.

——— de Mathématiques Pures et Appliquées.

——— de Pharmacie et de Chimie.

\* ——— de Physique.

——— des Savants.

——— für die Reine und Angewandte Mathematik (Crelle).

——— für Praktische Chemie.

——— of Anatomy and Physiology.

——— of Botany.

——— of Pathology and Bacteriology.

\* ——— of Physical Chemistry.

\* ——— of the Royal Society of Arts.

——— of the Society of Chemical Industry. (*Presented.*)

——— of the Washington Academy of Sciences.

Knowledge.

Manual of Conchology.

\* Mathematische und Naturwissenschaftliche Berichte aus Ungarn.

Mind.

Mineralogical Magazine. (*Presented.*)

Mineralogische und Petrographische Mittheilungen (Tschermak's).

Monist.

\* Nature.

——— (La).

Neues Jahrbuch für Mineralogie, Geologie, und Palæontologie.

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Nineteenth Century.

Notes and Queries.

Nuova Notarisa (De Toni).

\* Nuovo Cimento ; Giornale di Fisica, Chimica e Storia Naturale.

\* Nyt Magazin för Naturvidenskaberne.

Nyt Tidsskrift för Matematik.

Observatory.

Optical Society, London, Transactions.

Page's Engineering Weekly. (*Presented.*)

Palæontographical Society's Publications.

Petermann's Mittheilungen.

——— Ergänzungsheft.

\* Pharmaceutical Journal.

Philosophical Magazine. (London, Edinburgh, and Dublin.)

\* Photographic Journal.

\* Physical Review.

Physiological Abstracts.

Plankton-Expedition Ergebnisse.

Quarterly Journal of Microscopical Science.

——— of Experimental Physiology.

Quarterly Review.

Ray Society's Publications.

Registrar-General's Returns (Births, Deaths, and Marriages). (*Presented.*)

Resultate der Wissenschaftliche Erforschung der Balatonsees.

Review of Neurology and Psychiatry.

\* Revue Générale des Sciences Pures et Appliquées.

——— Philosophique de la France et de l'Etranger.

——— Politique et Littéraire. (Revue Bleue.)

——— Scientifique. (Revue Rose.)

\* ——— Semestrielle des Publications Mathématiques.

Saturday Review.

Science.

\* Science Abstracts.

——— Progress.

Scientia.

Scotsman.

Scottish Naturalist.

Symons's Meteorological Magazine.

Thesaurus Linguae Latinæ.

Times.

Zeitschrift für die Naturwissenschaften.

——— für Krystallographie und Mineralogie.

——— für Wissenschaftliche Zoologie.

Zoological Record.

Zoologische Jahrbücher. Abteilung für Anatomie und Ontogenie der Tiere.

——— Abteilung für Systematik, Geographie und Biologie der Tiere.

——— Abteilung für Allgemeine Zoologie und Physiologie der Tiere.

Zoologischer Anzeiger.

——— Jahresbericht.

#### ANNUALS (WORKS OF REFERENCE).

Annuaire du Bureau des Longitudes.

County Directory. (Scotland.)

Edinburgh and Leith Directory.

English Catalogue of Books.

Medical Directory.

Minerva (Jahrbuch der Gelehrten Welt).

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## INSTRUCTIONS TO AUTHORS.

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